



Additional Math with Ali Hashir

Proving Trigonometric Identities

Worksheet 3

O Level Additional Mathematics 4037 • Topic 10.4, 10.6

Summary. The symbol $\equiv$ means an equation is true for *every* value of the angle, not just special ones. Every identity on this worksheet is built from the three Pythagorean identities together with the quotient and reciprocal relations:

$$\begin{aligned} \sin^2 A + \cos^2 A &\equiv 1, & \sec^2 A &\equiv 1 + \tan^2 A, & \csc^2 A &\equiv 1 + \cot^2 A, \\ \tan A &\equiv \frac{\sin A}{\cos A}, & \cot A &\equiv \frac{\cos A}{\sin A}, & \sec A &\equiv \frac{1}{\cos A}, & \csc A &\equiv \frac{1}{\sin A}. \end{aligned}$$

Strategy.

- Work on the side that looks more complicated; simplify it until it matches the other side. Never move terms across the $\equiv$ sign.
- Rewrite everything in terms of sin and cos if you get stuck.
- If you have a sum of two fractions, combine them over a common denominator.
- Look for a difference of two squares, e.g. $1 - \cos^2 A = (1 - \cos A)(1 + \cos A)$, which often cancels neatly with another factor.

Instructions. Prove each identity, showing every line of working. Marks are for method as well as for the final line.

Q1

Prove that $\sin \theta \cot \theta \equiv \cos \theta$.

[2]

Q2

Prove that $(1 - \cos \theta)(1 + \cos \theta) \equiv \sin^2 \theta$.

[2]

Q3

Prove that $\csc \theta \tan \theta \equiv \sec \theta$.

[3]

Q4

Prove that $(\sec A - 1)(\sec A + 1) \equiv \tan^2 A$. [3]

Q5

Prove that $\cot \theta + \tan \theta \equiv \sec \theta \csc \theta$. [4]

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Q6

Prove that $\sin^4 A - \cos^4 A \equiv \sin^2 A - \cos^2 A$. [4]

Q7

Prove that $(1 + \tan^2 \theta)(1 - \sin^2 \theta) \equiv 1$. [4]

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Q8

Prove that

$$\frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} \equiv 2 \csc \theta.$$

[5]

Q9

Prove that $(\csc \theta - \cot \theta)^2 \equiv \frac{1 - \cos \theta}{1 + \cos \theta}.$

[5]

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Q10

Prove that $\tan^2 A - \sin^2 A \equiv \tan^2 A \sin^2 A$.

[5]

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Challenge 1

Prove that $(\sec \theta + \tan \theta)(1 - \sin \theta) \equiv \cos \theta$. [5]

Challenge 2

Prove that $(1 + \cot A - \csc A)(1 + \tan A + \sec A) \equiv 2$. [6]

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Answers and full worked solutions available separately.

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