Additional Math with Ali Hashir

Polynomial Long Division

Worksheet

CIE O Level Additional Mathematics 4037 • Total: 52 marks

Polynomial long division extends the long-division algorithm you already use for numbers. Dividing $f(x)$ by a divisor $d(x)$ of lower degree produces a *quotient* $Q(x)$ and a *remainder* $R(x)$. It turns a cubic or quartic into factors once one factor is known, and it underlies the remainder and factor theorems.

Key results

$$f(x) \equiv d(x)\,Q(x) + R(x), \qquad \deg R(x) < \deg d(x).$$

- $f(x)$ is the **dividend**, $d(x)$ the **divisor**, $Q(x)$ the **quotient** and $R(x)$ the **remainder**.
- A **linear** divisor leaves a constant remainder. A **quadratic** divisor leaves a remainder of the form $ax + b$.
- $R(x) = 0$ exactly when $d(x)$ divides $f(x)$ exactly.

The method. Repeat until what is left has lower degree than the divisor:

1. **Divide** the leading term of what is left by that of the divisor, giving the next term of the quotient.
2. **Multiply** the whole divisor by that term.
3. **Subtract** the product. The leading term must cancel.
4. **Bring down** the next term and repeat.

Before you start.

- Write $f(x)$ with *every* power present, inserting $0x^k$ where a term is missing, so like terms stay in their own columns. Do the same for the divisor.
- When subtracting, change the sign of *every* term of the product before adding. This is the single most common source of lost marks.
- If the divisor is not monic, divide by its full leading term — for $2x + 1$ that means dividing by $2x$, not by x.

Instructions. Answer all questions by long division only, stating each result in the form $f(x) = d(x)Q(x) + R(x)$; a remainder of 0 must still be written down. Marks are shown in brackets.

Q1

Divide $f(x) = x^3 + 2x^2 - 5x - 6$ by $d(x) = x - 2$.
Hence write $f(x)$ in the form $d(x)Q(x) + R$. [3]

Q2

Divide $f(x) = 2x^3 - 3x^2 + 4x - 1$ by $d(x) = x + 1$.
Hence write $f(x)$ in the form $d(x)Q(x) + R$. [3]

Q3

Divide $f(x) = x^3 - 4x^2 + 6x - 4$ by $d(x) = x - 2$, and write $f(x)$ in the form $d(x)Q(x) + R$.
Hence express $f(x)$ as the product of $x - 2$ and a quadratic. [3]

Q4

Divide $f(x) = 6x^3 + 11x^2 - 4x - 4$ by $d(x) = 2x + 1$.
Hence write $f(x)$ in the form $d(x)Q(x) + R$, and factorise $f(x)$ completely. [4]

Q5

Divide $f(x) = x^4 - 3x^2 + 2x + 5$ by $d(x) = x + 2$.
Hence write $f(x)$ in the form $d(x)Q(x) + R$. [4]

Q6

Divide $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 3$ by the quadratic divisor $d(x) = x^2 + x - 1$.
Hence write $f(x)$ in the form $d(x)Q(x) + R(x)$. [5]

Q7

Divide $f(x) = 2x^4 - x^3 + 3x - 7$ by $d(x) = x^2 + 2$.
Hence write $f(x)$ in the form $d(x)Q(x)+R(x)$, and state clearly the degree of your remainder. [5]

Q8

Divide $f(x) = 3x^4 - 2x^3 + x - 5$ by $d(x) = x^2 - 2x + 3$.
Hence write $f(x)$ in the form $d(x)Q(x) + R(x)$. [5]

Q9

The polynomial $f(x) = 2x^3 + ax^2 - 5x + 6$, where a is a constant, is divided by $d(x) = x - 3$. The remainder is 18.

(i) Carry out the long division of $f(x)$ by $x - 3$, keeping a in your working, and show that the remainder is $9a + 45$. [3]

(ii) Hence find the value of a, and write $f(x)$ in the form $d(x)Q(x) + R$ with all coefficients numerical. [2]

Q10

The polynomial $f(x) = x^4 + px^3 - 7x^2 + qx + 12$, where p and q are constants, is divided by $d(x) = x^2 - x - 2$. The division leaves no remainder.

(i) Carry out the long division, keeping p and q in your working, and show that the remainder is $(3p + q - 2)x + (2p + 4)$. [4]

(ii) Hence find the values of p and q, and write $f(x)$ in the form $d(x)Q(x) + R(x)$. [2]

(iii) By factorising both $d(x)$ and your quotient $Q(x)$, express $f(x)$ as a product of four linear factors. [2]

Extra Challenge

The polynomial $f(x) = 2x^4 - 3x^3 + ax^2 + bx - 6$, where a and b are constants, leaves a remainder of $3x + 7$ when it is divided by $d(x) = x^2 - 1$.

(i) Explain why the remainder on division by $x^2 - 1$ must be of the form $\alpha x + \beta$. [1]

(ii) By dividing $f(x)$ by $x^2 - 1$, keeping a and b in your working, show that the remainder is $(b-3)x + (a-4)$. Hence find the values of a and b, and write $f(x)$ in the form $d(x)Q(x) + R(x)$. [5]

(iii) Now divide your quotient $Q(x)$ by $x - 2$, and use the result to write $f(x)$ in the form

$$f(x) = (x^2 - 1)(x - 2)\, Q_2(x) + c\,(x^2 - 1) + R(x),$$

stating $Q_2(x)$ and the constant c. [3]

Answers available at the website.