Additional Math with Ali Hashir

Inverse Functions

Answers

CIE O Level Additional Mathematics 4037 • Total: 94 marks

Easy (Q1–Q2)

Q1. **(a)** $f^{-1} : x \mapsto \dfrac{x-7}{4}$ **(b)** $f^{-1} : x \mapsto \dfrac{5-2x}{x}, \quad x \neq 0$ **(c)** $f^{-1} : x \mapsto \dfrac{4x}{x-3}, \quad x \neq 3$

Q2. **(a)** $f^{-1} : x \mapsto \dfrac{x-8}{3}$ **(b)** $f^{-1}(23) = 5$

(c) f^{-1} is by definition the rule that undoes f; explicitly $f^{-1}f(x) = \dfrac{(3x+8)-8}{3} = x$

Medium (Q3–Q7)

Q3. **(a)** $f(x) = (x-3)^2 - 4$. The vertex is at $x = 3$, so the domain $x \geqslant 3$ lies entirely on the increasing branch — no two inputs share an output, hence one-one

(b) $f(x) \geqslant -4$

(c) $f^{-1} : x \mapsto 3 + \sqrt{x+4}$; domain $x \geqslant -4$, range $f^{-1}(x) \geqslant 3$

Q4. **(a)** $f^{-1} : x \mapsto \dfrac{x-5}{2}$; $g^{-1} : x \mapsto \dfrac{x+4}{x}, \quad x \neq 0$

(b) $x^2 - 7x - 8 = 0$, i.e. $(x-8)(x+1) = 0$, so $x = 8$ or $x = -1$

Q5. **(a)** $f^{-1} : x \mapsto \dfrac{3x}{2-x}, \quad x \neq 2$ **(b)** $f^{-1}(4) = -6$, so $-6 + a = 1$ and $a = 7$

(c) $x = 2$. Since the domain of f^{-1} is the range of f, the value 2 is *not* in the range of f — consistent with $f(x) = 2 - \dfrac{6}{x+3}$, which never reaches 2

Q6. **(a)** $3x^2 - 5x - 2 = 0 \Rightarrow (3x+1)(x-2) = 0$; reject $x = -\frac{1}{3}$, so $f^{-1}(5) = 2$

(b) $3x^2 + x - 2 = 0 \Rightarrow (3x-2)(x+1) = 0$; reject $x = -1$, so $f^{-1}(-1) = \frac{2}{3}$

(c) The range of f^{-1} equals the domain of f, which is $x > 0$. So f^{-1} can only ever output a positive number, and any negative root must be discarded

Q7. **(a)** $f^{-1} : x \mapsto \dfrac{b-2x}{x}, \quad x \neq 0$ **(b)** $\dfrac{b}{5} + (b-2) = 4 \Rightarrow \dfrac{6b}{5} = 6$, so $b = 5$

Hard (Q8–Q10)

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Q8. **(a)** $\mathrm{fg} : x \mapsto \dfrac{5-3x}{x-1},\ \ x \neq 1$; $(\mathrm{fg})^{-1} : x \mapsto \dfrac{x+5}{x+3},\ \ x \neq -3$

(b) $\mathrm{f}^{-1} : x \mapsto \dfrac{x+3}{2}$; $\mathrm{g}^{-1} : x \mapsto \dfrac{x+1}{x},\ \ x \neq 0$; $\mathrm{g}^{-1}\mathrm{f}^{-1} : x \mapsto \dfrac{x+5}{x+3},\ \ x \neq -3$

Yes — the two expressions agree, illustrating the general rule $(\mathrm{fg})^{-1} = \mathrm{g}^{-1}\mathrm{f}^{-1}$

Q9. **(a)** $\mathrm{f}^{-1} : x \mapsto \dfrac{2x+a}{x-3},\ \ x \neq 3$ **(b)** $\dfrac{10+a}{2} = 8$, so $a = 6$

(c) $(3x+6)(x-3) = (2x+6)(x-2)$ reduces to $x^2 - 5x - 6 = 0$, i.e. $(x-6)(x+1) = 0$, so $x = 6$ or $x = -1$

Q10. **(a)** $\mathrm{f}^{-1} : x \mapsto \dfrac{x+4}{3}$

(b) $\mathrm{f}^{-1}(2) = 2$, so $\mathrm{g}(2) = 0$ gives $2a + b = -7$; the remainder theorem gives $\mathrm{g}(3) = 24$, i.e. $3a + b = -3$. Hence $a = 4$, $b = -15$

(c) $\mathrm{g}(x) = (x-2)\left(x^2 + 6x - 3\right)$, so $x = 2$ or $x = -3 \pm 2\sqrt{3}$

Challenge 1

Ch. 1. **(a)** $y = \dfrac{ax+b}{cx-a} \Rightarrow x(cy-a) = ay + b \Rightarrow x = \dfrac{ay+b}{cy-a}$, which is the same rule, so $\mathrm{f}^{-1} = \mathrm{f}$

(b) $\mathrm{f}^2(x) = x$; $\mathrm{f}^5(x) = \mathrm{f}(x) = \dfrac{ax+b}{cx-a}$

(c) $a = 2$, $b = 5$, $c = 1$ (and $a^2 + bc = 9 \neq 0$). $\mathrm{f}(x) = x$ gives $x^2 - 4x - 5 = 0$, i.e. $(x-5)(x+1) = 0$, so $x = 5$ or $x = -1$

Challenge 2

Ch. 2. **(a)** $\mathrm{fg}\big(\mathrm{g}^{-1}\mathrm{f}^{-1}(x)\big) = \mathrm{f}\Big(\mathrm{gg}^{-1}\big(\mathrm{f}^{-1}(x)\big)\Big) = \mathrm{ff}^{-1}(x) = x$, and likewise $\mathrm{g}^{-1}\mathrm{f}^{-1}\big(\mathrm{fg}(x)\big) = x$. So $\mathrm{g}^{-1}\mathrm{f}^{-1}$ undoes fg both ways, hence $(\mathrm{fg})^{-1} = \mathrm{g}^{-1}\mathrm{f}^{-1}$

(b) fg applies g first and f last, so undoing it must remove the *last* step first — f^{-1} before g^{-1} (socks on first, shoes off first)

(c) h is: double, subtract 1, cube, multiply by 5, add 4. Reversing: subtract 4, divide by 5, cube root, add 1, halve:

$$\mathrm{h}^{-1}(x) = \frac{1}{2}\left(\sqrt[3]{\frac{x-4}{5}} + 1\right)$$

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