



Additional Math with Ali Hashir

Composite Functions

Worksheet

CIE O Level Additional Mathematics 4037 • Total: 78 marks

A **composite function** is what you get by feeding the output of one function straight into another. The notation is compact but unforgiving:

$$fg(x) = f(g(x))$$

— the function written *nearest to x* acts *first*. Read composites from the inside out, right to left.

Reading the notation

$fg(x)$	do g first, then apply f to the result
$gf(x)$	do f first, then apply g — a <i>different</i> function in general
$f^2(x)$	$f(f(x))$, not $[f(x)]^2$
$f^5(x)$	apply f five times over

The third row is where most marks are lost. If $f: x \mapsto x + 3$ then $f^2(x) = x + 6$, not $(x + 3)^2$.

Key results used in this worksheet

- 1. Order matters.** In general $fg \neq gf$. When a question produces $fg = gf$ that is a special property of those particular functions, never a rule you may assume.
- 2. Building a composite.** To form $fg(x)$, write out f with an empty bracket and drop the whole of $g(x)$ into it. Simplify only afterwards.
- 3. Recovering a hidden function.** If f and gf are known, rewrite $gf(x)$ so that every x appears inside a copy of $f(x)$ — completing the square usually does it — and then read off g . If instead f and fh are known, undo the last step of f instead.
- 4. Excluded values.** A composite inherits every restriction of the *inner* function and picks up new ones from the *outer* one after substitution. Both sets must be stated. Likewise $fg(x)$ only exists where $g(x)$ lands inside the domain of f .

Instructions

Answer **all** questions, showing full working — the method carries most of the marks. Give **exact** values: fractions and surds, never rounded decimals. State the excluded value(s) of x for every composite you form, and never write f^2 when you mean a square.

Composite Functions

Q1–Q10

Q1

Two functions are defined by

$$f : x \mapsto 3x - 1 \quad \text{and} \quad g : x \mapsto x^2 + 2.$$

- (a) Find $fg(2)$. [1]
- (b) Find $gf(2)$. [1]
- (c) Obtain an expression for $fg(x)$. [2]
- (d) Obtain an expression for $gf(x)$. [2]

Q2

Two functions are defined by

$$f : x \mapsto 2x - 3 \quad \text{and} \quad g : x \mapsto x^2 + 1.$$

- (a) Find the value of $fg(-2)$. [2]
- (b) Find the values of x for which $gf(x) = 17$. [3]

Q3

A function f is defined by $f : x \mapsto x - 3$.

- (a) Another function g is such that $gf : x \mapsto x^2 - 6x + 14$. Express $gf(x)$ in the form $(x - 3)^2 + c$, and hence write down an expression for $g(x)$. [4]
- (b) A third function h is such that $fh : x \mapsto 5x + 1$. Find an expression for $h(x)$. [2]
- (c) Using your answer to part (a), find an expression for $fg(x)$. [2]

Q4

A function f is defined by

$$f : x \mapsto \frac{2}{x} + k, \quad x \neq 0,$$

where k is a constant. Given that $f^2(2) = \frac{2}{3}f(1)$, calculate the possible values of k . [7]

Q5

Two functions are defined by

$$f : x \mapsto ax + 2 \quad \text{and} \quad g : x \mapsto 4x + a,$$

where a is a constant. Given that $fg(x) = gf(x)$ for *all* values of x , find the possible values of a . [6]

Q6

Given that $f : x \mapsto ax + b$ and $f^3 : x \mapsto 27x + 26$, find

(a) the value of a and of b , [4]

(b) an expression for f^4 . [2]

Q7

The function f is defined by

$$f : x \mapsto \frac{x+1}{x-1}, \quad x \neq 1.$$

Show that $f^2(x) = x$, $x \neq 1$, and hence express $f^5(x)$ and $f^{10}(x)$ in their simplest forms. [6]

Q8

Two functions are defined by

$$f : x \mapsto \frac{4}{x-1}, \quad x \neq 1 \quad \text{and} \quad g : x \mapsto ax + b,$$

where a and b are constants. It is given that $g(3) = 11$ and $fg(1) = 2$.

- (a) Find the value of a and of b . [4]
- (b) Obtain an expression for $gf(x)$, stating the value of x that must be excluded. [2]
- (c) Solve the equation $gf(x) = 3$. [2]

Q9

The functions f and g are defined by

$$f : x \mapsto x^2 - 4x + 7 \text{ for } x \geq 2, \quad \text{and} \quad g : x \mapsto 2x + 1.$$

- (a) Express $f(x)$ in the form $(x - p)^2 + q$ and hence state the range of f . [3]
- (b) Find an expression for $gf(x)$ and state the range of gf . [3]
- (c) Explain why $fg(x)$ is defined only for $x \geq \frac{1}{2}$, and find an expression for $fg(x)$. [3]

Q10

The function f is defined by

$$f : x \mapsto \frac{3}{x-2}, \quad x \neq 2.$$

- (a) Show that $f^2(x) = \frac{3x-6}{7-2x}$, and state the *two* values of x that must be excluded. [4]
- (b) Solve the equation $f^2(x) = 3$. [2]
- (c) Show that the equation $f^2(x) = x$ reduces to a quadratic equation, and solve it. Comment on why one of your answers also appeared in part (b). [3]

Challenge question

Challenge

A function f is defined by

$$f : x \mapsto \frac{x}{x+1}, \quad x \neq -1.$$

- (a) Prove that $f^2(x) = \frac{x}{2x+1}$, and state the values of x that must be excluded. [3]
- (b) Obtain a similar expression for $f^3(x)$, again stating the excluded values. [3]
- (c) Hence suggest a possible expression for $f^n(x)$, where n is a positive integer, and state the set of values that must be excluded from its domain. [2]

Answers and full worked solutions available at the website.



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