Further Math with Ali Hashir

Matrices

Further Pure Mathematics 1 – Chapter 4 • Answers

Final answers only. Full step-by-step working, including every multiplication, is in the companion *Worked Solutions* booklet.

QUESTIONS 1–7 • Matrix arithmetic, determinants and inverses

1 **(a)** $\begin{pmatrix} 6 & -17 \\ 10 & 5 \end{pmatrix}$ **(b)** $\mathbf{AB} = \begin{pmatrix} 2 & 14 \\ -8 & 14 \end{pmatrix}$, $\mathbf{BA} = \begin{pmatrix} 10 & 20 \\ -4 & 6 \end{pmatrix}$; matrix multiplication is not commutative **(c)** $\mathbf{AC} = \begin{pmatrix} 3 & 3 & -7 \\ 2 & 16 & 14 \end{pmatrix}$; $\mathbf{C}$ has 3 columns but $\mathbf{A}$ has 2 rows

2 **(a)** 26 **(b)** $\det = 0$, so the matrix has no inverse **(c)** $k = 4$ or $k = -1$

3 **(a)** -85 **(b)** $k = 3$ or $k = -1$

4 **(a)** $\mathbf{A}^{-1} = \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix}$, $\mathbf{B}^{-1} = \frac{1}{2}\begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$ **(b)** $\mathbf{X} = \begin{pmatrix} -8 & -18 \\ 5 & 11 \end{pmatrix}$ **(c)** $\mathbf{Y} = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$; multiplication is not commutative, so pre-multiplying and post-multiplying give different results

5 **(a)** $\det \mathbf{A} = 12 \neq 0$ **(b)** $\mathbf{A}^{-1} = \dfrac{1}{12}\begin{pmatrix} 5 & 2 & -3 \\ -1 & 2 & 3 \\ 2 & -4 & 6 \end{pmatrix}$ **(c)** first column of $\mathbf{AA}^{-1}$ is $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

6 **(a)** $\mathbf{AB} = \begin{pmatrix} 4 & -1 \\ 7 & -1 \end{pmatrix}$, $(\mathbf{AB})^{-1} = \frac{1}{3}\begin{pmatrix} -1 & 1 \\ -7 & 4 \end{pmatrix}$ **(b)** $\mathbf{B}^{-1}\mathbf{A}^{-1} = \frac{1}{3}\begin{pmatrix} -1 & 1 \\ -7 & 4 \end{pmatrix}$, the same matrix **(c)** $\mathbf{Q} = \mathbf{P}^{-1}\mathbf{R}^{-1}$

7 **(a)** $\mathbf{A}^2 = \begin{pmatrix} 19 & 18 \\ 6 & 7 \end{pmatrix}$ **(b)** $\mathbf{A}^{-1} = -\frac{1}{5}\mathbf{A} + \frac{6}{5}\mathbf{I} = \frac{1}{5}\begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$ **(c)** shown

QUESTIONS 8–14 • Transformations, areas and invariance

8 **(a)** rotation through 90° clockwise about the origin **(b)** enlargement, centre the origin, scale factor 5 **(c)** reflection in the line $y = x$ **(d)** stretch parallel to the y-axis, scale factor 4 (x-axis fixed) **(e)** shear, y-axis fixed, with $(1, 0)$ mapped to $(1, -2)$

9 **(a)** $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ **(b)** shown **(c)** $(2, 1)$, $(2, 3)$, $(8, 2)$; area 6 **(d)** $\mathbf{M}^{-1} = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & 0 \end{pmatrix}$

10 **(a)** 90 **(b)** $k = 4$ or $k = -4$ **(c)** $15\,\text{cm}^2$

11 **(a)** $A'(3, 1)$, $B'(4, 3)$, $C'(1, 2)$ **(b)** area 5; $\mathbf{M}^{-1} = \frac{1}{5}\begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$ **(c)** $\begin{pmatrix} 2 & 3 \\ -1 & 3 \end{pmatrix}$; area 9

12 **(a)** $\det(\mathbf{M}-\mathbf{I})=0$; the line is $y=2x$ **(b)** $\det(\mathbf{M}-\mathbf{I})=8\neq 0$, so $(\mathbf{M}-\mathbf{I})\begin{pmatrix}x\\y\end{pmatrix}=\mathbf{0}$ forces $x=y=0$ **(c)** $k=3$, line $y=-\frac{1}{2}x$; $k=-1$, line $y=\frac{1}{2}x$

13 **(a)** $y=-x$ and $y=\frac{1}{2}x$ **(b)** $y=-x$ and $y=\frac{1}{3}x$ **(c)** no line $y=mx$ is invariant; the only one is $x=0$

14 **(a)** shown **(b)** $\mathbf{M}^{-1}=\begin{pmatrix}1&0\\0&\frac{1}{2}\end{pmatrix}\begin{pmatrix}1&-3\\0&1\end{pmatrix}$ **(c)** $y=0$ and $y=\frac{1}{6}x$ **(d)** $y=0$; both transformations leave every point of the x-axis where it is

QUESTIONS 15–20 • Examination-standard problems

15 **(a)** $k=3$ **(b)** shown **(c)** $y=-\frac{1}{2}x$ and $y=x$

16 **(a)** $\det\mathbf{M}=-1$ **(b)** the product of the gradients is -1 **(c)** $\mathbf{M}=\begin{pmatrix}-\frac{1}{2}&\frac{\sqrt{3}}{2}\\\frac{\sqrt{3}}{2}&\frac{1}{2}\end{pmatrix}$; lines $y=\sqrt{3}\,x$ and $y=-\frac{1}{\sqrt{3}}x$ **(d)** $\mathbf{M}$ is a reflection, so applying it twice restores every point

17 **(a)** shown **(b)** two lines for $k>-2$ (with $k\neq 0$), one for $k=-2$, none for $k<-2$ **(c)** $y=\frac{1}{2}x$ and $x=0$

18 **(a)** $k\neq -2$ **(b)** $\mathbf{A}^{-1}=\frac{1}{10}\begin{pmatrix}2&0&2\\5&5&-5\\-1&5&-1\end{pmatrix}$ **(c)** $\mathbf{x}=\begin{pmatrix}1\\2\\-1\end{pmatrix}$

19 **(a)** shown **(b)** $p=4$, $q=-3$ **(c)** $\mathbf{M}$ is symmetric with a line of invariant points $y=\frac{1}{2}x$, which is the mirror **(d)** $(3,4)$

20 **(a)** shown **(b)** $(a-d)^2+4bc=0$ **(c)** $k=-1$; the line is $y=x$ **(d)** $\det(\mathbf{N}-\mathbf{I})=9\neq 0$ **(e)** $3\,\text{cm}^2$

Step-by-step working for every question is in the companion Worked Solutions booklet.