Additional Math with Ali Hashir

Coordinate Geometry

Advanced Mixed-Topic Worksheet

O Level Additional Mathematics • Total: 150 marks

Answer **all** questions. Show full working – in this topic the method carries most of the marks. Leave answers as exact fractions or surds unless told otherwise. Every question may draw on **more than one** coordinate-geometry idea, so read each part carefully before you start. Questions are arranged in **order of increasing difficulty**, not by subtopic. Where a question asks you to *show that* or *prove* a result, you must justify every step – a final answer alone will not earn full marks. A quick sketch is often the fastest way to check that an answer is sensible.

Key Concepts Summary

Distance & Midpoint

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M = \left(\tfrac{x_1+x_2}{2}, \tfrac{y_1+y_2}{2}\right)$$

Gradient & Lines

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = m(x - x_1)$$

Parallel / Perpendicular

$m_1 = m_2$ (parallel) $\quad m_1 m_2 = -1$ (perp.)

Perpendicular Bisector
Passes through the midpoint of AB with gradient $-1/m_{AB}$. Every point on it is equidistant from A and B – this *is* the locus of points equidistant from A and B.

Line meets Curve
Solve simultaneously to get $ax^2 + bx + c = 0$. The discriminant $b^2 - 4ac$ is > 0 (2 points), $= 0$ (tangent), < 0 (no intersection).

Area (Shoelace Formula)

$$\text{Area} = \tfrac{1}{2}\left|\sum (x_i y_{i+1} - x_{i+1} y_i)\right|$$

Q1

Points $A(1, 2)$ and $B(5, 10)$ are given.

(a) Find the midpoint M of AB. [1]

(b) Find the gradient of AB. [1]

(c) Hence find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$. [4]

Q2

Points $A(-2, 1)$ and $B(4, 9)$ are given. The point $C(k, 5)$ is equidistant from A and B.

(a) Find the equation of the perpendicular bisector of AB. [4]

(b) Hence find the value of k. [3]

Q3

Given the points $A(-1, -2)$, $B(2, 4)$ and $C(5, 10)$.

(a) Show that A, B and C are collinear. [3]

(b) The point $D(7, k)$ is such that the area of triangle ABD is 15 square units. Find the possible value(s) of k. [5]

Q4

The line l_1 passes through $(1, 1)$ and $(5, 9)$. The line l_2 is parallel to l_1 and passes through $(0, 6)$.

(a) Find the equation of l_2. [3]

(b) Line l_2 meets the x-axis at P and the y-axis at Q. Find the area of triangle OPQ, where O is the origin. [3]

(c) The line $l_3 : y = -2x + 2$ intersects l_2 at the point R. Find the coordinates of R. [2]

Q5

$A(-1,-4)$ and $B(5,2)$ are the endpoints of a diameter of a circle.

(a) Find the centre and radius of the circle. [3]

(b) Find the equation of the tangent to the circle at A (recall that a tangent is perpendicular to the radius at the point of contact). [4]

(c) This tangent meets the y-axis at the point T. Find the coordinates of T. [2]

Q6

The line $y = 2x + c$ is a tangent to the curve $y = x^2 + 3x + 1$.

(a) Show that the x-coordinates of any points of intersection of the line and the curve satisfy $x^2 + x + (1 - c) = 0$. [2]

(b) Given that the line is a tangent to the curve, find the value of c. [3]

(c) Find the coordinates of the point of contact, and find the area of the triangle formed by this tangent line and the two coordinate axes. [4]

Q7

The quadrilateral $PQRS$ has vertices $P(1,1)$, $Q(4,5)$, $R(8,2)$ and $S(5,-2)$, taken in order.

(a) Find the length of each of the four sides of $PQRS$. [4]

(b) Show that $PQRS$ is a square. [3]

(c) Find the area of $PQRS$ using the shoelace formula, and verify that it agrees with the area found from the side length. [3]

Q8

A point $P(x,y)$ moves so that it is always equidistant from $A(-1,3)$ and $B(5,-1)$.

(a) Show that the locus of P has equation $3x - 2y = 4$. [4]

(b) This locus intersects the line $2x - y = 1$ at the point Q. Find the coordinates of Q. [4]

(c) Verify, by calculation, that Q is equidistant from A and B. [2]

Q9

Triangle ABC has vertices $A(-4,1)$, $B(0,5)$ and $C(6,-1)$.

(a) Use the scalar (dot) product of vectors to show that angle $ABC = 90°$. [4]

(b) M is the midpoint of AC. Find the coordinates of M, and show that $BM = \frac{1}{2}AC$. [4]

(c) Find the area of triangle ABC using the shoelace formula. [2]

Q10

Triangle ABC has vertices $A(1,6)$, $B(7,8)$ and $C(9,2)$.

(a) Find the equation of the perpendicular bisector of AB. [4]

(b) Find the equation of the perpendicular bisector of BC. [4]

(c) Hence find the coordinates of the circumcentre N of triangle ABC (the point equidistant from all three vertices), and verify that N is indeed equidistant from A, B and C. [3]

Q11

$ABCD$ is a quadrilateral, taken in order, with $A(-3,-2)$, $B(3,0)$ and $C(5,6)$, such that $ABCD$ is a parallelogram.

(a) Find the coordinates of D. [3]

(b) Show that the diagonals AC and BD bisect each other, and find the coordinates of their point of intersection. [4]

(c) Determine, showing your reasoning fully, whether $ABCD$ is in fact a rhombus. [4]

Q12

A curve C has equation $y = x^2 - 6x + 11$. A line l has equation $y = 2x + c$.

(a) Show that the x-coordinates of the points of intersection of l and C satisfy $x^2 - 8x + (11 - c) = 0$. [2]

(b) Given that l is a tangent to C, find the value of c and the coordinates of the point of contact. [4]

(c) For this tangent line, find where it crosses each coordinate axis, and hence find the area of the triangle enclosed by the tangent and the two axes. [4]

(d) State, with a reason, the set of values of c for which the line l intersects the curve C at two distinct points. [2]

Q13

$A(-5,3)$ and $B(3,-5)$ are the endpoints of a diameter of a circle.

(a) Find the centre and radius of the circle. [3]

(b) Find the equations of the tangents to the circle at A and at B. [5]

(c) Show that these two tangents are parallel, and find the perpendicular distance between them. Comment briefly on how this distance relates to the circle. [4]

Q14

The points $A(2, 1)$, $B(5, 5)$, $C(1, 8)$ and $D(-2, 4)$ are the vertices of a quadrilateral $ABCD$, taken in order.

(a) Show, by calculating all four side lengths and using the scalar product, that $ABCD$ is a square. [5]

(b) Find the equations of both diagonals, AC and BD. [4]

(c) Show that the diagonals intersect at the centre of the square, and verify that this point of intersection is equidistant from all four vertices. [4]

Q15

$A(1, 7)$ and $C(9, 1)$ are opposite vertices of a rhombus $ABCD$.

(a) Find the coordinates of M, the midpoint of AC (this is the centre of the rhombus). [1]

(b) Find the length of AC, and the equation of the diagonal BD (recall that the diagonals of a rhombus are perpendicular bisectors of each other). [4]

(c) Given that $BD = 20$, and that B has the greater y-coordinate, find the coordinates of B and of D. [4]

(d) Find the area of the rhombus using the formula Area $= \frac{1}{2}d_1d_2$, and verify your answer by applying the shoelace formula to A, B, C, D in order. [5]

Answers available at the website.

Instagram: **@vectorspacepk**
Website: **www.vectorspace.pk**