Additional Math with Ali Hashir

Quadratics

Answers

CIE O Level Additional Mathematics 4037 • Total: 76 marks

Easy (Q1–Q3)

Q1. (a) minimum $y = 1$ at $x = 3$ (b) minimum $y = -5$ at $x = -1$ (c) maximum $y = 4$ at $x = 2$ (d) maximum $y = 9$ at $x = \frac{1}{2}$

Q2. Maximum $y = 1$ at $x = -3$; y-intercept $(0, -8)$

Q3. $x^2 + 8x + 21 = (x + 4)^2 + 5$; minimum value 5 at $x = -4$

Medium (Q4–Q10)

Q4. $2x^2 - 12x + 23 = 2(x - 3)^2 + 5$; minimum point $(3, 5)$; y-intercept $(0, 23)$

Q5. $3 + 9x - x^2 = \dfrac{93}{4} - \left(x - \dfrac{9}{2}\right)^2$ (so $q = \frac{93}{4}$, $r = -\frac{9}{2}$); maximum value $\frac{93}{4}$ at $x = \frac{9}{2}$

Q6. $a = 4$, $p = 2$, $q = -1$ (a) minimum point $(2, -1)$ (b) $x = \frac{3}{2}$ or $x = \frac{5}{2}$

Q7. Minimum value 0 at $x = -2$; on $-3 \leqslant x \leqslant 0$ the curve runs from $(-3, 3)$ down to $(-2, 0)$ and up to $(0, 12)$

Q8. $k = 1$; maximum point $(2, 9)$, y-intercept $(0, 1)$

Q9. $p = -6$, $q = 5$; the curve crosses the x-axis at $(1, 0)$ and $(5, 0)$

Q10. $5 - 6x - 3x^2 = 8 - 3(x + 1)^2$ (so $a = 3$, $p = 1$, $q = 8$); maximum value 8 at $x = -1$

Hard (Q11–Q14)

Q11. $y = 2(x + 2)^2 + 3 = 2x^2 + 8x + 11$

Q12. (i) $y = k\left(x - \dfrac{6}{k}\right)^2 + 3 - \dfrac{36}{k}$ (so $a = k$, $b = -\frac{6}{k}$, $c = 3 - \frac{36}{k}$) (ii) $k = 3$ (iii) $y = 3(x - 2)^2 - 9$: minimum $(2, -9)$, y-intercept $(0, 3)$, x-intercepts $\left(2 \pm \sqrt{3}, 0\right)$

Q13. (i) $2(x - 2)^2 - 5$ (ii) $(2, -5)$ (iii) greatest value $y = 3$ (at $x = 0$), least value $y = -5$ (at $x = 2$)

Q14. (i) $A = x(40 - 2x) = 40x - 2x^2$ (ii) $A = 200 - 2(x - 10)^2$ (iii) largest area $200\,\text{m}^2$, when $x = 10$

Extra Challenge

Challenge. $y = k\left(x - \dfrac{2}{k}\right)^2 + 3k - \dfrac{4}{k}$ (so $a = k$, $p = \frac{2}{k}$, $q = 3k - \frac{4}{k}$);

a maximum requires $k < 0$, and $3k - \dfrac{4}{k} = 4$ gives $3k^2 - 4k - 4 = 0$, so $k = -\frac{2}{3}$ or $k = 2$.

Rejecting $k = 2$ (which gives a minimum), $k = -\dfrac{2}{3}$