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Additional Math with Ali Hashir

Trigonometric Ratios & General Angles

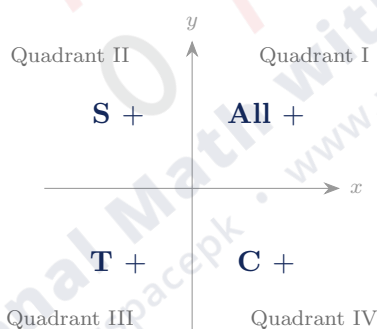
Worksheet 1

O Level Additional Mathematics 4037 • Topic 10.1, 10.4, 10.5

Summary. For any angle θ (measured anticlockwise from the positive x -axis, of any magnitude), the six trigonometric ratios are defined in terms of a point (x, y) on the unit circle:

$$\sin \theta = y, \quad \cos \theta = x, \quad \tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}.$$

The *ASTC rule* tells you which ratios are positive in each quadrant:



In quadrant I all six ratios are positive; in II only sine (and cosecant) are positive; in III only tangent (and cotangent); in IV only cosine (and secant). To solve an equation such as $\sin \theta = k$, first find the *basic (reference) angle* $\alpha = \sin^{-1} |k|$ using a calculator, then use the ASTC rule to place α in the correct quadrant(s) for the given domain.

Useful identity. $\sin^2 \theta + \cos^2 \theta \equiv 1$.

Instructions. Answer all questions. Give exact values where possible; otherwise round to 1 decimal place. Questions are ordered roughly by difficulty, with two challenge questions at the end.

Q1

The angle $\theta = 250^\circ$.

- (a) State which quadrant θ lies in. [1]
- (b) Hence state, giving a reason, whether each of $\sin \theta$, $\cos \theta$ and $\tan \theta$ is positive or negative. [3]

Q2

The angle A is acute and $\cos A = \frac{7}{25}$. Find the exact values of $\sin A$, $\tan A$, $\csc A$, $\sec A$ and $\cot A$. [5]

Q3

Find all the values of θ in the interval $0^\circ \leq \theta \leq 360^\circ$ for which $2 \cos \theta - 1 = 0$. [3]

Q4

Solve $\sec \theta = 2$ for $-180^\circ \leq \theta \leq 180^\circ$. [3]

Q5

Given that $\tan \theta = -\frac{4}{3}$ and that θ is obtuse, find the exact values of $\sin \theta$ and $\cos \theta$. [4]

Q6

Solve $3 \cot \theta = 1$ for $0^\circ \leq \theta \leq 180^\circ$, giving your answer correct to 1 decimal place. [3]

Q7

Find all values of x in the interval $0^\circ \leq x \leq 360^\circ$ for which $5 \sin x = 3 \cos x$, giving your answers correct to 1 decimal place. [3]

Q8

Find all the values of θ in the interval $0^\circ \leq \theta \leq 360^\circ$ for which $\cos 2\theta = -\frac{1}{2}$. [5]

Q9

Solve, for $0^\circ \leq \theta \leq 360^\circ$, the equation $2\sin^2 \theta = 1 + \sin \theta$. [5]

Q10

Given that $\csc \theta = -\sqrt{2}$, that θ is reflex, and that $\cos \theta < 0$, find the exact value of θ and the exact value of $\cot \theta$. [5]

Q11

Solve, for $0^\circ \leq x \leq 360^\circ$, the equation $2 \cos^2 x + \cos x - 1 = 0$. [5]

Q12

The angle θ is reflex and $2 \cos \theta + 1 = 0$. Find the exact values of $\sin \theta$, $\tan \theta$, $\cot \theta$, $\sec \theta$ and $\csc \theta$. [6]

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Q13

Solve, for $0^\circ \leq \theta \leq 360^\circ$, the equation $\cot^2 \theta - \cot \theta - 2 = 0$, giving your answers correct to 1 decimal place where necessary. [6]

Q14

Solve, for $0^\circ \leq \theta \leq 180^\circ$, the equation $\tan 3\theta = 1$. [5]

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Q15

Given that $\sin A = \frac{3}{5}$, where A is obtuse, and that $\cos B = \frac{5}{13}$, where B is acute, find the exact value of $\tan A + \tan B$. [5]

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Challenge 1

The angle θ is obtuse and $\sin \theta = k$, where $0 < k < 1$. Express $\cos \theta$ and $\tan \theta$ in terms of k .
[4]

Challenge 2

By first writing $\cos^2 x$ in terms of $\sin x$, solve the equation $2 \cos^2 x + \sin x = 2$ for $0^\circ \leq x \leq 360^\circ$.
[6]

Answers and full worked solutions available separately.

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