



Further Math with Ali Hashir

Polar Coordinates

Further Pure Mathematics 1 – Chapter 5 • Answers

Final answers only. Full step-by-step working, including every sketch, is in the companion *Worked Solutions* booklet.

QUESTIONS 1–7 • Coordinates, conversion and first areas

- 1 (a) $(-2, 2\sqrt{3})$ (b) $(3\sqrt{2}, \frac{3}{4}\pi)$, $(5, \frac{3}{2}\pi)$, $(2, \frac{5}{3}\pi)$
- 2 (a) $r = 5$ (b) $\theta = \frac{1}{3}\pi$ (c) $r = 6 \sin \theta$ (d) $r = \frac{4}{2 \cos \theta - 5 \sin \theta}$
- 3 (a) $x = 5$ (b) $(x - 2)^2 + (y - 1)^2 = 5$: a circle, centre $(2, 1)$, radius $\sqrt{5}$ (c) $3x^2 - y^2 - 12x + 9 = 0$
- 4 (a) 4, 3, 2, 3 (b) greatest 4 at $\theta = 0$; least 2 at $\theta = \pi$ (c) symmetrical about the initial line $\theta = 0$
- 5 (a) greatest $r = 2$ (at $\theta = 0, \pi$), least $r = 0$ at $\theta = \frac{1}{2}\pi, \frac{3}{2}\pi$ (c) $\frac{3}{2}\pi$
- 6 (a) (π^2, π) (b) $\frac{\pi^5}{10}$ (c) $\frac{31\pi^5}{320}$
- 7 (a) $(x - 3)^2 + y^2 = 9$: centre $(3, 0)$, radius 3 (c) $3\pi + \frac{9\sqrt{3}}{4}$

QUESTIONS 8–14 • Sketching, areas and greatest distances

- 8 (b) least distance $2\sqrt{2}$, at $\theta = \frac{1}{4}\pi$ (c) area 8; the triangle has vertices $(0, 0)$, $(4, 0)$, $(0, 4)$
- 9 (a) greatest 5 (at $\theta = \pi$), least 1 (at $\theta = 0$) (b) the initial line $\theta = 0$ (c) 11π
- 10 (a) the initial line $\theta = 0$ (c) $\frac{27\pi}{16}$ (d) $\frac{2\sqrt{3}}{3}$
- 11 (a) greatest 1 at $\theta = 0$; least $\frac{1}{3}$ at $\theta = \pi$ (c) $(\frac{1}{3}, 0)$
- 12 (a) C_1 : centre $(2, 0)$, radius 2; C_2 : centre $(0, 2)$, radius 2 (b) $(2\sqrt{2}, \frac{1}{4}\pi)$ (d) $2\pi - 4$
- 13 (a) shown; $\theta \approx 2.029$ (c) $\frac{\pi}{2}$

14 (a) $\theta = 1$ (c) $\frac{1}{4} \ln 5$

QUESTIONS 15–20 • Examination-standard problems

15 (a) centre $(0, \frac{3}{2})$, radius $\frac{3}{2}$ (b) $(\frac{3}{2}, \frac{1}{6}\pi)$ and $(\frac{3}{2}, \frac{5}{6}\pi)$ (d) $\frac{5\pi}{4}$

16 (a) $(2, 0)$ and $(1 + e^{-\pi}, \pi)$ (b) $\frac{\pi}{2} + \frac{5}{4} - e^{-\pi} - \frac{1}{4}e^{-2\pi}$ (c) shown; $\theta \approx 1.371$

17 (a) $(1, 0)$ and $(3, \ln 3)$ (c) $\alpha = \frac{1}{2} \ln 5$

18 (a) $x^2 = 1 - 2y$: a parabola with vertex $(0, \frac{1}{2})$ (b) least distance $\frac{1}{2}$, at $\theta = \frac{1}{2}\pi$ (d) $\frac{1}{3}$

19 (b) $2\pi + \frac{3\sqrt{3}}{2}$ (c) 1.76

20 (a) asymptote $x = 2$ (b) endpoint $(\sqrt{2}, \frac{1}{4}\pi)$ (d) $\frac{5}{2} - \frac{3\pi}{4}$

Step-by-step working for every question is in the companion Worked Solutions booklet.



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