



Additional Math with Ali Hashir

Quadratics: The Discriminant

Worksheet

CIE O Level Additional Mathematics 4037 • Total: 63 marks

Answer all questions. In every question, first rearrange the equation into the standard form $ax^2 + bx + c = 0$ and write down a , b and c clearly before using the discriminant $b^2 - 4ac$. Recall that for real a , b , c with $a \neq 0$:

$$\begin{aligned} b^2 - 4ac > 0 & \quad \text{two distinct real roots} \\ b^2 - 4ac = 0 & \quad \text{two equal real roots (a repeated root)} \\ b^2 - 4ac < 0 & \quad \text{no real roots} \end{aligned}$$

“Real roots” therefore means $b^2 - 4ac \geq 0$. Show all algebraic simplification.

Q1

What is the least value k can take if the roots of the equation $x^2 - 2kx + k^2 = 3 + x$ are real? [4]

Q2

Find the range of values of k for which the roots of the equation $x^2 + (2k + 1)x + k^2 - 3 = 0$ are not real. State the range of values of k for which the roots are real. [5]

Q3

Find the range of values of k for which the equation $x^2 - 2kx + k^2 - 2k = 6$ has real roots. Find the roots in terms of k . [6]

Q4

Show that the roots of the equation $2x^2 + p = 2(x - 1)$ are not real if $p > -\frac{3}{2}$. [4]

Q5

Given that the equation $px^2 + 3px + p + q = 0$, where $p \neq 0$, has two equal real roots, find q in terms of p . [4]

Q6

The equation $x(x - 2) + k^2 = k(2x - 1)$ is satisfied by two distinct real values of x . Find the range of values of k . [5]

Q7

Show that the solutions of the equation $x^2 + kx = 3 - k$ are real for all real values of k . [4]

Q8

Given that $x(x - 2) = t - 2$, find x in terms of t . Hence deduce the range of values of t for x to be real. [5]

Q9

The curve $y = 3x^2 - 2x + c - 1$ cuts the x -axis at 2 points. Find the range of values of c . [4]

Q10

Find the values of p for which the x -axis is a tangent to the curve $y = x^2 + px - p + 3$. For each of these values, find the coordinates of the point of tangency. [6]

Challenge 1

Show that $px^2 - 4x + p = 2px - 3x^2 - 1$ has real root(s) for all real values of p .

Read the wording of the question carefully before assuming the discriminant settles the matter on its own. [6]

Challenge 2

The line $y = kx + 1$ and the curve $y = x^2 + 3x + 5$ are drawn on the same axes, where k is a constant.

- (i) Show that the x -coordinate of any point of intersection satisfies $x^2 + (3 - k)x + 4 = 0$. [2]
- (ii) Find the two values of k for which the line is a tangent to the curve. [3]
- (iii) For each of these values of k , find the coordinates of the point of contact. [3]
- (iv) State, with a reason, the range of values of k for which the line does not meet the curve at all. [2]

Answers available at the website.

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