



Additional Math with Ali Hashir

Coordinate Geometry of the Circle

Worksheet 1

O Level Additional Mathematics 4037 • Topic 8.1–8.4

Summary. A circle with centre (a, b) and radius r can be written in *centre-radius form* or in *general form*:

$$(x - a)^2 + (y - b)^2 = r^2, \quad x^2 + y^2 + 2gx + 2fy + c = 0.$$

In the general form, the centre is $(-g, -f)$ and $r = \sqrt{g^2 + f^2 - c}$; to find these, complete the square on the x -terms and on the y -terms separately. Only the first form is given to you in the exam, so you must be able to move between the two.

Key facts and strategies.

- **Distance and midpoint.** For $P(x_1, y_1)$ and $Q(x_2, y_2)$, $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and the midpoint of PQ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- **Perpendicular gradients.** Two lines with gradients m_1 and m_2 are perpendicular exactly when $m_1 m_2 = -1$. The tangent to a circle at a point A is perpendicular to the radius drawn to A , so $m_{\text{tangent}} \times m_{\text{radius}} = -1$.
- **Circle through two points.** If AB is a *diameter*, the centre is the midpoint of AB and the radius is half of AB . If A and B merely lie *on* a circle, the centre lies on the perpendicular bisector of AB .
- **Line meets circle.** Substitute the line into the circle equation to get a quadratic. Two real roots means the line is a *chord*; a repeated root (discriminant = 0) means the line is a *tangent*; no real roots means the line *misses* the circle entirely. No calculus is needed for tangents.
- **Chord length.** If a line is a known perpendicular distance d from the centre of a circle of radius r , the half-chord has length $\sqrt{r^2 - d^2}$ (Pythagoras' theorem in the right triangle formed by the radius, the perpendicular, and the half-chord).
- **Two circles.** Subtracting one circle equation from the other eliminates the x^2 and y^2 terms and leaves the equation of the *common chord* (the line through the two points of intersection, if any).

Instructions. Answer all questions, showing full working. Leave answers as exact fractions or surds unless told otherwise. Questions are ordered roughly by difficulty, with two challenge questions at the end.

Q1

A circle has equation $(x - 2)^2 + (y + 5)^2 = 100$.

- (a) Write down the coordinates of the centre and the radius of the circle. [2]
- (b) Determine whether the point $(10, 1)$ lies inside the circle, on the circle, or outside the circle. [2]

Q2

A circle has equation $x^2 + y^2 - 6x + 10y + 18 = 0$. By completing the square, find the centre and the radius of the circle. [4]

Q3

A circle has centre $(3, -4)$ and passes through the point $(7, -1)$. Find the equation of the circle, giving your answer in the form $(x - a)^2 + (y - b)^2 = r^2$. [3]

Q4

The circle $(x - 5)^2 + (y + 2)^2 = 29$ meets the x -axis at the points P and Q . Find the coordinates of P and Q . [4]

Q5

The points $A(-3, 1)$ and $B(5, 7)$ are the endpoints of a diameter of a circle C . Find the equation of C . [4]

Q6

The point $A(9, 8)$ lies on the circle with centre $(1, 2)$ and radius 10.

- (a) Show that A lies on the circle. [2]
- (b) Find the equation of the tangent to the circle at A , giving your answer in the form $ax + by = c$. [4]

Q7

The circle with equation $x^2 + y^2 + 4x - 2y - 24 = 0$ meets the y -axis at the points $(0, m)$ and $(0, n)$. Find the values of m and n . [5]

Q8

The circle C has equation $(x - 2)^2 + (y - 3)^2 = 25$. The line l has equation $x + y = 10$.

(a) Show that l intersects C at two points. [2]

(b) Find the coordinates of these two points. [4]

Q9

A circle has equation $x^2 + y^2 - 4x + 6y - 12 = 0$. Determine, showing your method, whether the line with equation $3x + 4y = 19$ is a tangent to the circle, a chord of the circle, or does not intersect the circle. [4]

Q10

A circle with equation $(x - k)^2 + (y - 3k)^2 = 8$ passes through the point $(-1, 1)$.

(a) Find two possible values of k . [5]

(b) Given that $k > 0$, write down the equation of the circle. [1]

Q11

The line with equation $2x - y - 7 = 0$ does not intersect the circle with equation $x^2 + px + y^2 - 6y = 16$.

Show that $40 - 10\sqrt{15} < p < 40 + 10\sqrt{15}$. [6]

Q12

The circle C has equation $(x - 2)^2 + (y - 1)^2 = 20$. The lines l_1 and l_2 are each a tangent to the circle and intersect at the point $(2, 11)$.

- (a) Find the equations of l_1 and l_2 , giving your answers in the form $y = mx + c$. [7]
- (b) Find the coordinates of the points P and Q where l_1 and l_2 touch the circle, and hence find the length PQ . [4]

Q13

The circle C has equation $(x-1)^2 + (y+2)^2 = 169$. The line l has equation $5x + 12y - 46 = 0$. The line l intersects C at the points P and Q .

- (a) Find the perpendicular distance from the centre of C to l . [3]
- (b) Hence find the length of PQ . [3]

Q14

The circle C_1 has equation $x^2 + y^2 + 2y - 33 = 0$. The circle C_2 has equation $(x-6)^2 + (y-5)^2 = 10$. The circles intersect at the points P and Q .

- (a) Find the equation of the common chord PQ , giving your answer in the form $ax + by + c = 0$. [3]
- (b) Find the coordinates of P and Q . [5]

Q15

A circle has radius 6 and passes through the points $A(1, 7)$ and $B(7, 1)$. Find the two possible equations of the circle, giving each answer in the form $(x - a)^2 + (y - b)^2 = r^2$. [6]

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Challenge 1

The points $A(1, 8)$ and $B(9, 2)$ lie on a circle with centre C . M is the midpoint of AB , and the line l passes through M and C .

(a) Find an equation for l , giving your answer in the form $ax + by + c = 0$. [4]

Given that the x -coordinate of C is 8:

(b) find an equation of the circle, [4]

(c) find the area of triangle ABC . [3]

Challenge 2

The circle with equation $(x - 1)^2 + (y - 7)^2 = 50$, with centre A , intersects the circle with equation $(x - 10)^2 + (y + 5)^2 = 125$, with centre B , at the points P and Q .

- (a) Find the equation of the line containing P and Q , giving your answer in the form $ax + by + c = 0$. [4]
- (b) Find the coordinates of P and Q . [4]
- (c) Find the area of the kite $APBQ$. [4]

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Answers and full worked solutions available separately.

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