



Additional Math with Ali Hashir

Graphs of Trigonometric Functions

Worksheet 2

O Level Additional Mathematics 4037 • Topic 10.2, 10.3

Summary. For $y = a \sin bx + c$, $y = a \cos bx + c$ and $y = a \tan bx + c$ (with a a positive integer, b a simple fraction or integer, and c an integer):

	Amplitude	Period
$y = a \sin bx + c$ or $y = a \cos bx + c$	a	$\frac{360^\circ}{b}$ (or $\frac{2\pi}{b}$ rad)
$y = a \tan bx + c$	no amplitude	$\frac{180^\circ}{b}$ (or $\frac{\pi}{b}$ rad)

The constant c translates the whole graph vertically, so the range of $y = a \sin bx + c$ or $y = a \cos bx + c$ is $c - a \leq y \leq c + a$. The graph of $y = a \tan bx + c$ repeats every $\frac{180^\circ}{b}$ and has a vertical asymptote wherever bx is an odd multiple of 90° ; it has no amplitude and no maximum or minimum value. Sketches should show the correct shape, all axis intercepts, and (for tangent graphs) the equation of every asymptote in the given domain, clearly labelled.

Instructions. Answer all questions on graph-ruled or plain paper, using a ruler for axes. Only sin, cos and tan graphs appear on this worksheet. Two challenge questions appear at the end.

Q1

State the amplitude and the period of each of the following.

(a) $y = 5 \sin x$ [1]

(b) $y = 2 \cos 4x$ [1]

(c) $y = 3 \sin \frac{x}{2}$ [1]

Q2

Sketch the graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$, marking the coordinates of every point where the curve crosses the x -axis, and state the range of y . [4]

Q3

Sketch the graph of $y = 2 \cos x - 1$ for $0^\circ \leq x \leq 360^\circ$. State the maximum and minimum values of y , and the coordinates of every point where the curve crosses the coordinate axes. [5]

Q4

Sketch the graph of $y = \tan \frac{x}{2}$ for $0^\circ \leq x \leq 360^\circ$, stating the equation of any asymptote. [4]

Q5

The graph of $y = a \sin bx$ has amplitude 4 and period 90° . Find the values of a and b . [2]

Q6

Sketch the graph of $y = 3 \sin 2x + 1$ for $0^\circ \leq x \leq 180^\circ$, and state the range of y . [5]

Q7

Sketch the graph of $y = \cos x$ for $0 \leq x \leq 2\pi$ radians, marking the coordinates of every point where the curve meets the y -axis or crosses the x -axis, and state the period in radians. [4]

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Q8

On the same axes, sketch $y = \sin x$ and $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$. Hence state the number of solutions of the equation $\sin x = \cos x$ in this interval, and find them. [5]

Q9

Sketch the graph of $y = 2 \sin 2x - 1$ for $0^\circ \leq x \leq 360^\circ$. Hence, or otherwise, solve the equation $2 \sin 2x - 1 = 0$ in this interval. [6]

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Q10

Sketch the graph of $y = |\cos x|$ for $0^\circ \leq x \leq 360^\circ$. Hence solve the equation $|\cos x| = 0.5$ in this interval. [6]

Q11

Sketch the graph of $y = |\sin x|$ for $0^\circ \leq x \leq 360^\circ$. State the range of y , and the number of points in this interval where the curve touches the x -axis. [5]

Q12

Sketch the graph of $y = |\tan x|$ for $0^\circ \leq x \leq 360^\circ$, marking the equations of any asymptotes. Describe briefly how this graph differs in shape from the graph of $y = \tan x$. [5]

Challenge 1

The graph of $y = a \cos bx + c$ for $0^\circ \leq x \leq 360^\circ$ has maximum value 7, minimum value -1 , and completes exactly 3 full cycles in this interval. Find the values of a , b and c . [5]

Challenge 2

By sketching $y = \tan x$ and $y = 2 \cos x$ on the same axes for $0^\circ \leq x \leq 360^\circ$, state the number of solutions of the equation $\tan x = 2 \cos x$ in this interval. (You are not required to find the solutions.) [4]

Challenge 3

Sketch the graph of $y = |2 \sin x - 1|$ for $0^\circ \leq x \leq 360^\circ$. Hence solve the equation $|2 \sin x - 1| = 1$ in this interval. [6]

Answers and full worked solutions available separately.

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