



Additional Math with Ali Hashir

Circular Measure

Worksheet 1

O Level Additional Mathematics 4037 • Topic 9.1

Summary. A radian is the angle subtended at the centre of a circle by an arc equal in length to the radius. Once an angle is in radians, the arc length, sector area and segment area of a circle can all be found directly from the radius and the angle, without first converting to degrees. Chords, compound shapes and shaded regions are handled by breaking the figure into sectors, triangles and segments and combining (or subtracting) their areas.

$$180^\circ = \pi \text{ radians}, \quad 1 \text{ radian} = \frac{180^\circ}{\pi} \approx 57.3^\circ$$

$$\text{arc length } s = r\theta, \quad \text{sector area} = \frac{1}{2}r^2\theta, \quad \text{segment area} = \frac{1}{2}r^2(\theta - \sin \theta)$$

$$\text{chord } AB = 2r \sin\left(\frac{\theta}{2}\right), \quad \text{perimeter of sector} = 2r + r\theta, \quad \text{perimeter of segment} = r\theta + AB$$

(θ is always in **radians** in every formula above.)

Strategy.

- Convert every angle into radians before using any of the formulas above; keep exact values (multiples of π) wherever the question does not ask for a decimal.
- A segment is what is left of a sector once the triangle OAB is cut away, so segment area = sector area – triangle area.
- For a shape made of straight edges and arcs, the perimeter is the sum of the straight lengths and the arc lengths — never guess an arc length from a ruler; always compute it from $r\theta$.
- For a shaded/compound region, identify a sector, triangle or segment that can be added or subtracted to leave exactly the shaded area, then write this as a single equation before calculating.
- Sketch every diagram given (or re-draw the one provided) and mark on it every length and angle you find as you go — this catches sign and labelling errors before they cost you the whole question.

Instructions. Answer every question. Unless told otherwise, give non-exact numerical answers correct to 3 significant figures, and angles in radians correct to 3 decimal places. Diagrams are not drawn to scale.

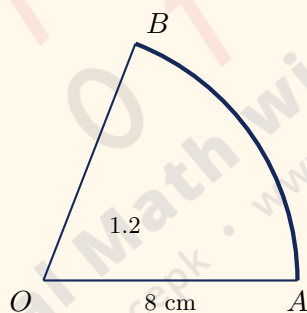
Q1

- (a) Convert 108° into radians, giving your answer as an exact multiple of π . [1]
- (b) Convert $\frac{7\pi}{9}$ radians into degrees. [1]
- (c) Convert 2.4 radians into degrees, correct to 1 decimal place. [1]
- (d) Convert 50° into radians, correct to 3 significant figures. [1]

Q2

A sector of a circle has radius 8 cm and angle 1.2 radians.

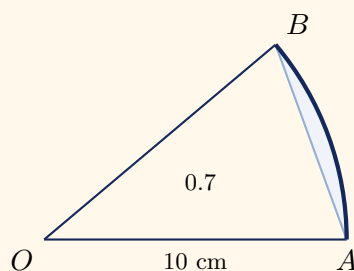
- (a) Find the length of the arc. [2]
- (b) Find the perimeter of the sector. [2]
- (c) A different sector of the *same* circle (radius 8 cm) has an arc length of 10 cm. Find the angle of this sector, in radians. [2]



Q3

A sector OAB of a circle, centre O , has radius 10 cm and angle $AOB = 0.7$ radians.

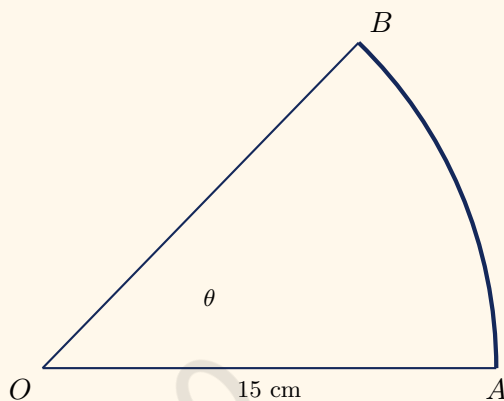
- Find the area of the sector OAB . [2]
- Find the area of triangle OAB . [2]
- Hence find the area of the segment cut off by the chord AB (shaded). [2]



Q4

A sector of a circle has radius 15 cm and perimeter 42 cm.

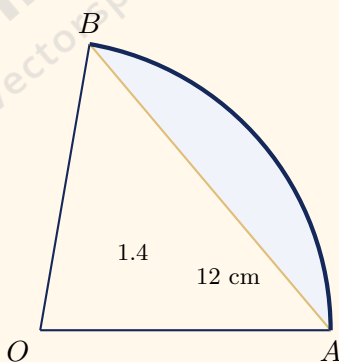
- (a) Find the length of the arc of the sector. [1]
- (b) Find the angle of the sector, in radians. [2]
- (c) Find the area of the sector. [2]



Q5

The diagram shows a sector OAB of a circle, centre O , radius 12 cm, with angle $AOB = 1.4$ radians.

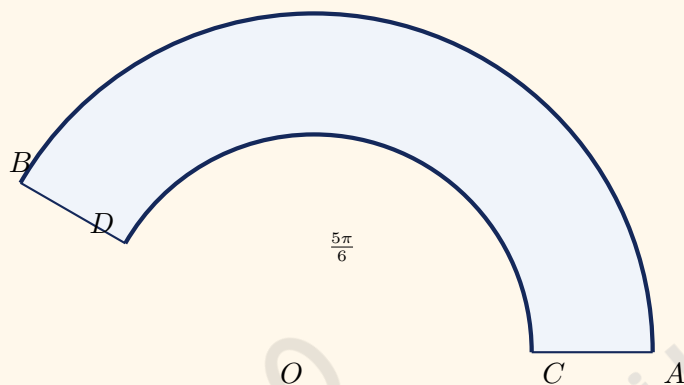
- (a) Calculate the length of the chord AB . [3]
- (b) Calculate the area of the segment cut off by the chord AB (shaded). [3]



Q6

The diagram shows two concentric arcs AB and CD , centre O , with radii 14 cm and 9 cm respectively. Angle $AOB = \frac{5\pi}{6}$.

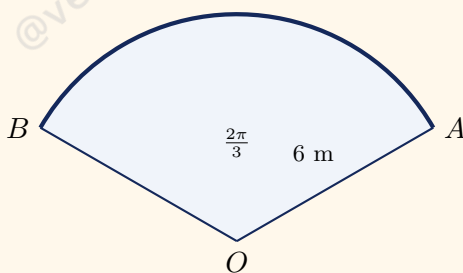
- (a) Calculate the perimeter of the shaded region $ABDC$. [4]
- (b) Calculate the exact area of the shaded region $ABDC$, giving your answer in the form $k\pi$. [3]



Q7

A garden sprinkler is fixed at a point O . It sprays water in a straight jet of length 6 m, and rotates back and forth so that the jet sweeps out an angle of $\frac{2\pi}{3}$ radians.

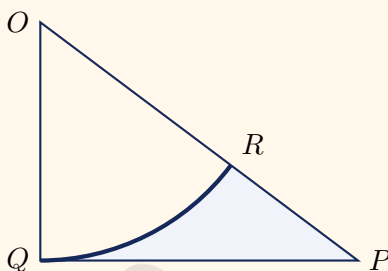
- (a) Find, in exact form, the length of the arc traced out by the end of the jet in one sweep. [2]
- (b) Find, in exact form, the area of the garden watered in one sweep. [3]



Q8

In the diagram, OQP is a right-angled triangle, right-angled at Q , with $OQ = 9$ cm and $QP = 12$ cm. The arc QR , centre O and radius 9 cm, meets OP at R .

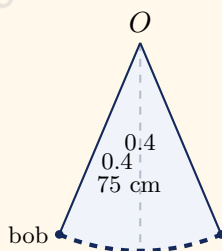
- Show that $OP = 15$ cm. [1]
- Find the angle POQ , in radians, correct to 3 decimal places. [2]
- Find the perimeter of the shaded region QRP . [2]
- Find the area of the shaded region QRP . [3]



Q9

A pendulum of length 75 cm hangs from a fixed point O . It swings so that the string makes an angle of 0.4 radians with the vertical on each side of the vertical.

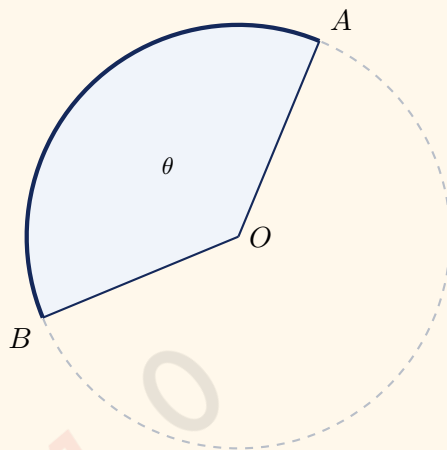
- Find the total distance travelled by the bob along its arc in one complete swing (from one extreme position to the other and back again). [2]
- Find the area swept out by the string in travelling from one extreme position to the other (one direction only). [2]



Q10

The angle of a sector is such that the area of the sector is $\frac{3}{8}$ of the area of the whole circle. The radius of the circle is 20 cm.

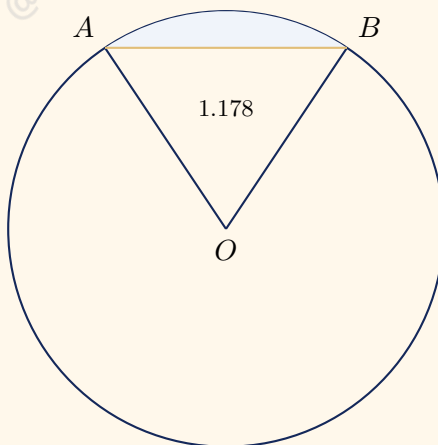
- (a) Find the angle of the sector, in radians, as an exact multiple of π . [3]
- (b) Find the perimeter of the sector. [3]



Q11

The cross-section of a uniform cylindrical rod has centre O and radius 18 cm. Points A and B lie on the circumference, and the chord $AB = 20$ cm.

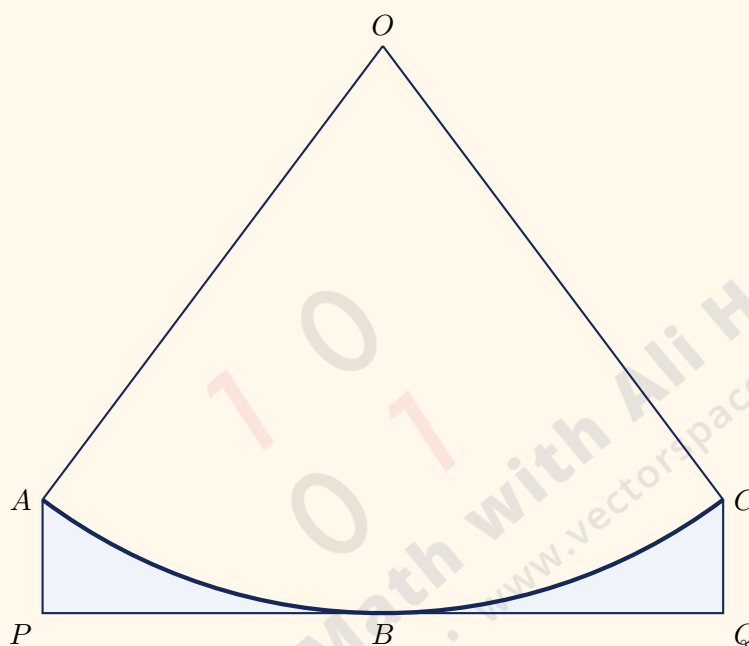
- (a) Show that angle $AOB = 1.178$ radians, correct to 3 decimal places. [3]
- (b) Find the perimeter of the minor segment cut off by the chord AB . [2]
- (c) Find the area of the minor segment cut off by the chord AB (shaded). [3]



Q12

ABC is the segment of a circle, centre O . The segment is enclosed in a rectangle $APQC$, where B is the point at which the arc touches PQ . Given that $AC = 30$ cm and $AP = 5$ cm, calculate

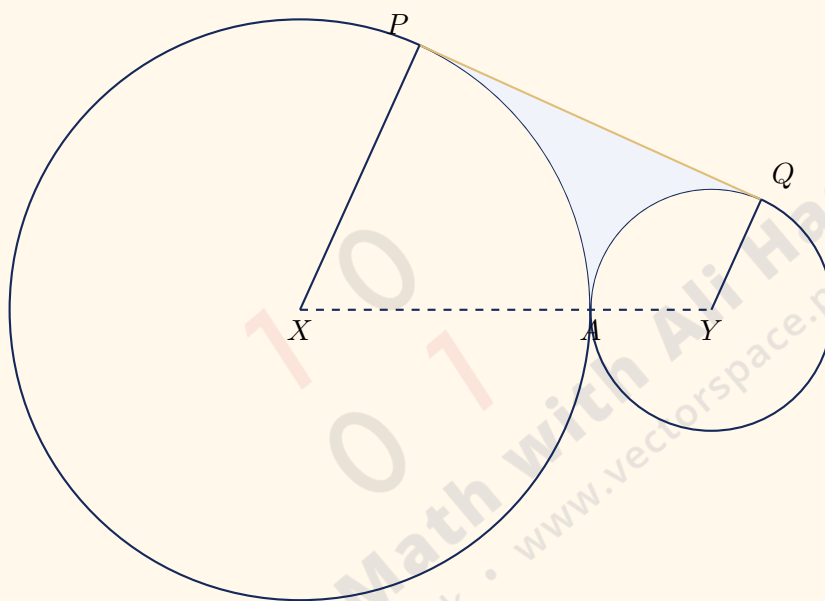
- the radius of the circle, [3]
- the angle AOC , in radians, correct to 3 decimal places, [3]
- the area of the shaded region (inside the rectangle but outside the segment). [4]



Q13

The diagram shows two circles, C_1 and C_2 , touching at A . Circle C_1 has radius 12 cm and centre X ; circle C_2 has radius 5 cm and centre Y . A common tangent touches the circles at P and Q respectively.

- Show that the length of PQ is 15.5 cm, correct to 3 significant figures. [3]
- Show that angle $PXY = 1.146$ radians, correct to 3 decimal places. [2]
- Find the length of the minor arc AP of circle C_1 and the minor arc AQ of circle C_2 . [3]
- Find the area of the shaded region APQ . [3]

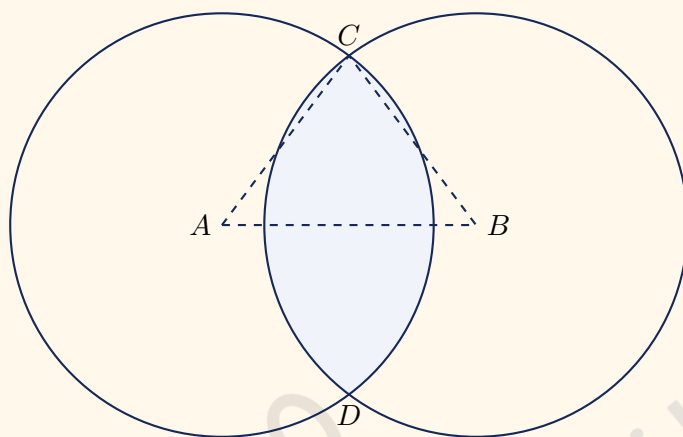


Q14

Two circles of equal radius 10 cm have centres A and B , where $AB = 12$ cm. The circles intersect at C and D .

(a) Show that angle $CAB = 0.927$ radians, correct to 3 decimal places. [3]

(b) Find the area common to both circles (shaded). [4]



Q15

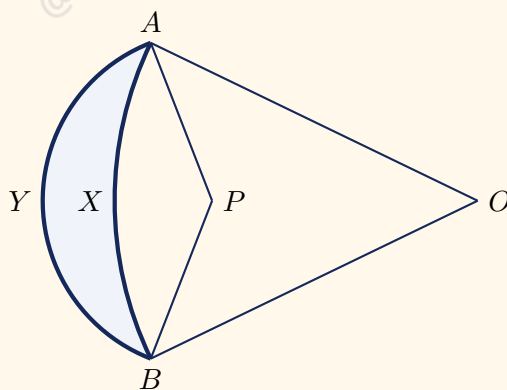
In the diagram, AXB is an arc of a circle centre O and radius 15 cm, with angle $AOB = 0.9$ radians. AYB is an arc of a circle centre P and radius 7 cm, with angle $APB = \varphi$.

(a) Calculate the length of the chord AB . [2]

(b) Calculate the value of φ , in radians, correct to 3 decimal places. [3]

(c) Calculate the difference in length between the arcs AYB and AXB . [2]

(d) Calculate the area of the shaded region bounded by the two arcs. [3]



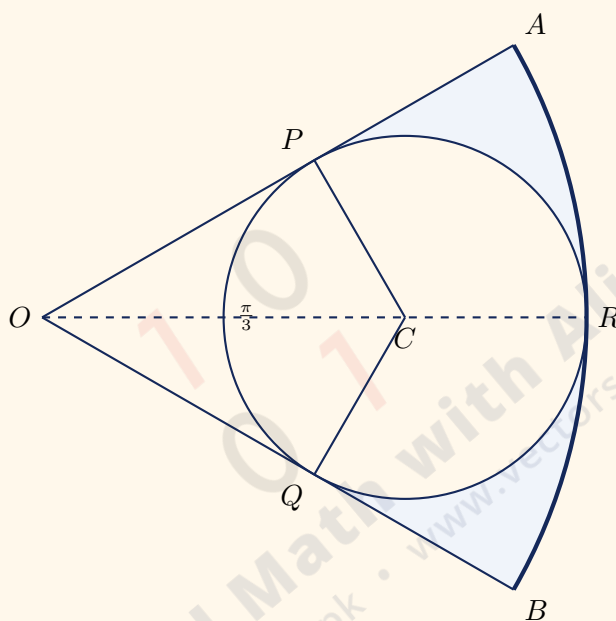
Q16 (challenge)

In the figure, OAB is a sector of a circle with centre O and radius 24 cm, with angle $AOB = \frac{\pi}{3}$. The circle PQR , with centre C and radius r cm, is inscribed in the sector so that it touches OA at P , touches OB at Q , and touches the arc AB at R . Show that $r = 8$ and that $OQ = 8\sqrt{3}$ cm. Calculate, in exact form,

(a) the area of the sector OAB , [2]

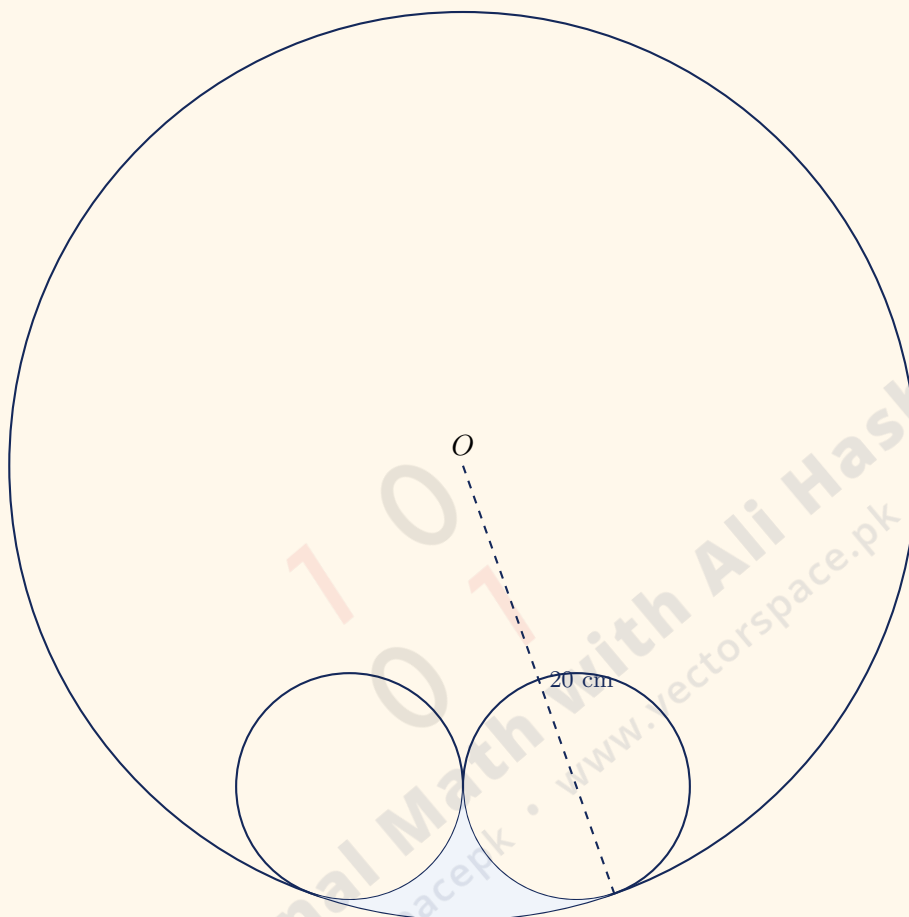
(b) the area of the quadrilateral $OQCP$. [3]

Hence show that the area of the shaded region is $\frac{32}{3} (5\pi - 6\sqrt{3})$ cm². [5]



Q17 (challenge)

Two circles of equal size, each of radius 5 cm, touch each other externally. Both of them touch another circle, centre O and radius 20 cm, internally, as shown in the diagram. Calculate the area of the shaded region. [9]



Answers and full worked solutions available separately.

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