Further Math with Ali Hashir

Inference Using Normal and t-Distributions

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/21, 9231/22 • May/June 2015 • Question 6

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Mark scheme.

Equate pooled estimate of σ^2 to 12: $(11-5^2/N+160-10^2/10)/(N+10-2) = 12$.	**M1 A1**
Formulate and solve relevant quadratic equation for N: $12N^2 - 65N + 25 = 0$, $N = 5$.	**M1 A1**
	Total: 4

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Q2

9231/21, 9231/22 • May/June 2015 • Question 7

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Mark scheme.

Find Σx via sample mean $\bar{x}$: $\Sigma x = 8\bar{x} = 8 \times \frac{1}{2}(1.17 + 2.03) = 8 \times 1.6 = 12.8$.	**M1 A1**
Find estimate of population variance s^2: $t\sqrt{s^2/8} = \frac{1}{2}(2.03 - 1.17)$ $[= 0.43]$.	**M1**
Use of correct tabular value (1.96 leads to 23.2): $t_{7,0.975} = 2.36[5]$.	**A1**
(to 3 d.p.) $s^2 = 0.2645$ or $32/121$ or 0.5143^2.	**A1**
Find Σx^2 from s^2: $s^2 = \{\Sigma x^2 - (\Sigma x)^2/8\}/7$ (M0 for $s^2 = \{\ldots\}/8$), $\Sigma x^2 = 7 \times 0.2645 + 12.8^2/8 = 22.3$.	**M1 A1**
	Total: 7

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Q3

9231/21, 9231/22 • May/June 2015 • Question 10

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Mark scheme.

Calculate gradient b_1 in $y - \bar{y} = b_1(x - \bar{x})$: $S_{xy} = 313.28 - 50.8 \times 56.9/10 = 24.228$, $S_{xx} = 284.16 - 50.8^2/10 = 26.096$, $b_1 = S_{xy}/S_{xx} = 0.928$ (PA−1 so max 4/5 for 0.93, giving $y = 7.48$).	**M1 A1**
Find y when $x = 7$ from regression line of y on x: $y = 56.9/10 + b_1(7 - 50.8/10) = 5.69 + 0.928(7 - 5.08)$.	**M1 A1**
$y = 7.47$ (allow 7.48 or 7.5).	**A1**
Find differences (e.g. $y - x$) and sample mean: 1, 0.7, 1.3, 0.2, 0.1, 0.9, 0.8, 0.5, 0.1, 0.5; $\bar{d} = 6.1/10 = 0.61$.	**M1 A1**
Estimate population variance (to 3 s.f.): $s^2 = (5.19 - 6.1^2/10)/9 = 0.1632$ or 0.404^2 (allow biased here: 0.1469 or 0.3833^2).	**B1**
State hypotheses (AEF; B0 for $\bar{x}$), e.g. $H_0 : \mu_y - \mu_x = 0.4$, $H_1 : \mu_y - \mu_x > 0.4$.	**B1**
Calculate value of t: $t = (\bar{d} - 0.4)/(s/\sqrt{10}) = 1.64$.	**M1 A1**
State or use correct tabular t-value: $t_{9,0.95} = 1.83[3]$ (or can compare $\bar{d}$ with 0.634).	**B1**
Consistent conclusion (AEF, ft on both t-values): accept H_0, no improvement of more than 0.4. (Wrong test can earn only B1 for hypotheses and B1 for conclusion.)	**B1**
	Total: 13

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Q4

9231/23 • May/June 2015 • Question 8

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Mark scheme.

Calculate sample mean: $\bar{x} = 45.29/7 = 6.47$.	**M1**
Estimate population variance: $s_1^2 = (295.36 - 45.29^2/7)/6 = 0.389$ or 0.6236^2 (allow biased here: 0.3334 or 0.5774^2).	**M1**
State hypotheses (AEF; B0 for $\bar{x}$): $H_0 : \mu = 6.2$, $H_1 : \mu > 6.2$.	**B1**
Calculate value of t (or $-t$; to 3 s.f.): $t = (\bar{x} - 6.2)/(s_1/\sqrt{7}) = 1.15$.	**M1 A1**
State or use correct tabular t-value (to 3 s.f.): $t_{6,0.95} = 1.94[3]$ (or compare $\bar{x}$ with $6.2 + 0.458 = 6.66$).	**B1**
Consistent conclusion (AEF, ft on both t-values): accept H_0, population mean not greater than 6.2.	**B1**
Find estimate of population mean: $\frac{1}{2}(5.89 + 6.75) = 6.32$.	**M1 A1**
Find estimate of population variance s_2^2: $t\sqrt{s_2^2/8} = \frac{1}{2}(6.75 - 5.89)$ [$= 0.43$] (SR allow M1 for $t\sqrt{s_2^2/7}$, max 4/5).	**M1**
Use of correct tabular value: $t_{7,0.975} = 2.36[5]$ (use of 2.36 may lose final A1); [$s_2 = 0.5143$ or $4\sqrt{2}/11$].	**A1**
(to 3 d.p.) $s_2^2 = 0.2645$ or $32/121$.	**A1**
	Total: 12

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Q5

9231/21, 9231/22, 9231/23 • October/November 2015 • Question 5

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Mark scheme.

Find sample mean $\bar{x} = 222.8/10 = 22.28$ and estimate population variance $s^2 = 4.12/9 = 0.458$.	**M1**
Confidence interval: $22.28 \pm t\sqrt{0.458/10}$.	**M1A1**
Use correct tabular value $t_{9,0.975} = 2.262$.	**A1**
Evaluate: 22.3 ± 0.484 or $[21.8, 22.8]$.	**A1**
	Total: 5

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Q6

9231/21, 9231/22 • May/June 2016 • Question 7

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Mark scheme.

Calculate sample mean: $\bar{x} = 24.6/9 = 2.733$	**B1**
Estimate population variance: $s^2 = (68.5 - 24.6^2/9)/8 = 63/400$ or 0.1575 or 0.3969^2	**M1 B1**
State hypotheses: $H_0 : \mu = 2.5$, $H_1 : \mu > 2.5$	**M1 A1**
Calculate value of t: $t = (\bar{x} - 2.5)/(s/\sqrt{9}) = 1.76$	**B1**
State or use correct tabular t-value: $t_{8,0.95} = 1.860$	**M1**
Conclusion: $1.76 < 1.86$ so population mean not greater than 2.5	**A1**
	Total: 8

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Q7

9231/23 • May/June 2016 • Question 8

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Mark scheme.

State assumption: differences are normally distributed	**B1**
State hypotheses: $H_0 : \mu_{Before} - \mu_{After} = 0$, $H_1 : \mu_{Before} - \mu_{After} > 0$	**B1**
Consider differences (Before − After): 0.7, 3.0, −0.6, 0.2, 0.3, 0.9, 0, 1.5	**M1**
Calculate sample mean $\bar{d} = 6/8 = 0.75$ and estimate population variance: $s^2 = (13.04 - 6^2/8)/7 = 61/50$ or 1.22 or 1.105^2 (allow biased here: 1.0675 or 1.033^2)	**M1**
Calculate value of t: $t = \bar{d}/(s/\sqrt{8}) = 1.92$	**M1 A1**
State or use correct tabular t-value: $t_{7,0.975} = 2.365$	**B1**
Consistent conclusion: accept H_0: no improvement in athletes' times	**B1**
	Total: 8

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Q8

9231/21, 9231/22, 9231/23 • October/November 2016 • Question 6

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Mark scheme.

Estimate population variance: $s^2 = 0.784/7 = 0.112$ or $14/125$ or 0.3347^2 (allow biased here: 0.098 or 0.3130^2)	**M1**
State hypotheses (AEF; B0 for $\bar{x}$): $H_0 : \mu = 6.44$, $H_1 : \mu < 6.44$	**B1**
Calculate value of t (either sign; to 3 s.f.): $t = (6.44 - \bar{x})/(s/\sqrt{8}) = 1.64$	**M1 A1**
State or use correct tabular t-value (to 3 s.f.): $t_{7,0.95} = 1.895$ (or can compare $\bar{x}$ with $6.4 - 0.224 = 6.176$)	**B1**
State or imply valid method for conclusion: accept H_0 if $t <$ tabular value	**M1**
Conclusion (AEF, requires both values correct): $1.64 < 1.895$ so population mean not less than 6.44	**A1**
	Total: 7

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Q9

9231/21, 9231/22, 9231/23 • October/November 2016 • Question 8

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Mark scheme.

EITHER:	**(i)**
Estimate P's population variance (to 3 d.p.): $s_P^2 = (1\,560\,000 - 9600^2/60)/59$ [= 406.78] (allow biased here: 400)	**M1**
Estimate Q's population variance (to 3 d.p.): $s_Q^2 = (1\,052\,500 - 7200^2/50)/49$ [= 320.41] (allow biased here: 314)	**M1**
Find pooled estimate of common variance: $s^2 = (59s_P^2 + 49s_Q^2)/108 = 367.6$ or 368 or $9925/27$	**M1 A1**
OR: (M3 A1) Find pooled estimate directly: $s^2 = (1\,560\,000 - 9600^2/60 + 1\,052\,500 - 7200^2/50)/108 = 367.6$ or 368 or $9925/27$	
(ii)	
Find confidence interval for the difference: $9600/60 - 7200/50$ [$= 160 - 144$] $\pm zs\sqrt{(60^{-1} + 50^{-1})}$	**M1 A1**
Use appropriate tabular value (to 2 d.p.): $z_{0.975} = 1.96$ or $t_{120,0.975} = 1.98$	**A1**
Evaluate confidence interval (AEF, to 1 d.p.): 16 ± 7.2 or $[8.8, 23.2]$ or 16 ± 7.3 or $[8.7, 23.3]$	**M1 A1**
	Total: 9

Q10

9231/21, 9231/22 • May/June 2017 • Question 7

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Mark scheme.

Find sample mean: $\bar{x} = 7.2$.	**B1**
Estimate population variance (allow biased here: 2.36 or 1.536^2): $s^2 = (542 - 72^2/10)/9$ [= 118/45 or 2.622 or 1.619^2].	**M1**
State hypotheses (B0 for $\bar{x}\ldots$): $H_0 : \mu = 6.2$, $H_1 : \mu > 6.2$ (AEF).	**B1**
State or use correct tabular t-value: $t_{9,0.95} = 1.833$.	**B1**
Find value of t (or compare $\bar{x}$ with $6.2+0.939 = 7.14$): $t = (\bar{x}-6.2)/(s/\sqrt{10}) = 1.95$. Consistent conclusion.	**M1,A1**
Claim (of mean mass increased) is justified (AEF) (FT on both t-values).	**B1,FT**
	Total: 7

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Q11

9231/21, 9231/22 • May/June 2017 • Question 9

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Mark scheme.

State hypotheses (B0 for $\bar{x}\ldots$): $H_0 : \mu_X = \mu_Y$, $H_1 : \mu_X \neq \mu_Y$ (AEF).	**B1**
Find sample means (values to 3 s.f. throughout): $\bar{x} = 13.4/8 = 1.67[5]$, $\bar{y} = 2.02$.	**B1**
Estimate or imply population variances (allow biased: 0.2819 or 0.5309^2; 0.3796 or 0.6161^2): $s_X^2 = (24.7 - 13.4^2/8)/7 = 451/1400 = 0.3221 = 0.5678^2$; $s_Y^2 = (44.6 - 20.2^2/10)/9 = 949/2250 = 0.4218 = 0.6494^2$.	**M1**
Estimate (pooled) common variance (note s_X^2, s_Y^2 not needed explicitly): $s^2 = (7s_X^2 + 9s_Y^2)/16$ or $(24.7 - 13.4^2/8 + 44.6 - 20.2^2/10)/16$.	**M1,A1**
$= 6051/16000 = 0.3782 = 0.6150^2$.	**A1**
State or use correct tabular t value: $t_{16,0.95} = 1.746$.	***B1**
Find value of t (or compare $\bar{y} - \bar{x} = 0.345$ with 0.509): $[-]t = (\bar{y} - \bar{x})/(s\sqrt{1/8 + 1/10}) = 1.18$.	**M1,A1**
Correct conclusion (FT on t, dep *B1): $t < 1.75$ so mean masses are the same (AEF).	**DB1,FT**
Implicitly taking s_X^2, s_Y^2 as unequal population variances: $Z = (\bar{y} - \bar{x})/\sqrt{(s_X^2/8 + s_Y^2/10)} = 0.345/\sqrt{0.078} = 1.20$ (may also earn first B1 B1 M1).	**Special ruling: (B1)**
$Z < 1.645$ so mean masses are the same (AEF) (comparison with $Z_{0.95}$ and conclusion, FT on Z; can earn at most 5/10 via this route).	**Special ruling: (B1),FT**
	Total: 10

Q12

9231/23 • May/June 2017 • Question 6

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Mark scheme.

State or find expression for pooled estimate of σ^2 (confusing biased/unbiased estimates may still earn M1): $(10-4^2/N+102-8^2/2N)/(N+2N-2)$ (AEF).	**M1,A1**
Equate to 10 and rearrange as quadratic: $112-48/N=10(3N-2)$, $15N^2-66N+24=0$.	**M1,A1**
Solve quadratic for N, rejecting root 0.4: $N=(66\pm54)/30=4$.	**A1**
	Total: 5

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Q13

9231/23 • May/June 2017 • Question 8

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Mark scheme.

Find both sample means: $\bar{x} = 28.32$, $\bar{y} = 22.2$.	**B1**
Estimate both population variances.	**M1**
$s_A^2 = 20.39$, $s_B^2 = 164/15$ or 10.93 (to 3 s.f.) (allow biased: 19.98, 10.66).	**A1**
EITHER: estimate combined variance $s^2 = s_A^2/50 + s_B^2/40 = 0.681$ or 0.825^2 (if biased values used wrongly, giving $zs = 1.60$, only this A1 lost).	**M1,A1**
Find confidence interval for difference: $\bar{x} - \bar{y} \pm zs$.	**M1**
Use appropriate tabular value: $z_{0.975} = 1.96$.	**A1**
Evaluate semi-interval length: $zs = 1.62$.	**A1**
State confidence interval: 6.12 ± 1.62 or $[4.50, 7.74]$.	**A1**
OR: state assumption of equal population variances and find pooled estimate $s^2 = (49s_A^2 + 39s_B^2)/88 = 16.2$ or 4.02^2.	**B1**
Find CI for difference: $\bar{x} - \bar{y} \pm zs\sqrt{(1/50 + 1/40)}$.	**M1**
$z_{0.975} = 1.96$ (or appropriate t-value).	**A1**
Evaluate semi-interval length: ($t_{88,0.975} = 1.9873$ or 1.99) 1.67 (1.70).	**A1**
State CI: 6.12 ± 1.67 or $[4.45, 7.79]$ (6.12 ± 1.70 or $[4.42, 7.82]$).	**A1**
	Total: 9

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Q14

9231/21, 9231/22, 9231/23 • October/November 2017 • Question 10

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Mark scheme.

(i)	
Find both sample means: $\bar{x} = 0.91$, $\bar{y} = 1.205$.	**B1**
Estimate both population variances: $s_X^2 = 19.56/49 = 0.3992$, $s_Y^2 = 30.25/59 = 0.5127$.	**M1**
Estimate combined variance: $s^2 = s_X^2/50 + s_Y^2/60 = 0.01653$.	**M1 A1**
Find confidence interval for difference $Y - X$: $(\bar{y} - \bar{x}) \pm zs$.	**M1**
Use appropriate tabular value $z_{0.95} = 1.645$; evaluate: 0.295 ± 0.211.	**A1**
Confidence interval: $[0.084,\ 0.506]$.	**A1**
(ii)	
Find value of z: $z = (1.205 - 0.91)/s = 0.295/s = 2.294$.	**M1, A1**
Find $\Phi(z) = 0.9891$.	**A1**
Find limiting value for α, based on two-tail test: $100 \times (1 - 0.989) \times 2 = 2.2$.	**M1, A1**
Find set of possible values of α: $\alpha < 2.2$.	**A1**
	Total: 13

Q15

9231/21, 9231/22 • May/June 2018 • Question 7

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Mark scheme.

Find sample mean $\bar{x} = 681.6/7 = 97.37$ (allow 97.4)	**B1**
Estimate population variance $s^2 = (66\,536.1 - 681.6^2/7)/6 = 27.96$ (allow biased 23.96)	**M1**
State hypotheses: $H_0 : \mu = 94$, $H_1 : \mu \neq 94$	**B1**
Find value of t: $t = (\bar{x} - 94)/(s/\sqrt{7}) = 1.69$	**M1 A1**
State or use correct tabular t-value: $t_{7,0.95} = 1.943$	**B1**
Consistent conclusion (FT): accept H_0, mean mass is equal to 94 kg	**B1**
	Total: 7

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Q16

9231/21, 9231/22 • May/June 2018 • Question 10

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Mark scheme.

State hypotheses: $H_0 : \mu_P = \mu_Q$, $H_1 : \mu_P > \mu_Q$	**B1**
Estimate both population variances: $s_P^2 = (138\,200 - 2620^2/50)/49 = 18.61$; $s_Q^2 = (157\,000 - 3060^2/60)/59 = 15.93$ (allow biased 18.24, 15.67)	**M1 A1**
Estimate combined variance: $s^2 = s_P^2/50 + s_Q^2/60 = 0.6378$	**M1 A1**
State/use correct tabular value: $z_{0.9} = 1.282$	**B1**
Calculate $z = (\bar{y} - \bar{x})/s = (52.4 - 51)/s = 1.75$; $z >$ tabular value	**M1 A1**
Conclusion: college P students take more time (FT) (ii)	**B1**
Find confidence interval: $\bar{x} - \bar{y} \pm zs$	**M1**
Use appropriate tabular value: $z_{0.95} = 1.645$	**A1**
Evaluate: 1.4 ± 1.31, i.e. $[0.09, 2.71]$	**A1**
	Total: 12

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Q17

9231/23 • May/June 2018 • Question 10

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Mark scheme.

State hypotheses: $H_0 : \mu_x - \mu_y = 1$, $H_1 : \mu_x - \mu_y > 1$	**B1**
Consider differences $d_i = x_i - y_i$: 1.1, 2.1, 0.7, 0.4, 2.0, 2.8, 0.2, 1.9, 1.4	**M1**
Find sample mean $\bar{d} = 12.6/9 = 1.4$	**B1**
Estimate population variance $s^2 = (23.72 - 12.6^2/9)/8 = 0.76$ (allow biased 0.6756)	**M1**
State or use correct tabular t-value: $t_{8,0.9} = 1.397$	**B1**
Find $t = (\bar{d}-1)/(s/\sqrt{9}) = 1.38$ (or compare $\bar{d}-1 = 0.4$ with $t_{8,0.9}s/\sqrt{9} = 0.406$); accept H_0: no evidence for the coach's belief	**M1 A1**
Consistent conclusion (FT): time will not decrease by more than 1 second (ii)	**B1**
Estimate population variance using y's sample: $s_y^2 = (11\,395.79 - 319.5^2/9)/8 = 6.693$	**B1**
Find confidence interval: $319.5/9 \pm t\sqrt{s_y^2/9}$	**M1**
State or use correct tabular value: $t_{8,0.975} = 2.306$	**A1**
Evaluate: 35.5 ± 1.99, i.e. $[33.5, 37.5]$	**A1**
	Total: 12

Q18

9231/21, 9231/23 • October/November 2018 • Question 8

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Mark scheme.

(i)

$s_A^2 = (531\,000 - 5120^2/50)/49$ [= 6712/49] and $s_B^2 = (375\,135 - 3760^2/40)/39$ [= 21695/39] (estimate both population variances)	**M1**
$s_A^2 = 136.98$ and $s_B^2 = 556.28$ (to 3 s.f.) (allow biased here: 134.24 and 542.4)	**A1**
EITHER: $s^2 = s_A^2/50 + s_B^2/40 = 16.65$ or 4.08^2 (estimate combined variance)	**M1A1**
$z_{0.975} = 1.96$ (state or use correct tabular z value)	***B1**
$z = (\bar{y} - \bar{x})/s = (102.4 - 94)/s = 2.06$; $z >$ tabular value [accept H_1] (or compare $\bar{y} - \bar{x} = 8.4$ with 8.0)	**M1A1**
So $\mu_A \neq \mu_B$ or salaries do differ (AEF) (correct conclusion, FT on z, dep *B1)	**B1**

(ii)

No assumption [of normality] needed since large samples or due to central limit theorem	**B1**

Total: 9

Q19

9231/21, 9231/23 • October/November 2018 • Question 9

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Mark scheme.

(i)	
$\bar{x} = 13.88/8 = 1.735$ (allow 1.73 or 1.74) (find sample mean)	**B1**
$s^2 = (24.1182 - 13.88^2/8)/7$ [= 13/2500 or 0.0052 or 0.07211^2] (estimate population variance; allow biased here: 0.00455 or 0.06745^2)	**M1**
$H_0 : \mu = 1.7$, $H_1 : \mu > 1.7$ (AEF) (B0 for $\bar{x}\ldots$)	**B1**
$t = (\bar{x} - 1.7)/(s/\sqrt{8}) = 1.37$ (find value of t)	**M1A1**
$t_{7,0.95} = 1.89$[5] (state or use correct tabular t-value; or compare $\bar{x}$ with $1.7 + 0.048 = 1.75$)	**B1**
[Accept H_0:] Mean ht. not greater than 1.7 m (AEF) (consistent conclusion, FT on both t-values)	**B1**
(ii)	
$13.88/8 \pm t\sqrt{(s^2/8)}$ (find confidence interval)	**M1**
$t_{7,0.975} = 2.36$[5] (state or use correct tabular value of t)	**A1**
1.73[5] ± 0.06 or $[1.67, 1.80]$ (evaluate confidence interval, either form)	**A1**
	Total: 10

Q20

9231/22 • October/November 2018 • Question 6

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Mark scheme.

$\bar{x} = 90.3/8 = 11.2875$ (to 4 s.f.) (find sample mean)	**B1**
$s^2 = (1043.67 - 90.3^2/8)/7$ (estimate population variance)	**M1**
$= 19\,527/5600$ or 3.487 [or 1.867^2] (to 4 s.f.) (allow biased here: 3.051 or 1.747^2)	**A1**
$90.3/8 \pm t\sqrt{(s^2/8)}$ (find confidence interval)	**M1**
$t_{7,0.975} = 2.365$ (to 4 s.f.) (state or use correct tabular value of t)	**A1**
11.3 ± 1.6 or $[9.7, 12.8[5]]$ (evaluate C.I., either form)	**A1**
	Total: 6

Q21

9231/21, 9231/22 • May/June 2019 • Question 8

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Mark scheme.

State both hypotheses: $H_0 : \mu_b - \mu_e = 0.05$, $H_1 : \mu_b - \mu_e > 0.05$	**B1**
Consider differences d_i from e.g. $x_b - y_e$: d_i : 0.10, 0.07, 0.03, 0.03, 0.21, 0.19	**M1**
Find sample mean: $\bar{d} = 0.63/6 = 0.105$	**B1**
Estimate population variance (allow biased): $s^2 = (0.0969 - 0.63^2/6)/5 = 123/20\,000$ or 0.00615	**M1**
State or use correct tabular t-value: $t_{5,0.9} = 1.476$	**B1**
Find value of t (or compare $\bar{d} - 0.05 = 0.055$ with $t_{5,0.9}s/\sqrt{6} = 0.0473$): $t = (\bar{d} - 0.05)/(s/\sqrt{6}) = 1.72$	**M1 A1**
FT on both t-values; consistent conclusion: reject H_0, evidence for organiser's belief that times improve by more than 0.05 min	**B1**
	Total: 8

Q22

9231/23 • May/June 2019 • Question 9

↑ Back to contents

Mark scheme.

Find sample mean: $\bar{x} = 0.785$	**B1**
Estimate population variance (allow biased): $s^2 = (6.19 - 7.85^2/10)/9 = 37/12\,000$ or 0.003083	**M1**
State hypotheses: $H_0 : \mu = 0.75$, $H_1 : \mu > 0.75$	**B1**
State or use correct tabular t-value: $t_{9,0.95} = 1.83[3]$	**B1**
Find value of t; consistent conclusion: $t = (\bar{x} - 0.75)/(s/\sqrt{10}) = 1.99$, reject H_0 and accept H_1	**M1 A1**
Claim (of yield per plant increased) is justified (ii)	**B1**
Find confidence interval (must be a t value): $\bar{x} \pm t\sqrt{s^2/10}$	**M1**
State or use correct tabular value of t: $t_{9,0.975} = 2.26[2]$	**B1**
Evaluate confidence interval: 0.785 ± 0.04 or $[0.745, 0.825]$	**A1**
	Total: 10

Q23

9231/21, 9231/22, 9231/23 • October/November 2019 • Question 6

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Mark scheme.

(i)

Find estimate s^2 of population variance (must be t): $t\sqrt{s^2/9} = \frac{1}{2}(1.85-1.65) = 0.1$.	**M1**
Use correct tabular t-value: $t_{8,0.975} = 2.306$ (3 s.f.).	**A1**
$s^2 = 9 \times 0.04337^2 = 0.0169$ or 0.1301^2.	**A1**

(ii)

Find sample mean $\bar{x}$: $\bar{x} = \frac{1}{2}(1.65 + 1.85) = 1.75$, or $\Sigma x = 9 \times 1.75 = 15.75$.	**M1 A1**
$s^2 = (\Sigma x^2 - 9\bar{x}^2)/8$ or $\{\Sigma x^2 - (\Sigma x)^2/9\}/8$.	**M1**
Find Σx^2 from s^2: $\Sigma x^2 = 8 \times 0.0169 + 15.75^2/9 = 27.7$.	**A1**

Total: 7

Q24

9231/21, 9231/22, 9231/23 • October/November 2019 • Question 8

↑ Back to contents

Mark scheme.

Find sample mean for A: $\bar{x}_A = 32.4/8 = 4.05$.	**B1**
Estimate or imply population variance for A: $s_A^2 = (131.82 - 32.4^2/8)/7 = 3/35$ (or 0.08571 or 0.2928^2, to 3 s.f.).	**M1**
State hypotheses (AEF): $H_0 : \mu_A = \mu_B$, $H_1 : \mu_A \neq \mu_B$.	**B1**
Estimate (pooled) common variance: $s^2 = (7s_A^2 + 9s_B^2)/16 = 0.12497$ or 0.3535^2 or 3999/32000.	**M1 A1**
State or use correct tabular t value: $t_{16,0.95} = 1.746$.	**B1***
$t = (\bar{x}_A - \bar{x}_B)/(s\sqrt{1/8 + 1/10})$.	**M1**
Find value of t (or compare $\bar{x}_A - \bar{x}_B = 0.27$ with 0.293): $t = 0.27/0.1677 = 1.61$.	**A1**
Correct conclusion (FT on t, dep B1*), AEF: $t < 1.75$ so accept H_0, mean masses are the same.	**DB1**
	Total: 9

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Q25

9231/41, 9231/42 • May/June 2020 • Question 4

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Mark scheme.

$H_0 : \mu_x = \mu_y$, $H_1 : \mu_x \neq \mu_y$	**B1**
$s_x^2 = \frac{1}{49}\left(5.1 - \frac{15.2^2}{50}\right) = 0.0097796$; $s_y^2 = \frac{1}{39}\left(4.8 - \frac{13.4^2}{40}\right) = 0.007974$	**M1A1**
$s^2 = \frac{0.00977959}{50} + \frac{0.007974}{40} = 0.0003949$	**M1A1**
$z = \frac{0.304 - 0.335}{\sqrt{0.0003949}} = (-)1.56$	**M1A1**
Compare with 1.645	**M1**
Accept H_0: insufficient evidence to reject manager's claim	**A1**
	Total: 9

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Q26

9231/41, 9231/42 • May/June 2020 • Question 5

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Mark scheme.

(a)

$\sum x = 183.6$, $\sum x^2 = 4249.08$, $\bar{x} = 22.95$	**B1**
$s^2 = \frac{1}{7}\left(4249.08 - \frac{183.6^2}{8}\right) = 5.066$	**M1**
$H_0 : \mu = 22.0$, $H_1 : \mu > 22.0$	**B1**
$t = \frac{22.95 - 22.0}{\sqrt{s^2/8}} = 1.194$	**M1A1**
Compare t with correct tabular value 1.895	**M1**
Accept H_0: mean distance thrown is not more than 22.0 m	**A1**

(b)

$22.95 \pm t\sqrt{s^2/8}$	**M1**
With $t = 2.365$	**B1**
$[21.1, 24.8]$	**A1**

Total: 10

Q27

9231/43 • May/June 2020 • Question 2

↑ Back to contents

Mark scheme.

$s^2 = \frac{10.2}{40} + \frac{11.1}{50} = 0.477$	**M1A1**
$\text{CI} = (24.4 - 17.2) \pm zs$	**M1**
$= (24.4 - 17.2) \pm 1.645\sqrt{0.477}$	**A1**
$= [6.06, 8.34]$	**A1**
	Total: 5

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Q28

9231/43 • May/June 2020 • Question 5

↑ Back to contents

Mark scheme.

(a)

$\bar{x} = \frac{828}{10} = 82.8$, $s^2 = \frac{1}{9}\left(68622 - \frac{828^2}{10}\right) = 7.0667$	**B1**
$\frac{82.8 - k}{\sqrt{s^2/10}} \geq t$ where $t = 1.833$ (M1 if equality, A1 for inequality and correct t value)	**M1A1**
$k \leq 81.259$, i.e. $k \leq 81.3$	**A1**

(b)

$H_0 : \mu_A = \mu_B$, $H_1 : \mu_A \neq \mu_B$	**B1**
Pooled variance $= \frac{9 \times 7.0667 + 7 \times 9.966}{10 + 8 - 2} = s_p^2$	**M1**
$= 8.335$	**A1**
$t = \frac{82.8 - 79.8}{s_p\sqrt{\frac{1}{10} + \frac{1}{8}}} = 2.19$	**M1A1**
Compare with 2.12 $(t(16, 0.975))$ and reject H_0	**M1**
Population means are not the same	**A1**
	Total: 11

Q29

9231/41, 9231/43 • October/November 2020 • Question 1

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Mark scheme.

$s_x^2 = \frac{1}{39}\left(1587.0 - \frac{242.0^2}{40}\right) = 3.15128$; $s_y^2 = \frac{1}{59}\left(2532.6 - \frac{373.2^2}{60}\right) = 3.58129$ (either unsimplified; both correct)	**M1 A1**
$s^2 = \frac{3.15128}{40} + \frac{3.58129}{60} = 0.13847$ (pooled variance is M0A0, or $s = 0.3721$)	**M1 A1**
$6.05 - 6.22 \pm zs$ (FT their s, must be a z value)	**M1**
$= 0.17 \pm 1.645\sqrt{0.13847}$ (with 1.645)	**A1**
$= [-0.442,\ 0.782]$ or $[-0.782,\ 0.442]$	**A1**
	Total: 7

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Q30

9231/41, 9231/43 • October/November 2020 • Question 4

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Mark scheme.

Assume (population) differences are normally distributed	**B1**
$H_0 : \mu_X - \mu_Y = 0.3$, $H_1 : \mu_X - \mu_Y > 0.3$	**B1**
Signed differences: 0.9, 0.4, 0.7, 0.5, −0.2, 0, −0.1, 1.2, 0.3	**M1**
$\sum d = 3.7$, $\sum d^2 = 3.29$; $\bar{d} = 0.411$, $s^2 = \frac{1}{8}\left(3.29 - \frac{3.7^2}{9}\right) = 0.2211$ ($s = 0.4702$)	**M1**
$t = \frac{0.411 - 0.3}{\sqrt{0.2211/9}} = 0.708$ or 0.709	**M1 A1**
Compare their 0.708 with 1.397	**M1**
Accept H_0; insufficient evidence to support claim	**A1 FT**
	Total: 8

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Q31

9231/42 • October/November 2020 • Question 1

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Mark scheme.

$s^2 = \dfrac{313.1}{10} = 31.3$ (can be implied)	**B1**
$t = \dfrac{176.2 - 172.5}{s/\sqrt{11}}$	**M1**
$= 2.19$	**A1**
Compare their t value with 2.228	**M1**
Accept H_0: insufficient evidence to reject mean height is 172.5, or sufficient evidence to accept mean height is 172.5	**A1 FT**
	Total: 5

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Q32

9231/42 • October/November 2020 • Question 6

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Mark scheme.

B1 $\bar{x} = \frac{1}{2}(4.22 + 3.36) = 3.79$	**(a)**
$4.22 - 3.36 = \frac{2ts}{\sqrt{10}}$ (using z value implies M0)	**M1**
With $t = 1.833$	**A1**
$(s = 0.7418)$ variance $= 0.55(0)$	**A1**
B1 $H_0 : \mu_A = \mu_B$, $H_1 : \mu_A > \mu_B$ (not $\bar{x}$)	**(b)**
$s^2 = \frac{1}{7}\left(98.02 - \frac{27.86^2}{8}\right) = 0.142(507)$	**M1**
Pooled estimate $= \frac{9 \times 0.550316 + 7 \times 0.142507}{16}$	**M1**
$= 0.372$	**A1**
$t = \frac{27.86/8 - 3.79}{\sqrt{0.372}\sqrt{1/10 + 1/8}} = (-)1.063$	**M1 A1**
Compare their value with 1.337	**M1**
Accept H_0: Nassa's claim is not supported	**A1 FT**
	Total: 12

Q33

9231/41, 9231/42 • May/June 2021 • Question 1

↑ Back to contents

Mark scheme.

(a)

$\bar{x} = \dfrac{56.56}{7} = 8.08$ (accept unsimplified)	**B1**
$s^2 = \dfrac{1}{6}\left(457.275 - \dfrac{56.56^2}{7}\right) = 0.045033 \left[= \dfrac{1351}{30000}\right]$ (accept unsimplified)	**M1**
$t = \dfrac{8.08 - 8.22}{\sqrt{s^2/7}} = -1.745$ (1.74 – 1.75)	**M1 A1**
Tabular value 1.440; $1.745 > 1.440$, so reject H_0 (comparison with 1.440 and correct FT conclusion)	**M1**
There is insufficient evidence to support the hypothesis that the (population) mean is 8.22 OR sufficient evidence to suggest mean is less than 8.22 OR sufficient evidence to suggest $\mu < 8.22$ OE	**A1**

(b)

Underlying distribution is normal / population is normal	**B1**

Total: 7

Q34

9231/41, 9231/42 • May/June 2021 • Question 4

↑ Back to contents

Mark scheme.

$s_x^2 = \frac{1}{49}\left(1596 - \frac{282^2}{50}\right) = 0.11265 \left[= \frac{138}{1225}\right]$ (one correct, unsimplified)	**M1**
$s_y^2 = \frac{1}{59}\left(1808 - \frac{328^2}{60}\right) = 0.25311 \left[= \frac{224}{885}\right]$ (both correct to 3sf)	**A1**
$s^2 = \frac{0.11265}{50} + \frac{0.25312}{60} = 0.006471\ldots$ (may be implied)	**M1 A1**
$z = \frac{282/50 - 328/60}{\sqrt{\text{their } s^2}}$	**M1**
$[z] = 2.155$ (tolerance 2.15 – 2.16)	**A1**
Comparison with tabular value 1.96 OE; $2.155 > 1.96$, reject H_0	**M1**
Sufficient evidence to reject population means are equal OR sufficient evidence to accept population means are not equal OR sufficient evidence to suggest $\mu_A \neq \mu_B$ OE	**A1**
	Total: 8

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Q35

9231/43 • May/June 2021 • Question 1

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Mark scheme.

$s^2 = \frac{1}{9}\left(2313 - \frac{152^2}{10}\right) = 0.28889 \left[= \frac{13}{45}\right]$ (accept unsimplified)	**M1**
$H_0 : \mu = 14.8$, $H_1 : \mu > 14.8$ (must see 14.8, must be $=$ and $>$)	**B1**
$[t =] \frac{152/10 - 14.8}{s/\sqrt{10}}$ (using unbiased estimate)	**M1**
$[t =] 2.35$ (rounds to 2.35)	**A1**
Tabular value $= 1.833$; $2.35 > 1.833$, reject H_0 (comparison with 1.833 and correct FT conclusion)	**M1**
Sufficient evidence to accept Farmer B's claim OE	**A1**
	Total: 6

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Q36

9231/43 • May/June 2021 • Question 3

↑ Back to contents

Mark scheme.

$\bar{x} = \frac{89}{50} = 1.78$, $\bar{y} = \frac{67.2}{40} = 1.68$ (implied by 0.1 in the CI formula)	**B1**
$\sigma_x^2 = \frac{1}{49}\left(159.4 - \frac{89^2}{50}\right) = 0.02 \left[= \frac{1}{50}\right]$ (one variance correct unsimplified)	**M1**
$\sigma_y^2 = \frac{1}{39}\left(113.1 - \frac{67.2^2}{40}\right) = 0.0052308 \left[= \frac{17}{3250}\right]$ (both correctly calculated)	**A1**
$\sigma^2 = \frac{0.02}{50} + \frac{0.0052308}{40} = \left(\frac{69}{130000}\right)$ (unsimplified expression must be seen, 40 and 50 in correct places)	**M1**
$[\sigma^2 =] 0.0005308$ (may be implied by correct final answer)	**A1**
CI: $1.78 - 1.68 \pm z \times \sqrt{0.0005308}$ (correct form for CI with a z-value)	**M1**
with $z = 1.96$	**A1**
0.10 ± 0.0452 or $[0.0548, 0.145]$ (allow either form)	**A1**
	Total: 8

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Q37

9231/41, 9231/43 • October/November 2021 • Question 1

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Mark scheme.

$s^2 = \frac{78.5}{9} = 8.7222 = \frac{157}{18}$.	**B1**
Correct expression: CI : $25.6 \pm t\sqrt{\frac{s^2}{10}}$.	**M1**
With correct t value $t = 1.833$.	**A1**
$25.6 \pm 1.71(2)$ or $[23.9, 27.3]$. Accept in either form, ISW. Accept inequality form.	**A1**

$$\boxed{[23.9, 27.3]}$$

(b)

$\frac{25.6 - k}{\sqrt{s^2/10}} \geqslant t$.	**M1**
With $t = -1.383$ (allow 1.383).	**A1**
26.9 (CAO).	**A1**

Total: 7

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Q38

9231/41, 9231/43 • October/November 2021 • Question 4

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Mark scheme.

$H_0 : \mu_B - \mu_A = 2$ and $H_1 : \mu_B - \mu_A > 2$ (or use of μ_d).	**B1**
Differences: 5, 8, 2, 0, 4, 7, -2, 1, 10, -3 (allow one error).	**M1**
Sample mean and variance: $\sum d = 32$, $\sum d^2 = 272$; $\bar{d} = 3.2$, $s^2 = \frac{1}{9}\left(272 - \frac{32^2}{10}\right) = 18.84 = \frac{848}{45}$.	**M1**
$t = \dfrac{3.2 - 2}{\sqrt{s^2/10}} = 0.874$.	**M1 A1**
Compare their value with tabular value 1.833: $0.874 < 1.833$, accept H_0.	**M1**
Insufficient evidence to support claim. Correct conclusion, in context, with level of uncertainty in language.	**A1**
Assumption: population differences are normally distributed.	**B1**
	Total: 8

Q39

9231/42 • October/November 2021 • Question 1

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Mark scheme.

One correct unsimplified: $s_x^2 = \frac{1}{39}\left(14320 - \frac{752^2}{40}\right) = 4.677 = \frac{304}{65}$.	**M1**
Both correct to at least 3 s.f.: $s_y^2 = \frac{1}{59}\left(40200 - \frac{1548^2}{60}\right) = 4.434 = \frac{1308}{295}$.	**A1**
$s^2 = \frac{4.6769}{40} + \frac{4.4339}{60}$.	**M1**
$0.1908\ (= \frac{3659}{19175})$.	**A1**
CI $= \left(\frac{752}{40} - \frac{1548}{60}\right) \pm 1.96s$; correct form with a z-value, $1.96s$ can be unsimplified.	**M1 A1**
$-7.0 \pm 0.856 = [-7.86, -6.14]$. Accept in either form, ISW. Accept $[6.14, 7.86]$. Accept inequality form.	**A1**
	Total: 7

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Q40

9231/42 • October/November 2021 • Question 6

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Mark scheme.

$H_0 : \mu_A = \mu_B$ and $H_1 : \mu_A > \mu_B$.	**B1**
$\sum x = 21.2$, $\sum x^2 = 47.92$; $s_x^2 = \frac{1}{9}\left(47.92 - \frac{21.2^2}{10}\right) = 0.33067 = \frac{124}{375}$; $s_y^2 = \frac{1}{11}\left(53.75 - \frac{24.48^2}{12}\right) = 0.34644 = \frac{9527}{27500}$.	**M1 A1**
$s^2 = \dfrac{9 \times 0.33067 + 11 \times 0.34644}{10 + 12 - 2} = 0.3393.$	**M1 A1**
$t = \dfrac{2.12 - 2.04}{s\sqrt{\frac{1}{10} + \frac{1}{12}}} = 0.321$ (accept 0.321–0.322).	**M1 A1**
Compare their value with critical value 1.325: $0.321 < 1.325$, accept H_0.	**M1**
Insufficient evidence that mean of A is greater than mean of B.	**A1**
Assumption: distributions are normal and equal variances (or distributions are normal), consistent with method used.	**B1**
	Total: 10

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Q41

9231/41, 9231/42 • May/June 2022 • Question 1

↑ Back to contents

Mark scheme.

Attempt at differences, allow one error: 0.6, 1.3, 0.0, 0.7, 0.6, 0.6, 1.8, −0.2, −0.2, 0.4.	**M1**
$\sum d = 5.6$, $\sum d^2 = 6.74$, $s^2 = \frac{1}{9}\left(6.74 - \frac{5.6^2}{10}\right) = 0.400(4)$.	**M1**
$H_0 : \mu_B - \mu_A = 0.4$ and $H_1 : \mu_B - \mu_A > 0.4$ (allow use of μ_d).	**B1**
$t = \frac{0.56 - 0.4}{\sqrt{0.4004/10}}$	**M1**
$t = 0.7996$ (0.800)	**A1**
Critical value $= t_{0.90}(9) = 1.383$. Compare: $0.7996 < 1.383$, accept H_0.	**M1**
Insufficient evidence to support manager's claim (correct conclusion in context, with level of uncertainty in language). **(b)**	**A1**
Distribution of population differences is normal.	**B1**
	Total: 8

Q42

9231/41, 9231/42 • May/June 2022 • Question 5

↑ Back to contents

Mark scheme.

$\sum x = 44.4$, $\sum x^2 = 247.48$.	**B1**
$s^2 = \frac{1}{7}\left(247.48 - \frac{44.4^2}{8}\right) = 0.1514 \ \left(= \frac{53}{350}\right)$.	**M1**
CI: $\frac{44.4}{8} \pm 1.895\sqrt{\frac{0.1514}{8}}$ (correct formula with a t value).	**M1**
1.895 used.	**B1**
$5.55 \pm 0.261 = [5.29, 5.81]$ (either form). **(b)**	**A1**
$H_0 : \mu = 5.9$ and $H_1 : \mu < 5.9$.	**B1**
$t = \frac{5.55 - 5.9}{\sqrt{s^2/8}} = -2.544$.	**M1**
Critical value $t_{0.975}(7) = 2.365$. Compare: $-2.544 < -2.365$, reject H_0.	**M1**
Sufficient evidence to support Raman's claim (correct conclusion in context).	**A1**
	Total: 9

Q43

9231/43 • May/June 2022 • Question 1

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Mark scheme.

$s^2 = \dfrac{94.5}{7}$ $(= 13.5)$.	**B1**
CI: $33.8 \pm 2.365\sqrt{\dfrac{s^2}{8}}$ (correct formula with a t value, must be 33.8).	**M1**
2.365 used.	**B1**
$33.8 \pm 3.072 = [30.7, 36.9]$ (either form).	**A1**
	Total: 4

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Q44

9231/43 • May/June 2022 • Question 6

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Mark scheme.

$s_x^2 = \frac{1}{39}\left(31400 - \frac{1120^2}{40}\right) = 1.0256,\ s_y^2 = \frac{1}{49}\left(37600 - \frac{1370^2}{50}\right) = 1.2653$ (both required).	**M1**
$s^2 = \frac{1.0256}{40} + \frac{1.2653}{50}$.	**M1**
$s^2 = 0.050947$ (may be implied by 2.66 as z value).	**A1**
$H_0 : \mu_a - \mu_b = 0$ and $H_1 : \mu_a - \mu_b \neq 0$.	**B1**
$z = \frac{\frac{1120}{40} - \frac{1370}{50}}{\sqrt{0.050947}} = \frac{0.6}{\sqrt{0.050947}}$.	**M1**
$z = 2.66$.	**A1**
Critical value of $z = 2.326$. $2.66 > 2.326$, reject H_0 (compare their z value with 2.326, correct FT conclusion).	**M1**
Sufficient evidence to support difference in population means (correct conclusion in context). **(b)**	**A1**
$z = \frac{0.6 - 0.25}{s}$ (condone 1.54).	**M1**
$z = 1.55$.	**A1**
0.9392–0.9395 or 0.0605–0.0608.	**A1**
$\alpha \geqslant 6.05$ (condone 6.05–6.08, accept $>$).	**A1**
	Total: 12

Q45

9231/41, 9231/43 • October/November 2022 • Question 1

↑ Back to contents

Mark scheme.

$\bar{x} = \dfrac{1625}{50} = 32.5,\ \ s_x^2 = \dfrac{1}{49}\left(53200 - \dfrac{1625^2}{50}\right) = 7.908$	**M1**
$\bar{y} = \dfrac{1854}{60} = 30.9,\ \ s_y^2 = \dfrac{1}{59}\left(57900 - \dfrac{1854^2}{60}\right) = 10.3627$ (both correct)	**A1**
$s^2 = \dfrac{7.908}{50} + \dfrac{10.3627}{60} = 0.33087$	**M1 A1**
CI: $(32.5 - 30.9) \pm 1.96s$ – correct form with a z-value; correct with 1.96	**M1 A1**
$1.6 \pm 1.1274 = [0.473, 2.73]$ (at least 3 s.f.)	**A1**
	Total: 7

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Q46

9231/41, 9231/43 • October/November 2022 • Question 6

↑ Back to contents

Mark scheme.

$s_x^2 = \frac{1}{11}\left(3.26 - \frac{6.24^2}{12}\right) = 0.001382$; $s_y^2 = \frac{1}{9}\left(2.6124 - \frac{5.1^2}{10}\right) = 0.001267$	**M1**
Both correct ($= \frac{19}{13750}, \frac{19}{15000}$)	**A1**
Find pooled variance: $s^2 = \frac{11 \times 0.001382 + 9 \times 0.001267}{12 + 10 - 2} = 0.001330$	**M1 A1**
Use correct formula for t: $t = \frac{\frac{6.24}{12} - \frac{5.1}{10}}{s\sqrt{\frac{1}{12} + \frac{1}{10}}} = 0.640$	**M1 A1**
$H_0 : \mu_a = \mu_b$, $H_1 : \mu_a > \mu_b$	**B1**
Critical value is 2.086 ($t_{0.975}(20)$); $0.640 < 2.086$, accept H_0 (compare calculated value with 2.086 and give conclusion)	**M1**
There is insufficient evidence to support inspector's claim.	**A1**
	Total: 9

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Q47

9231/42 • October/November 2022 • Question 1

↑ Back to contents

Mark scheme.

$2.02 = \bar{x} + 1.833\sqrt{\frac{s^2}{10}}$, $1.78 = \bar{x} - 1.833\sqrt{\frac{s^2}{10}}$; add: $\bar{x} = \frac{2.02 + 1.78}{2} = 1.90$	**M1**
$\sum x = 19$	**A1**
Subtract: $2.02 - 1.78 = 2 \times 1.833\sqrt{\frac{s^2}{10}}$ (allow 1.372, 1.383, 1.812 instead of 1.833)	**M1**
$s^2 = 0.042859$ (may be implied; $s = 0.20702$)	**A1**
$s^2 = \frac{1}{9}\left(\sum x^2 - \frac{(\sum x)^2}{10}\right)$	**M1**
$\sum x^2 = 9s^2 + \frac{19^2}{10} = 36.5$ (36.486)	**A1**
	Total: 6

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Q48

9231/42 • October/November 2022 • Question 3

↑ Back to contents

Mark scheme.

$s_x^2 = \frac{1}{49}\left(115.2 - \frac{75.5^2}{50}\right) = 0.02439$; $s_y^2 = \frac{1}{79}\left(172.6 - \frac{116.8^2}{80}\right) = 0.02623$	**M1**
Both correct ($= \frac{239}{9800}$, $\frac{259}{9875}$)	**A1**
$s^2 = \frac{0.02439}{50} + \frac{0.02623}{80} = [0.00081565]$ (may be implied by 1.75 for z)	**M1 A1**
$z = \frac{\frac{75.5}{50} - \frac{116.8}{80}}{s}$	**M1**
$z = 1.75$	**A1**
Compare with 1.645: $1.75 > 1.645$, reject null hypothesis (or using areas, $0.04 < 0.05$)	**M1**
Sufficient evidence to suggest that the population mean in country X is greater than the population mean in country Y.	**A1**
	Total: 8

Q49

9231/41, 9231/42 • May/June 2023 • Question 1

↑ Back to contents

Mark scheme.

$\overline{x} = \dfrac{91.2}{12} = 7.6, \quad s_x^2 = \dfrac{1}{11}\left(695.8 - \dfrac{91.2^2}{12}\right) = 0.2436 \left(\dfrac{67}{275}\right)$. Both.	**M1**
CI: $7.6 \pm t_{11}(0.975) \times \sqrt{\dfrac{0.2436}{12}}$. With a t-value.	**M1**
2.201 seen.	**B1**
$7.6 \pm 0.3136 = [7.29, 7.91]$.	**A1**
	Total: 4

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Q50

9231/41, 9231/42 • May/June 2023 • Question 2

↑ Back to contents

Mark scheme.

$\overline{x} = \dfrac{99.6}{12} = 8.3, \quad s_x^2 = \dfrac{21.5}{11} = 1.9545$ or $\dfrac{43}{22}$. Both SOI.	**M1**
$H_0 : \mu_P = \mu_Q, \quad H_1 : \mu_P > \mu_Q$.	**B1**
Pooled estimate $= \dfrac{9 \times 2.16 + 11 \times 1.9545}{10 + 12 - 2}$. FT their 1.9545 only.	**M1 FT**
2.047 (2.05). May be implied, allow 2.04.	**A1**
$t = \dfrac{9.12 - 8.3}{\sqrt{2.047} \times \sqrt{\frac{1}{10} + \frac{1}{12}}}$. FT their pooled estimate.	**M1 FT**
1.339 (or 1.338). Accept 1.34.	**A1**
$'1.339' < 1.725$. Accept H_0 / not significant. Compare with correct tabular value 1.725 and conclusion without context. Condone 'reject H_1'.	**M1**
Insufficient evidence to suggest that the (mean) time taken at P is greater than the (mean) time taken at Q. Correct conclusion in context, following correct work, level of uncertainty in language (not 'prove', not 'there is no evidence'), no contradictions. A0 if H_0 and H_1 reversed.	**A1**
	Total: 8

Q51

9231/43 • May/June 2023 • Question 2

↑ Back to contents

Mark scheme.

$\overline{x} + 1.771\sqrt{\dfrac{s^2}{14}} = 1.14$ or $\overline{x} - 1.771\sqrt{\dfrac{s^2}{14}} = 1.11$. SOI. Allow incorrect t-value, not z-value.	**M1**
$\left[\text{Add: } \overline{x} = \dfrac{1}{2}(1.14 + 1.11) = 1.125\right]$ $\sum x = 15.75$. Does not depend on use of a t-value.	**B1**
Subtract or substitute: $\sqrt{\dfrac{s^2}{14}} = \dfrac{1}{2}\left(\dfrac{1.14 - 1.11}{1.771}\right)$. Allow incorrect t-value, but not a z-value.	**M1**
$s^2 = 0.00100[4]$ or $s = 0.0316[9]$. $\dfrac{450}{448063}$, implied by correct final answer.	**A1**
$\sum x^2 = 13s^2 + \dfrac{(\sum x)^2}{14}$. OE.	**M1**
$\sum x^2 = 17.7(3)$. CWO.	**A1**
	Total: 6

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Q52

9231/43 • May/June 2023 • Question 4

↑ Back to contents

Mark scheme.

$H_0 : \mu_x = \mu_y$; $H_1 : \mu_x < \mu_y$.	**B1**
$s_x^2 = \frac{1}{79}\left(338.1 - \frac{164^2}{80}\right)$ [= 0.02405] and $s_y^2 = \frac{1}{59}\left(261.1 - \frac{124.8^2}{60}\right)$ [= 0.02569]. Both. Implied by $\frac{19}{790}$, $\frac{379}{14750}$ or 3sf.	**B1**
$s^2 = \frac{0.02405}{80} + \frac{0.02569}{60}$.	**M1**
$s^2 = 0.0007289$ or $s = 0.026998$ or 0.0270. Implied by $z = -1.11$.	**A1**
$z = \frac{\frac{164.0}{80} - \frac{124.8}{60}}{s}$. FT their value for s.	**M1 FT**
-1.11.	**A1**
'1.11' < 1.282. Accept H_0/not significant. Compare with correct z-value 1.282 and consistent signs. Condone 'reject H_1'. Using probabilities, $P(Z > 1.11) = 0.1333 > 0.1$.	**M1**
Insufficient evidence to support the inspector's suspicion / Insufficient evidence that the (mean) lengths of rods from machine X are shorter than the (mean) lengths of rods from machine Y. Correct conclusion in context, level of uncertainty in language, no contradictions. A0 if hypotheses wrong way round.	**A1**
(b)	
Large sample sizes OR central limit theorem applies.	**B1**
	Total: 9

Q53

9231/41, 9231/43 • October/November 2023 • Question 1

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Mark scheme.

$H_0 : \mu = 4.22$, $H_1 : \mu < 4.22$.	**B1**
$\bar{x} = \frac{42.05}{10} = 4.205$, $s^2 = \frac{1}{9}\left(176.83 - \frac{42.05^2}{10}\right) = \frac{13}{12000} = 0.0010833$.	**M1**
$t = \dfrac{4.205 - 4.22}{\sqrt{s^2/10}}$, using their 4.205 and s^2.	**M1**
$t = -1.44$ (condone sign).	**A1**
Tabular value = 1.833: $\lvert -1.44 \rvert < 1.833$, accept H_0 (allow 'not significant').	**M1**
There is insufficient evidence to support Maya's belief (correct conclusion in context; A0 if hypotheses wrong way round or missing).	**A1**
	Total: 6

Q54

9231/41, 9231/43 • October/November 2023 • Question 3

↑ Back to contents

Mark scheme.

$H_0 : \mu_B - \mu_E = 2$, $H_1 : \mu_B - \mu_E > 2$ (may use μ_d, consistent with direction of differences).	**B1**
Differences: 8, 8, 15, −2, −3, 20, −3, 0, 5, 4 (at most 2 errors).	**M1**
$\sum d = 52$, $\sum d^2 = 816$, $s^2 = \frac{1}{9}\left(816 - \frac{52^2}{10}\right) = \frac{2728}{45} = 60.62$.	**M1**
$t = \dfrac{\frac{52}{10} - 2}{\sqrt{s^2/10}}$.	**M1**
$t = 1.29967 = 1.30$.	**A1**
Tabular value = 2.262: $1.30 < 2.262$, accept H_0 (allow 'not significant').	**M1**
Insufficient evidence to suggest that cholesterol level has reduced by more than 2 (correct conclusion in context; A0 if hypotheses wrong way round or missing).	**A1**
For part (b):	
Population differences are normally distributed.	**B1**

Total: 8

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Q55

9231/42 • October/November 2023 • Question 1

↑ Back to contents

Mark scheme.

$s_x^2 = \frac{243.8}{59} = 4.132$, $s_y^2 = \frac{384.9}{74} = 5.201$ (implied by correct s or pooled estimate).	**B1**
$s^2 = \dfrac{4.132}{60} + \dfrac{5.201}{75} = 0.1382$ or $s = 0.3718$ (using their sample variances; pooled estimate M0).	**M1**
Correct value.	**A1**
CI: $\dfrac{6345}{60} - \dfrac{7614}{75} \pm 1.96 \times 0.3718$ (with a z-value).	**M1**
With 1.96 (with their s).	**A1**
$[3.50, 4.96]$ or $(3.5, 4.96)$ (condone $[4.96, 3.5]$ etc.).	**A1**
	Total: 6

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Q56

9231/42 • October/November 2023 • Question 5 (parts b, c)

↑ Back to contents

Mark scheme.

$H_0 : \mu_x = \mu_y$, $H_1 : \mu_x < \mu_y$.	**B1**
$\sum x = 39.7$, $\sum x^2 = 199.33$, $\sum y = 49.5$, $\sum y^2 = 275.47$; $s_x^2 = \frac{1}{7}\left(199.33 - \frac{39.7^2}{8}\right) = 0.33125$.	**B1**
$s_y^2 = \frac{1}{8}\left(275.47 - \frac{49.5^2}{9}\right) = 0.4025$.	**B1**
Pooled variance $s^2 = \dfrac{7(0.33125) + 8(0.4025)}{8 + 9 - 2}$.	**M1**
$s^2 = 0.36925$.	**A1**
$t = \dfrac{\frac{39.7}{8} - \frac{49.5}{9}}{s\sqrt{\frac{1}{8} + \frac{1}{9}}}$.	**M1**
$t = -1.82$.	**A1**
Tabular value $= 1.753$: $1.82 > 1.753$.	**M1**
Reject H_0; sufficient evidence that mean for machine X is less than mean for machine Y (CWO, correct conclusion in context). For part (c):	**A1**
The t-test assumes a normal distribution with equal variances; this may not be true, so there is no reason to expect the results to be the same. Outliers affect part (b) but not part (a).	**B1**
	Total: 10

Q57

9231/41, 9231/42 • May/June 2024 • Question 1

↑ Back to contents

Mark scheme.

$s^2 = \dfrac{941.5}{13} = 72.423.$	**B1**
CI: $42.8 \pm 2.160\sqrt{\dfrac{s^2}{14}}$. Correct formula with a t value; 2.160 seen in a CI formula for A1.	**M1 A1**
$[37.9, 47.7]$.	**A1**
	Total: 4

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Q58

9231/41, 9231/42 • May/June 2024 • Question 6

↑ Back to contents

Mark scheme.

Signed differences: 4.1, 1.1, −0.4, 0.2, 3.9, 2.6, −0.6, 3.5, 0.1, 0. Allow one error, must be signed differences.	**M1**
$\sum d = 14.5$, $\sum d^2 = 52.81$, $s^2 = \frac{1}{9}\left(52.81 - \frac{14.5^2}{10}\right) = 3.532$.	**M1**
H_0: $\mu_B - \mu_A = 1$; H_1: $\mu_B - \mu_A > 1$.	**B1**
$t = \dfrac{1.45 - 1}{\sqrt{s^2/10}}$. The -1 must be present in the numerator.	**M1**
0.757.	**A1**
Critical value is 1.383: '0.757'< 1.383 Accept H_0.	**M1**
Insufficient evidence to support Jade's claim. (b)	**A1**
Distribution of population differences is normal.	**B1**
	Total: 8

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Q59

9231/43 • May/June 2024 • Question 2

↑ Back to contents

Mark scheme.

$\bar{x} = \frac{1}{2}(68.66 + 64.22) = 66.44$.	**M1**
$\sum x = 12 \times 66.44 = 797.28$. May come from use of incorrect t value; accept AWRT 797.	**A1**
$\dfrac{ts}{\sqrt{12}} = \frac{1}{2}(68.66 - 64.22)$. Must be with a t value (not z value).	**M1**
With $t = 2.718$ gives $s^2 = 8.0055$ (accept AWRT 8.01; $s = 2.8294$).	**A1**
Use $s^2 = \frac{1}{11}\left(\sum x^2 - \frac{(\sum x)^2}{12}\right)$ to find an expression for $\sum x^2$.	**M1**
$\sum x^2 = 53059$ (AWRT 53100 but not from wrong working). (b)	**A1**
Population is normally distributed.	**B1**
	Total: 7

Q60

9231/43 • May/June 2024 • Question 6

↑ Back to contents

Mark scheme.

$s_x^2 = \frac{1}{49}\left(23480 - \frac{1080^2}{50}\right)$ $(= 3.102)$; $s_y^2 = \frac{1}{39}\left(22220 - \frac{940^2}{40}\right)$ $(= 3.333)$.	**M1**
$s^2 = \dfrac{3.102}{50} + \dfrac{3.333}{40}$ $(= 0.1454)$.	**M1**
$s^2 = \dfrac{2137}{14700} = 0.1454$.	**A1**
CI: $\dfrac{1080}{50} - \dfrac{940}{40} \pm 1.645\sqrt{s^2}$. Use of correct formula with a z value; B1 for 1.645 used in CI formula.	**M1 B1**
$[1.27, 2.53]$. (b)	**A1**
$z = \dfrac{1.9 - 1.1}{s}$.	**M1**
2.098 (AWRT 2.10).	**A1**
'2.098'$> 1.28(2)$ Reject H_0 (or $\Phi(2.098) = 0.9821 > 0.9$ Reject H_0).	**M1**
Sufficient evidence to suggest that the lengths of tails of wallabies in region Y are, on average, more than 1.1cm longer than the tails of wallabies in region X.	**A1**
	Total: 10

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Q61

9231/41, 9231/43 • October/November 2024 • Question 1

↑ Back to contents

Mark scheme.

$s_x^2 = \frac{1}{49}\left(54850 - \frac{1654^2}{50}\right) = 2.769.$	**B1**
$s^2 = \frac{4.95}{60} + \frac{2.769}{50} = 0.1379.$	**M1**
Correct value for s^2.	**A1**
CI: $35.6 - \frac{1654}{50} \pm 1.96s$ – correct formula with a z value.	**M1**
1.96 seen.	**B1**
$[1.79,\ 3.25]$ (accept inequality).	**A1**
	Total: 6

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Q62

9231/41, 9231/43 • October/November 2024 • Question 6

↑ Back to contents

Mark scheme.

$s_x^2 = \frac{1}{7}\left(519.46 - \frac{64.2^2}{8}\right) = 0.607857$; $s_y^2 = \frac{1}{8}\left(567.13 - \frac{71.1^2}{9}\right) = 0.680$.	**M1**
Pooled variance $s^2 = \frac{8(0.680) + 7(0.607857)}{9+8-2} = 0.64633$ $(= \frac{1939}{3000})$.	**M1 A1**
H_0: $\mu_X - \mu_Y = 0$; H_1: $\mu_X - \mu_Y > 0$.	**B1**
$t = \frac{8.025 - 7.9}{s\sqrt{\frac{1}{8}+\frac{1}{9}}} = \frac{0.125}{0.3906} = 0.320$ (dependent on pooled variance being used).	**M1 A1**
Compare with 1.341: $0.320 < 1.341$, accept H_0.	**M1**
Insufficient evidence to support Ansal's suspicion / insufficient evidence that wingspan from region X is greater than wingspan from region Y.	**A1**
	Total: 8

Q63

9231/42 • October/November 2024 • Question 1

↑ Back to contents

Mark scheme.

$\sum x = 30.6$, $\sum x^2 = 120.52$; $s^2 = \frac{1}{7}\left(120.52 - \frac{30.6^2}{8}\right) = 0.49643$.	**M1**
CI $= \frac{30.6}{8} \pm 1.895\sqrt{\frac{s^2}{8}}$ – correct formula with a t value.	**M1**
1.895 seen.	**B1**
[3.35, 4.30] (accept with inequality signs or open brackets).	**A1**
	Total: 4

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Q64

9231/42 • October/November 2024 • Question 5

↑ Back to contents

Mark scheme.

$s_x^2 = \frac{1}{34}\left(29.0 - \frac{30.8^2}{35}\right) = 0.055765$; $s_y^2 = \frac{1}{59}\left(76.8 - \frac{62.4^2}{60}\right) = 0.201763$ (allow B1 for correct expression for pooled variance).	**B1**
$s^2 = \frac{0.055765}{35} + \frac{0.201763}{60} = 0.004956$ (pooled variance: M0A0).	**M1 A1**
H_0: $\mu_x = \mu_y$; H_1: $\mu_x < \mu_y$.	**B1**
$z = \frac{\frac{30.8}{35} - \frac{62.4}{60}}{s} = \frac{-0.25}{\sqrt{0.004956}} = -2.27$.	**M1 A1**
Corresponding probability is 0.9885 (FT their z).	**M1**
One tail: $100 \times (1 - 0.9885) = 1.15$ (allow for $1 - 0.9885$).	**M1**
$\alpha > 1.15$ (allow $\geq$).	**A1**
	Total: 9

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Q65

9231/41, 9231/42 • May/June 2025 • Question 1

↑ Back to contents

Mark scheme.

$H_0 : \mu_x = \mu_y$, $H_1 : \mu_x \neq \mu_y$ (population means).	**B1**
$s_x^2 = 103.218\ldots$, $s_y^2 = 171.139\ldots$ (unbiased estimates, SOI).	**M1**
$\dfrac{s_x^2}{120} + \dfrac{s_y^2}{80} = 2.999$ (not pooled).	**A1**
$z = \dfrac{\frac{10470}{120} - \frac{6560}{80}}{\sqrt{\frac{s_x^2}{120} + \frac{s_y^2}{80}}} = 3.03.$	**M1**
Correct value of z (use of pooled s scores M0 A0).	**A1**
$3.03 > 2.576$, reject H_0 (significant). [OR $P(Z > 3.03) = 0.00122 < 0.005$, reject H_0.]	**M1**
Correct conclusion in context: there is sufficient evidence to suggest the mean spend of customers is different on weekends compared to weekdays.	**A1**
	Total: 7

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Q66

9231/41, 9231/42 • May/June 2025 • Question 4

↑ Back to contents

Mark scheme.

M1 $\sum(x-\bar{x})^2 = 40104 - \frac{1}{10}(612)^2 = 2649.6$; $\sum(y-\bar{y})^2 = 27460 - \frac{1}{8}(444)^2 = 2818$. **(a)**

A1 $s_p^2 = \dfrac{2649.6 + 2818}{10 + 8 - 2} = 341.725$.

B1 $t_{16} = 2.120$ (SOI).

M1 Correct form: $(61.2 - 55.5) \pm 2.120\sqrt{s_p^2(\frac{1}{10} + \frac{1}{8})}$.

A1 Correct un-simplified CI: $5.7 \pm 2.12 \times 8.7686$.

A1 $[-12.9,\ 24.3]$.

B1 Because zero lies inside the confidence interval. **(b)**

Total: 7

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Q67

9231/43 • May/June 2025 • Question 2

↑ Back to contents

Mark scheme.

Correct method to find one unbiased estimate: $s_x^2 = \frac{1}{39}(720 - \frac{168^2}{40}) = 0.36923$; $s_y^2 = \frac{1}{59}(900 - \frac{228^2}{60}) = 0.56949$.	**M1**
Both unbiased estimates correct to 3sf or better.	**A1**
$s^2 = \dfrac{0.36923}{40} + \dfrac{0.56949}{60}$ (M0 for pooled estimate).	**M1**
$s^2 = 0.0187$ (SOI).	**A1**
Correct form with any z-value: $\left(\dfrac{168}{40} - \dfrac{228}{60}\right) \pm 1.645\sqrt{s^2}$.	**M1**
$z = 1.645$ (accept 1.64 or 1.65) used.	**B1**
$[0.175,\ 0.625]$.	**A1**
	Total: 7

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Q68

9231/43 • May/June 2025 • Question 5

↑ Back to contents

Mark scheme.

B1 $H_0 : \mu_B - \mu_A = k$, $H_1 : \mu_B - \mu_A > k$. **(a)**

M1 Differences, d: 19, 17, 8, 16, 9, 23, 6, 5 (allow one error).

M1 $\sum d = 103$, $\sum d^2 = 1641$; $\bar{d} = 12.875$, $s_d^2 = \frac{1}{7}(1641 - \frac{1}{8}(103^2)) = 44.982$.

M1 Correct form for t: $t = \dfrac{12.875 - k}{\sqrt{s_d^2/8}}$.

A1 Correct expression.

M1 $\dfrac{12.875 - k}{\sqrt{s_d^2/8}} \geqslant 1.895$.

A1 $0 \leqslant k \leqslant 8.38$.

B1 Population of differences are normally distributed. **(b)**

Total: 8

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Q69

9231/44 • May/June 2025 • Question 1

↑ Back to contents

Mark scheme.

$t = \pm\dfrac{10.24 - 10.6}{\sqrt{0.52/12}}$ (correct form).	**M1**
$(\pm)1.729$ (CWO, AWRT ± 1.73).	**A1**
$-1.729 < -1.363$, reject H_0 (significant); compare their t with ± 1.363 and appropriate conclusion.	**M1**
Sufficient evidence to support/suggest that the population mean is less than 10.6.	**A1**

Total: 4

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Q70

9231/44 • May/June 2025 • Question 6

↑ Back to contents

Mark scheme.

B1 $\sum x = 30.4$. **(a)**

B1 $t_9 = 2.262$ seen.

M1 Subtracting/substituting to form equation involving s_x: $s_x = \dfrac{\sqrt{10}(3.13 - 2.95)}{2 \times 2.262}$.

A1 FT $s_x^2 = 0.0158$ or $s_x = 0.126$.

A1 FT $\sum x^2 = 9 \times 0.0158307 + \dfrac{30.4^2}{10} = 92.6$ (CWO).

B1 $H_0 : \mu_Y = \mu_X$, $H_1 : \mu_Y > \mu_X$. **(b)**

M1 $76.98 - \dfrac{24.8^2}{8}$ (may be embedded).

M1 $s_p^2 = \dfrac{9s_x^2 + 7s_y^2}{10 + 8 - 2} = 0.015155$ (correct form).

A1 SOI (can be implied by $|t|$ AWRT 1.02 or 1.03).

M1 Correct form: $t = \dfrac{3.04 - 3.1}{s_p\sqrt{\frac{1}{10} + \frac{1}{8}}} = -1.028$.

A1 AWRT ± 1.02 or ± 1.03.

M1 Compare their t with critical value ± 1.746 (18 d.f. as stated on the paper; note $10 + 8 - 2 = 16$) and appropriate conclusion.

A1 $|-1.028| < 1.746$, accept H_0 (not significant); insufficient evidence to support Mona's claim.

Total: 13

Q71

9231/41, 9231/43 • October/November 2025 • Question 1

↑ Back to contents

Mark scheme.

$s^2 = \frac{1}{9}\left(71314 - \frac{844^2}{10}\right) = \frac{134}{15} = 8.933.$	**M1**
2.262 seen.	**B1**
Correct form, must be a t-value: $\frac{844}{10} \pm 2.262\sqrt{\frac{8.933}{10}}$.	**M1**
$[82.3, 86.5]$.	**A1**
Assumption: the distribution of estimates of angles is normal (OR estimates are a random sample from some population / independent). (b)	**B1**
Population unlikely to be normal as θ is close to a right-angle / data is skewed (OR estimates may not be independent, e.g. due to collusion / no indication sample is random). Must refer to context.	**B1**
	Total: 6

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Q72

9231/41, 9231/43 • October/November 2025 • Question 3

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Mark scheme.

$\sum(x-\bar{x})^2 = 9 \times 0.36 = 3.24.$	**B1**
$55.8 - \frac{16.6^2}{5}$ (implied by 0.172 or 0.688).	**M1**
Correct use of pooled variance formula: $\frac{3.24+0.688}{10+5-2}$ $(= 0.302)$.	**M1**
$s = 0.550$ (CAO).	**A1**
	Total: 4

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Q73

9231/41, 9231/43 • October/November 2025 • Question 6

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Mark scheme.

$H_0 : \mu_B = \mu_A$, $H_1 : \mu_B > \mu_A$ (population means).	**B1**
Attempt at signed differences: 5, 0, −1, 6, 1, −2, 8, 7, 15.	**M1**
$s_d^2 = \frac{1}{8}\left(405 - \frac{39^2}{9}\right) = 29.5$.	**M1**
Correct form $\dfrac{39/9}{\sqrt{29.5/9}}$.	**M1**
$= 2.393$ (AWRT 2.39).	**A1**
Compare $2.393 > 1.860$, reject H_0 / significant.	**M1**
Sufficient evidence to suggest new training programme results in reduced times / to support the coach's belief. (b)	**A1**
The population of differences may not be normally distributed / may not be symmetrical. Must mention population/distribution and differences. (c)	**B1**
A Wilcoxon matched-pairs signed-rank test (OR a paired-sample sign test).	**B1**
	Total: 9

Q74

9231/42 • October/November 2025 • Question 2

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Mark scheme.

$s^2 = \frac{1}{11}\left(476117 - \frac{2390^2}{12}\right) = 9.8788.$	**M1**
3.106 seen.	**B1**
Correct form, must be a t-value: $\frac{2390}{12} \pm 3.106\sqrt{\frac{9.8788}{12}}$.	**M1**
$[196, 202]$. (b)	**A1**
± 1.796 seen.	**B1**
Correct form, must be a t-value, consistent signs: $\dfrac{199.17 - k}{\sqrt{9.8788/12}} > -1.796.$	**M1**
$k < 200.8$ (accept 200.7, 200, or 201).	**A1**
	Total: 7

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Q75

9231/42 • October/November 2025 • Question 5

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Mark scheme.

$H_0 : \mu_A = \mu_B$, $H_1 : \mu_A \neq \mu_B$ (population means).	**B1**
$1331848 - \frac{3264^2}{8}$ [= 136] (implied by $s_A^2 = 19.43$).	**M1**
$964964 - \frac{2406^2}{6}$ [= 158] (implied by $s_B^2 = 31.6$).	**M1**
$s_p^2 = \dfrac{136 + 158}{8 + 6 - 2} = 24.5.$	**M1**
OE, e.g. $\frac{49}{2}$.	**A1**
$t = \dfrac{\frac{3264}{8} - \frac{2406}{6}}{s_p\sqrt{\frac{1}{8} + \frac{1}{6}}} = 2.62.$	**M1**
AWRT 2.62.	**A1**
Compare $2.62 > 2.179$, reject H_0 / significant.	**M1**
Sufficient evidence to suggest a difference in the mean tensile strengths of steel rods produced by the two machines.	**A1**
	Total: 9

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Q76

9231/44 • October/November 2025 • Question 1

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Mark scheme.

$\sum x = 723$, $\sum x^2 = 42261$.	**B1**
$s^2 = \frac{1}{12}\left(42261 - \frac{723^2}{13}\right) = 170.9231$.	**M1**
1.782 seen.	**B1**
Correct form, must be a t-value: $\frac{723}{13} \pm 1.782\sqrt{\frac{170.9231}{13}}$.	**M1**
$[49.2, 62.1]$. (b)	**A1**
The population (of car speeds) is normally distributed / underlying distribution is normal.	**B1**
	Total: 6

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Q77

9231/44 • October/November 2025 • Question 3

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Mark scheme.

$H_0 : \mu_Y = \mu_X$, $H_1 : \mu_Y > \mu_X$ (population means).	**B1**
Attempt at signed differences: 3, −1, 6, 2, 3, 3, −1, −2, 3, 3.	**M1**
$s_d^2 = \frac{1}{9}\left(91 - \frac{19^2}{10}\right) = 6.1$.	**M1**
Correct form using their s_d^2: $\dfrac{19/10}{\sqrt{6.1/10}} = 2.43$.	**M1**
AWRT 2.43.	**A1**
Compare $2.43 < 2.821$, accept H_0 / not significant.	**M1**
Insufficient evidence to support the manufacturer's claim / that machine Y extracts more juice on average than machine X. (b)	**A1**
Population of differences is normally distributed / underlying distribution of differences is normal. (c)	**B1**
Comment recognising the differences remain unchanged (or redo test: $2.43 < 2.821$).	***B1**
The conclusion remains the same.	**DB1**
	Total: 10

Written by Ali Hashir.