Additional Math with Ali Hashir

Quadratic Inequalities

Answers

CIE O Level Additional Mathematics 4037 • Total: 70 marks

Easy (Q1–Q2)

Q1. (a) $(x-2)(x-3) > 0 \Rightarrow x < 2$ or $x > 3$

(b) $(2x-1)(x+3) \leqslant 0 \Rightarrow -3 \leqslant x \leqslant \frac{1}{2}$

(c) $x^2 > 9 \Rightarrow x < -3$ or $x > 3$

Q2. $mx^2+8x+(m-6) = 0$; no real $x \Rightarrow b^2-4ac < 0$: $-4m^2+24m+64 < 0 \Rightarrow (m-8)(m+2) > 0$

$m < -2$ or $m > 8$

Medium (Q3–Q7)

Q3. $2x^2 + (4-p)x + (2-3p) = 0$; $b^2 - 4ac = p^2 + 16p > 0 \Rightarrow p < -16$ or $p > 0$

Q4. $b^2 - 4ac = -8p^2 - 8p + 160 > 0 \Rightarrow (p+5)(p-4) < 0$, so $-5 < p < 4$, $p \neq -2$

Q5. $2x^2 + (4+m)x + (2+2m) = 0$; $b^2 - 4ac = m^2 - 8m \geqslant 0 \Rightarrow m \leqslant 0$ or $m \geqslant 8$

Q6. $(3k-5)x^2 + (k-5)x - 2 = 0$; $b^2 - 4ac = k^2 + 14k - 15 < 0 \Rightarrow -15 < k < 1$

Q7. $px^2 - 2(p+2)x + (10-p) = 0$. If $p = 0$ the equation is linear with root $x = \frac{5}{2}$.

If $p \neq 0$: $b^2 - 4ac = 8p^2 - 24p + 16 \geqslant 0 \Rightarrow (p-1)(p-2) \geqslant 0 \Rightarrow p \leqslant 1$ or $p \geqslant 2$.

Hence p cannot lie strictly between 1 and 2

Hard (Q8–Q10)

Q8. Substituting gives $3x^2 - cx + 3 = 0$; no intersection $\Rightarrow c^2 - 36 < 0 \Rightarrow -6 < c < 6$

Q9. Substituting $y = 2x - k$ gives $3x^2 - 3kx + (k^2-1) = 0$; $b^2 - 4ac = 12 - 3k^2 > 0 \Rightarrow -2 < k < 2$

Q10. Minimum point $\Rightarrow k - 6 > 0 \Rightarrow k > 6$.

Cuts the x-axis twice $\Rightarrow b^2 - 4ac = -4k^2 + 24k + 64 > 0 \Rightarrow -2 < k < 8$.

Both conditions together: $6 < k < 8$

Challenge 1

Ch. 1. $y < 0$ only between the roots, so the roots are 2 and k.

Product of roots: $2k = \dfrac{16}{2} = 8 \Rightarrow k = 4.$ Sum of roots: $2 + k = -\dfrac{p}{2} \Rightarrow p = -12.$

$p = -12,\ \ k = 4 \quad (\text{so } y = 2(x-2)(x-4))$

Challenge 2

(i). $b^2 - 4ac = 0$: $(m+n)^2 - 4m^2n = 0$, which expands to $n^2 + 2\left(m - 2m^2\right)n + m^2 = 0$

(ii). Treating this as a quadratic in n, n real requires $4\left(m-2m^2\right)^2 - 4m^2 \geqslant 0$, i.e. $4m^2\left(4m^2 - 4m\right) \geqslant 0$.

Since the original equation is quadratic, $m \neq 0$, so $m^2 > 0$ and we may divide: $4m(m-1) \geqslant 0$.

$m < 0 \ \text{ or } \ m \geqslant 1$