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Additional Math with Ali Hashir

Trigonometry – Mixed Hard Practice

Worksheet 4

O Level Additional Mathematics 4037 • Topic 10.1–10.6

Summary. This worksheet combines every idea from Worksheets 1–3: ratios of angles of any magnitude, the graphs of sin, cos and tan (including their modulus graphs), and the three Pythagorean identities. Every question is multi-part, and each part draws on a different one of these areas, so before writing anything decide which tools apply to each part.

$$\sin^2 A + \cos^2 A \equiv 1, \qquad \sec^2 A \equiv 1 + \tan^2 A, \qquad \csc^2 A \equiv 1 + \cot^2 A,$$
$$\tan A \equiv \frac{\sin A}{\cos A}, \qquad \sec A \equiv \frac{1}{\cos A}, \qquad \csc A \equiv \frac{1}{\sin A}.$$

Strategy.

- If an equation mixes tan and sec (or cot and csc), use the matching Pythagorean identity to rewrite it in a single ratio, then treat the result as a quadratic.
- If an angle has been shifted, e.g. $(x - 30^\circ)$, substitute $\varphi = x - 30^\circ$, solve for φ over the shifted domain, then convert back to x at the very end.
- To sketch $y = |f(x)|$, sketch $y = f(x)$ first and reflect any part below the x-axis upward; the curve can never dip below $y = 0$.
- Once you know $\sec\theta \pm \tan\theta$, the identity $\sec^2\theta - \tan^2\theta \equiv 1$ factorises as a difference of two squares and often hands you the other bracket for free.

Instructions. Every question on this worksheet is challenge level and has several parts. Show full working, and use earlier parts of a question to help you answer the later parts.

Q1

(i) Using the identity $\sec^2\theta \equiv 1 + \tan^2\theta$, solve, for $0° \leqslant \theta \leqslant 360°$, the equation

$$2\sec^2\theta - \tan\theta = 8,$$

giving your answers correct to 1 decimal place where necessary. [6]

(ii) Sketch $y = \tan\theta$ for $0° \leqslant \theta \leqslant 360°$, marking clearly the four values of θ found in part (i). [3]

Q2

(i) The angle θ is reflex and $5\cos\theta + 3 = 0$. Find the exact value of $\tan\theta + \cot\theta$. [5]

(ii) Sketch $y = \cos\theta$ for $0° \leqslant \theta \leqslant 360°$, and use your sketch to explain why there is exactly one reflex value of θ satisfying $\cos\theta = -\frac{3}{5}$. [3]

Q3

(i) Solve, for $0° \leqslant x \leqslant 360°$, the equation

$$2\cos^2(x - 30°) = 1 + \sin(x - 30°).$$

[7]

(ii) Using the periodicity of cosine and sine, write down one value of x outside the interval $0° \leqslant x \leqslant 360°$ that also satisfies the equation. [1]

Q4

The graph of $y = a\sin bx + c$, for $0° \leqslant x \leqslant 360°$, has amplitude 5, passes through $(0, 2)$, and completes exactly 2 full cycles in this interval.

(i) Find the values of a, b and c. [3]

(ii) Hence find the coordinates of every minimum point of the graph in this interval. [4]

(iii) Sketch $y = |a\sin bx + c|$, using your values of a, b and c, for $0° \leqslant x \leqslant 360°$, and state its range. [3]

Q5

Given that $\sec\theta - \tan\theta = \dfrac{1}{4}$ and that θ is acute,

(i) use the identity $\sec^2\theta - \tan^2\theta \equiv 1$ to find the exact value of $\sec\theta + \tan\theta$, [2]

(ii) hence find the exact values of $\sec\theta$, $\tan\theta$, $\sin\theta$ and $\cos\theta$, [5]

(iii) hence find the exact value of $\sin\theta + \cos\theta$. [1]

Q6

(i) Given that $x + y = 140°$, $\tan(x - y) = 1$, $x > y$, and $0° \leqslant x, y \leqslant 180°$, find the values of x and y. [5]

(ii) Verify that your values of x and y satisfy both given equations. [2]

Q7

(a) Prove that

$$\frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} \equiv 2\sec\theta.$$

[4]

(b) Hence solve, for $0^\circ \leqslant \theta \leqslant 360^\circ$, the equation

$$\frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} = 4.$$

[3]

Q8

(i) By sketching $y = |\cos x|$ and $y = \tan x$ on the same axes for $0^\circ \leqslant x \leqslant 360^\circ$, state the number of solutions of the equation $|\cos x| = \tan x$ in this interval. (You are not required to find the solutions.) [4]

(ii) State, with a reason based on the sign of $\tan x$, the two quadrants in which these solutions must lie. [2]

Q9

Given that $0^\circ \leqslant \theta \leqslant 360^\circ$ and

$$2\cot^2\theta - 7\csc\theta + 2 = 0,$$

(i) use the identity $\csc^2\theta \equiv 1 + \cot^2\theta$ to form a quadratic equation in $\csc\theta$, and hence find the values of θ, correct to 1 decimal place, [6]

(ii) state the quadrant in which each value of θ lies, and verify this using the sign of $\cot\theta$ in each case. [2]

Q10 (capstone)

The curve $y = a\cos x + b$, for $0° \leqslant x \leqslant 360°$, has a maximum value of 6 at $x = 0°$ and a minimum value of -2.

(i) Find the values of a and b. [3]

(ii) Show that the equation $a\cos x + b = 3\sec x$, with your values of a and b, can be written as

$$4\cos^2 x + 2\cos x - 3 = 0.$$

[3]

(iii) Hence solve $a\cos x + b = 3\sec x$ for $0° \leqslant x \leqslant 360°$, giving your answers correct to 1 decimal place. [5]

(iv) Sketch $y = |a\cos x + b|$, using your values of a and b, for $0° \leqslant x \leqslant 360°$, and state its range. [3]

Answers and full worked solutions available separately.