Additional Math with Ali Hashir

Composite Functions

Answers

CIE O Level Additional Mathematics 4037 • Total: 78 marks

Easy (Q1–Q2)

Q1. **(a)** $\text{fg}(2) = \text{f}(6) = 17$ **(b)** $\text{gf}(2) = \text{g}(5) = 27$ **(c)** $\text{fg}(x) = 3x^2 + 5$ **(d)** $\text{gf}(x) = 9x^2 - 6x + 3$

Q2. **(a)** $\text{fg}(-2) = \text{f}(5) = 7$

(b) $(2x-3)^2 + 1 = 17 \Rightarrow (2x-3)^2 = 16 \Rightarrow 2x - 3 = \pm 4$, so $x = \frac{7}{2}$ or $x = -\frac{1}{2}$

Medium (Q3–Q7)

Q3. **(a)** $\text{gf}(x) = (x-3)^2 + 5$, so $c = 5$ and $\text{g}(x) = x^2 + 5$ **(b)** $\text{h}(x) = 5x + 4$ **(c)** $\text{fg}(x) = x^2 + 2$

Q4. $\dfrac{2}{1+k} + k = \dfrac{2}{3}(2+k)$ reduces to $k^2 - 3k + 2 = 0$, i.e. $(k-1)(k-2) = 0$, so $k = 1$ or $k = 2$

Q5. $\text{fg}(x) = 4ax + a^2 + 2$ and $\text{gf}(x) = 4ax + a + 6$. Equating constants gives $a^2 - a - 6 = 0$, i.e. $(a-3)(a+2) = 0$, so $a = 3$ or $a = -2$

Q6. **(a)** $\text{f}^3(x) = a^3x + b(a^2 + a + 1)$, so $a^3 = 27$ and $13b = 26$, giving $a = 3$, $b = 2$ **(b)** $\text{f}^4 : x \mapsto 81x + 80$

Q7. $\text{f}^2(x) = \dfrac{2x}{2} = x$. Since f^2 is the identity, $\text{f}^5 = \text{f}^4\text{f} = \text{f}$, so $\text{f}^5(x) = \dfrac{x+1}{x-1}$, and $\text{f}^{10} = (\text{f}^2)^5$, so $\text{f}^{10}(x) = x$

Hard (Q8–Q10)

Q8. **(a)** $3a + b = 11$ and $a + b = 3$, so $a = 4$, $b = -1$, i.e. $\text{g}(x) = 4x - 1$

(b) $\text{gf}(x) = \dfrac{16}{x-1} - 1 = \dfrac{17-x}{x-1}$, $x \neq 1$ **(c)** $x = 5$

Q9. **(a)** $\text{f}(x) = (x-2)^2 + 3$; range $\text{f}(x) \geqslant 3$

(b) $\text{gf}(x) = 2(x-2)^2 + 7 = 2x^2 - 8x + 15$; range $\text{gf}(x) \geqslant 7$

(c) f only accepts inputs $\geqslant 2$, so $\text{g}(x) \geqslant 2$, i.e. $2x + 1 \geqslant 2$, giving $x \geqslant \frac{1}{2}$; $\text{fg}(x) = (2x-1)^2 + 3 = 4x^2 - 4x + 4$

Q10. **(a)** $\mathrm{f}^2(x) = \dfrac{3x-6}{7-2x}$; exclude $x = 2$ (from the inner f) and $x = \frac{7}{2}$ (from the outer one)

(b) $x = 3$

(c) $3x - 6 = 7x - 2x^2 \Rightarrow 2x^2 - 4x - 6 = 0 \Rightarrow x^2 - 2x - 3 = 0$, i.e. $(x-3)(x+1) = 0$, so $x = 3$ or $x = -1$. $x = 3$ also solved (b) because $\mathrm{f}(3) = 3$: a fixed point of f is automatically a fixed point of f^2, and its image is itself

Challenge

Ch.. **(a)** $\mathrm{f}^2(x) = \dfrac{x}{2x+1}$; exclude $x = -1$ and $x = -\frac{1}{2}$

(b) $\mathrm{f}^3(x) = \dfrac{x}{3x+1}$; exclude $x = -1$, $x = -\frac{1}{2}$ and $x = -\frac{1}{3}$

(c) $\mathrm{f}^n(x) = \dfrac{x}{nx+1}$; exclude $x = -1, -\frac{1}{2}, -\frac{1}{3}, \ldots, -\frac{1}{n}$, that is $x \neq -\dfrac{1}{k}$ for $k = 1, 2, \ldots, n$

Full worked solutions available at the website.

Instagram: **@vectorspacepk**
Website: **www.vectorspace.pk**