Further Math with Ali Hashir

Roots of Polynomial Equations

Further Pure Mathematics 1 – Chapter 1 • Answers

Final answers only. Full step-by-step working is in the companion *Worked Solutions* booklet.

QUESTIONS 1–10 • Foundations: reading the coefficients

1 **(a)** $\alpha+\beta=7,\ \alpha\beta=5$ **(b)** $\alpha+\beta=-\frac{8}{3},\ \alpha\beta=-\frac{2}{3}$ **(c)** $\alpha+\beta=0,\ \alpha\beta=-\frac{4}{5}$

2 **(a)** $\sum\alpha=-4,\ \sum\alpha\beta=-2,\ \alpha\beta\gamma=-9$ **(b)** $\frac{5}{2},\ \frac{1}{2},\ -\frac{3}{2}$ **(c)** $0,\ \frac{7}{4},\ \frac{1}{4}$

3 $\sum\alpha=\frac{2}{3},\quad \sum\alpha\beta=\frac{5}{3},\quad \sum\alpha\beta\gamma=-\frac{1}{3},\quad \alpha\beta\gamma\delta=-\frac{8}{3}$

4 **(a)** $2x^2+5x-12=0$ **(b)** $x^3-6x^2+3x+10=0$ **(c)** $x^4-2x^2+16x-15=0$

5 **(a)** -7 **(b)** 3

6 **(a)** $\frac{4}{5}$ **(b)** $\frac{6}{5}$

7 **(a)** $2x^2+6x+5=0$ **(b)** $2x^2-30x+125=0$ **(c)** $4x^2-16x+25=0$

8 **(a)** $p=-5$ **(b)** $q=6$ **(c)** $\frac{1}{2}$

9 **(a)** $4y^3-5y^2+2y-3=0$ **(b)** $\frac{9}{16}$

10 **(a)** $y^3-y^2-10y+23=0$ **(b)** -23

QUESTIONS 11–20 • Substitutions and power sums

11 **(a)** $y^3-14y^2-47y-36=0$ **(b)** 14 **(c)** 290

12 **(a)** $S_1=\frac{1}{2},\ S_2=-\frac{15}{4}$ **(b)** shown **(c)** $S_3=\frac{13}{8},\ S_4=\frac{145}{16}$

13 **(a)** $\frac{5}{4}$ **(b)** $\frac{9}{16}$ **(c)** $-\frac{43}{64}$

14 $1,\ 4,\ 7$

15 **(a)** $\frac{3}{4},\ \frac{3}{2},\ 3$ **(b)** $k=63$

16 **(a)** $\frac{2}{3}$ **(b)** $\frac{14}{9}$

17 **(a)** $y^3+15y^2+67y+125=0;\ \sum\alpha^3=-15$ **(b)** 91 **(c)** $-\frac{67}{125}$

18 **(a)** $y^4-4y^3+10y^2-y+9=0;\ \sum\alpha^2=4$ **(b)** -4 **(c)** $\frac{1}{9}$

19 **(a)** $x^3-4x^2+5x-2=0$ **(b)** $1,\ 1,\ 2$

20 $\{\alpha,\beta,\gamma\}=\{1,\ 2,\ 3\}$ (in any order)

QUESTIONS 21–30 • Examination-standard problems

21 **(a)** $4y^3 - 11y^2 + 13y - 8 = 0$ **(b)** $\frac{11}{4}$

22 **(a)** shown **(b)** 158 **(c)** $(\alpha+1)(\beta+1)(\gamma+1) = -1$; $\sum(\beta+1)(\gamma+1) = 12$

23 **(a)** shown **(b)** $S_2 = 12$ **(c)** $S_4 = -428$

24 **(a)** shown **(b)** $q = 16$; roots 2, 2, -4

25 **(a)** $a = c = 0$, $b = -(\alpha^2 + \beta^2)$, $d = \alpha^2\beta^2$ **(b)** $x^4 - 10x^2 + 9 = 0$; roots ± 1, ± 3

26 **(a)** -6 **(b)** $\sum\alpha^2 = -6 < 0$, but a sum of squares of real numbers cannot be negative; so the roots are not all real **(c)** $k = 10$; roots -2, $\mathrm{i}\sqrt{5}$, $-\mathrm{i}\sqrt{5}$

27 **(a)** -4 **(b)** $a = 7$, $b = -3$

28 **(a)** shown **(b)** $c = 31$; roots 2, 3, 5

29 **(a)** shown **(b)** $\alpha\beta = 2$; roots 1, 2, $\frac{1}{2}(1 + \mathrm{i}\sqrt{7})$, $\frac{1}{2}(1 - \mathrm{i}\sqrt{7})$

30 **(a)** shown **(b)** $\displaystyle\sum \frac{1}{(1+\alpha)^2} = \frac{6}{25}, \quad \sum \frac{1}{(1+\alpha)^3} = \frac{21}{125}$ **(c)** $-\frac{3}{5}$

Step-by-step working for every question is in the companion Worked Solutions booklet.