Additional Math with Ali Hashir

Absolute Value (Modulus) Functions

Answers

CIE O Level Additional Mathematics 4037 • Total: 150 marks

Easy (Q1–Q2)

Q1. **(a)** $f(-2) = 13$, $f(1) = 1$, $f(3) = 7$ **(b)** $x = 3$ or $x = -\frac{1}{2}$

(c) A modulus is a distance and can never be negative, so $|4x - 5| = -2$ is impossible

(d) A "V" with vertex $\left(\frac{5}{4}, 0\right)$ and y-intercept $(0, 5)$ **(e)** $f(x) \geqslant 0$

Q2. **(a)** $x = 5$ or $x = -\frac{7}{3}$ **(b)** $x = -7$ or $x = \frac{5}{3}$

(c) $x = \frac{5}{2}$. Only zero has a single number with that modulus, so $|A| = 0$ gives the one equation $A = 0$ rather than the usual pair $A = \pm k$

(d) $1 - x = -(x - 1)$, and a number and its negative have the same modulus, so $|1 - x| = |-(x - 1)| = |x - 1|$

Medium (Q3–Q7)

Q3. **(a)** $x = 4$ or $x = -2$ (both valid)

(b) $x = 5$ only; $x = \frac{11}{3}$ is rejected because it makes the right-hand side $2x - 8 = -\frac{2}{3} < 0$

(c) **No solutions**: the cases give $x = -1$ and $x = \frac{3}{2}$, and each makes $x - 4$ negative

(d) The line $y = x + 5$ cuts the "V" twice, at $(-2, 3)$ and $(4, 9)$ — two intersections, matching the two roots

Q4. **(a)** $x + 3 = x^2 + 1$ gives $x^2 - x - 2 = 0$, so $x = 2$ or $x = -1$; $-(x + 3) = x^2 + 1$ gives $x^2 + x + 4 = 0$, whose discriminant is -15, so no real roots. Hence $x = 2$ or $x = -1$

(b) Intersections at $(-1, 2)$ and $(2, 5)$ **(c)** $-1 < x < 2$

Q5. **(a)** Vertex $(3, 0)$, endpoints $(0, 6)$ and $(5, 4)$ **(b)** $0 \leqslant f(x) \leqslant 6$

(c) Not one-one: e.g. $f(2) = f(4) = 2$ **(d)** $k = 3$ **(e)** $x = 1$ or $x = 5$

Q6. **(a)** Vertex $(-2, -4)$; x-intercepts $\left(-\frac{10}{3}, 0\right)$ and $\left(-\frac{2}{3}, 0\right)$; y-intercept $(0, 2)$ **(b)** $f(x) \geqslant -4$

(c) $-5 \leqslant x \leqslant 1$ **(d)** $x < -4$ or $x > 0$

(e) (i) 0 (ii) 1 (iii) 2

Q7. **(a)** $x = \frac{3}{2}$, $y = \frac{1}{2}$ — one intersection only ($x = 1$ is rejected, as it lies on the wrong branch)

(b) $y = x - 4$ is parallel to the right branch $y = x - 2$ so never meets it; the left branch gives $x = 3$, which is outside $x < 2$. Hence no intersections

(c) $(6, 13)$ and $\left(-\frac{8}{3}, \frac{13}{3}\right)$ **(d)** $(2, 2)$ and $(-2, 2)$

Hard (Q8–Q10)

Q8. **(a)** $x^2 - 5x = 6$ gives $x = 6$ or $x = -1$; $x^2 - 5x = -6$ gives $x = 2$ or $x = 3$. Four values: $x = -1, 2, 3, 6$

(b) $x = 0, x = 4, x = 6$

(c) x-intercepts $(0, 0)$ and $(5, 0)$; highest point of the arch $\left(\frac{5}{2}, \frac{25}{4}\right)$

(d) (i) 4 (ii) 3 (iii) 2

Q9. **(a)** $|1 - a| = |9 - a|$ forces $a = 5$, and then $b = 3$ **(b)** Least value 3, at $x = 5$

(c) A "V" with vertex $(5, 3)$ passing through $(1, 7)$ and $(9, 7)$ **(d)** $x = 12$ or $x = -2$ **(e)** $k > 3$

Q10. **(a)** x-intercepts $(\pm 2, 0)$; y-intercept $(0, 4)$; endpoints $(\pm 3, 5)$ **(b)** $0 \leqslant \mathrm{f}(x) \leqslant 5$

(c) (i) 2 (ii) 4 (iii) 2

(d) $x = \pm 1$ and $x = \pm\sqrt{7}$ **(e)** $-\sqrt{7} < x < -1$ or $1 < x < \sqrt{7}$

Challenge 1

Ch. 1. **(a)** $x < -2$: $-2x - 1 = 5$ gives $x = -3$ ✓; $-2 \leqslant x < 1$: the left side is the constant $3 \neq 5$, no solutions; $x \geqslant 1$: $2x + 1 = 5$ gives $x = 2$ ✓. Hence $x = -3$ or $x = 2$

(b) For $-2 \leqslant x \leqslant 1$ the sum is $(1 - x) + (x + 2) = 3$, a constant. In words: $|x - 1|$ and $|x + 2|$ are the distances from x to 1 and to -2; when x lies between them the two distances always add up to the gap between the points, which is 3

(c) (i) $k < 3$: no solutions, as the sum is never less than 3; (ii) $k = 3$: infinitely many, namely every x in $-2 \leqslant x \leqslant 1$; (iii) $k > 3$: exactly two, one on each side

Challenge 2

Ch. 2. **(a)** $(2x - 1)^2 = (x + 4)^2$ gives $3x^2 - 12x - 15 = 0$, i.e. $x^2 - 4x - 5 = 0$, so $x = 5$ or $x = -1$

(b) $(x + 2)^2 = (x - 4)^2$ gives $12x = 12$, so $x = 1$. But then $|1 + 2| = 3$ while $1 - 4 = -3$, and $3 \neq -3$. Not a solution — in fact the equation has none

(c) $|A| = |B|$ and $A^2 = B^2$ are equivalent statements, so no information is gained or lost in (a). In (b) squaring also captures $|A| = -B$, an equation that is *not* the one asked. Whenever only one side is a modulus, every root must be substituted back, and a root is valid only if it makes the non-modulus side $\geqslant 0$

(d) $(3x - 1)^2 = (x + 5)^2$ gives $x^2 - 2x - 3 = 0$, so $x = 3$ or $x = -1$; and $|3x - 1| < |x + 5|$ requires $(x - 3)(x + 1) < 0$, giving $-1 < x < 3$

Full worked solutions available at the website.

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