Additional Math with Ali Hashir

Quadratics: The Discriminant

Answers

CIE O Level Additional Mathematics 4037 • Total: 63 marks

Easy (Q1–Q2)

Q1. Rearranges to $x^2 - (2k+1)x + (k^2 - 3) = 0$; $b^2 - 4ac = 4k + 13 \geqslant 0$, so $k \geqslant -\frac{13}{4}$. Least value $k = -\frac{13}{4}$

Q2. $b^2 - 4ac = 4k + 13$. Not real: $k < -\frac{13}{4}$. Real: $k \geqslant -\frac{13}{4}$

Medium (Q3–Q7)

Q3. $b^2 - 4ac = 8k + 24 \geqslant 0$, so $k \geqslant -3$; roots $x = k \pm \sqrt{2k+6}$

Q4. Rearranges to $2x^2 - 2x + (p+2) = 0$; $b^2 - 4ac = -8p - 12$, and $-8p - 12 < 0 \iff p > -\frac{3}{2}$

Q5. $q = \dfrac{5p}{4}$

Q6. Rearranges to $x^2 - (2k+2)x + (k^2 + k) = 0$; $b^2 - 4ac = 4k + 4 > 0$, so $k > -1$

Q7. $x^2 + kx + (k-3) = 0$; $b^2 - 4ac = k^2 - 4k + 12 = (k-2)^2 + 8 \geqslant 8 > 0$ for all real k, so the solutions are always real (and distinct)

Hard (Q8–Q10)

Q8. $x = 1 \pm \sqrt{t-1}$; real when $t - 1 \geqslant 0$, i.e. $t \geqslant 1$

Q9. $b^2 - 4ac = 16 - 12c > 0$, so $c < \dfrac{4}{3}$

Q10. $p^2 + 4p - 12 = 0$, so $p = -6$ or $p = 2$.
$p = -6$: $y = (x-3)^2$, point of tangency $(3, 0)$.
$p = 2$: $y = (x+1)^2$, point of tangency $(-1, 0)$

Challenge 1

Ch. 1. Rearranges to $(p+3)x^2 - (2p+4)x + (p+1) = 0$.

If $p = -3$ the coefficient of x^2 is zero, so the equation is *linear*: $2x - 2 = 0$, giving the real root $x = 1$.

If $p \neq -3$, $b^2 - 4ac = 4 > 0$ for every p, so there are two distinct real roots.
Both cases together $\Rightarrow$ real root(s) for all real p

Challenge 2

(i). $x^2 + 3x + 5 = kx + 1 \Rightarrow x^2 + (3-k)x + 4 = 0$

(ii). Tangent $\Rightarrow b^2 - 4ac = 0$: $(3-k)^2 = 16$, so $k = -1$ or $k = 7$

(iii). $k = -1$: contact at $(-2, 3)$. $k = 7$: contact at $(2, 15)$

(iv). No intersection $\Rightarrow b^2 - 4ac < 0$: $(3-k)^2 < 16$, giving $-1 < k < 7$