Additional Math with Ali Hashir

Basic Functions

Answers

CIE O Level Additional Mathematics 4037 • Total: 123 marks

Section A — Easy (Q1–Q3)

Q1. **(a)** Function. Every vertical line meets the straight line exactly once, so each x has one image.

(b) Not a function. The curve is $x = y^2 - 1$; the vertical line $x = 0$ meets it at $y = 1$ and $y = -1$, so the single input 0 has *two* images (one-to-many).

(c) Function. Every vertical line meets the cubic exactly once. A *horizontal* line meets it three times, but that only means f is many-to-one, which is allowed

Q2. **(a)** $\{11, 5, 2, -7\}$ **(b)** $p = 6$

Q3. **(a)** $\mathrm{g}(0) = 3,\ \mathrm{g}(3) = 6,\ \mathrm{g}(6) = \frac{9}{2}$ **(b)** $x = 4$

Section A — Medium (Q4–Q8)

Q4. **(a)** $a = -3$, $b = 7$ **(b)** $\{10, 7, -5\}$ **(c)** $x = \frac{7}{4}$

Q5. **(a)** $a = 1$, $b = 2$, $c = -3$, so $\mathrm{f}(x) = x^2 + 2x - 3$ **(b)** 3 **(c)** $\{-5, 3\}$

Q6. **(a)** $a = 2$, $b = 1$, so $\mathrm{f}(x) = \dfrac{6}{2x+1}$ **(b)** $x = \frac{1}{4}$

(c) A fraction is zero only when its *numerator* is zero. Here the numerator is the constant 6, which is never zero, so $\mathrm{f}(x) = 0$ has no solution

Q7. **(a)** (i) $-4a - 1$ (ii) $5 + 2a - 2a^2$ (iii) $\dfrac{5a-2}{a}$ (iv) $\dfrac{3a-5}{a-1}$ **(b)** $a = 0$ or $a = 1$

Q8. $3x^2 - 11x - 4 = 0$, i.e. $(3x+1)(x-4) = 0$, so $x = -\frac{1}{3}$ or $x = 4$

Section A — Hard (Q9–Q10)

Q9. **(a)** $\mathrm{f}(x) = x$ reduces to $x^2 - 2x + 1 = 0$, i.e. $(x-1)^2 = 0$ — a *repeated* root, so there is exactly one such value: $x = 1$

(b) $3x^2 - 2x - 1 = 0$, i.e. $(3x+1)(x-1) = 0$, so $x = -\frac{1}{3}$ or $x = 1$. The value $x = 1$ reappears because $\mathrm{f}(1) = 1$ from part (a), and $\frac{1}{1} = 1$ as well — any fixed point equal to 1 or -1 must also satisfy $\mathrm{f}(x) = \frac{1}{x}$

Q10. $b = 1$ and $a = 1$ or $a = -1$. (The second root $b = -\frac{1}{2}$ is rejected: it forces $a^2 + 2 = -6$, which has no real solution)

Section B — Medium (Q11–Q13)

Q11. **(a)** $x \neq \frac{7}{2}$

(b) $x^2 - x - 6 \geqslant 0 \Rightarrow (x-3)(x+2) \geqslant 0$, so $x \leqslant -2$ or $x \geqslant 3$ (*not* $-2 \leqslant x \leqslant 3$)

(c) $x < 5$ (not $x \leqslant 5$ — the root is in the denominator) **(d)** $x \neq 3$ and $x \neq -3$

Q12. **(a)** $\mathrm{f}(x) = (x-3)^2 - 4$ **(b)** $-4 \leqslant \mathrm{f}(x) \leqslant 5$ **(c)** $1 \leqslant x \leqslant 6$

Q13. **(a)** $1 \leqslant x \leqslant 2$

(b) (i) $1 \leqslant \mathrm{g}(x) \leqslant 16$ (the vertex $x = 2$ is outside this domain, so the endpoints alone give the answer)

(ii) $0 \leqslant \mathrm{g}(x) \leqslant 9$ (the vertex $x = 2$ *is* inside this domain, so the minimum is 0, not $\mathrm{g}(3) = 1$)

Section B — Hard (Q14–Q15)

Q14. **(a)** $\mathrm{f}(x) = 2(x-2)^2 + 3$ **(b)** $\mathrm{f}(x) \geqslant 3$ **(c)** $k = 2 + 2\sqrt{2}$

Q15. **(a)** $2(x-3) + 7 = 2x + 1$, so $\dfrac{2x+1}{x-3} = 2 + \dfrac{7}{x-3}$

(b) $\mathrm{f}(x) = 2$ would require $\dfrac{7}{x-3} = 0$, and a fraction with a non-zero numerator is never zero

(c) $3 \leqslant \mathrm{f}(x) \leqslant 9$ **(d)** $3 < x \leqslant \dfrac{16}{3}$

Challenge 1

Ch. 1. **(a)** Replacing x by $-x$ gives $\mathrm{f}(-x) + 2\,\mathrm{f}(x) = -3x + 9$. Solving this simultaneously with $\mathrm{f}(x) + 2\,\mathrm{f}(-x) = 3x + 9$ gives $\mathrm{f}(x) = 3 - 3x$

(b) $3 - 3x = x \Rightarrow x = \frac{3}{4}$

Challenge 2

Ch. 2. **(a)** $(x-k)^2 + 3k = x^2 - 2kx + k^2 + 3k$ ✓

(b) $k > 0$: the vertex $x = k$ lies *inside* $x \geqslant 0$, so $\mathrm{f}(x) \geqslant 3k$

(c) $k < 0$: the vertex $x = k$ lies *outside* $x \geqslant 0$, so f is increasing throughout the domain and the least value is at the endpoint $x = 0$. Hence $\mathrm{f}(x) \geqslant k^2 + 3k$

(d) $k \geqslant 0$ or $k \leqslant -3$

Full worked solutions available at the website.

Instagram: **@vectorspacepk**
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