Further Math with Ali Hashir

Continuous Random Variables

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/21, 9231/22 • May/June 2015 • Question 9

Cotton cloth is sold from long rolls of cloth. The number of flaws on a randomly chosen piece of cloth of length a metres has a Poisson distribution with mean $0.8a$. The random variable X is the length of cloth, in metres, between two successive flaws.

(i) Explain why, for $x \geqslant 0$, $\mathrm{P}(X > x) = e^{-0.8x}$. [2]

(ii) Find the probability that there is at least one flaw in a 4 metre length of cloth. [2]

(iii) Find

(a) the distribution function of X, [1]

(b) the probability density function of X, [2]

(c) the interquartile range of X. [4]

Q2

9231/23 • May/June 2015 • Question 9

The continuous random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} 0 & x < 2, \\ ae^{-(x-2)} & x \geqslant 2, \end{cases}$$

where a is a constant. Show that $a = 1$. [1]

Find the distribution function of X and hence find the median value of X. [5]

The random variable Y is defined by $Y = e^X$. Find

(i) the probability density function of Y, [4]

(ii) $\mathrm{P}(Y > 10)$. [2]

Q3

9231/21, 9231/22, 9231/23 • October/November 2015 • Question 7

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{21}x^2 & 1 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable Y is defined by $Y = X^2$. Show that Y has probability density function given by

$$g(y) = \begin{cases} \frac{1}{42}y^{1/2} & 1 \leq y \leq 16, \\ 0 & \text{otherwise.} \end{cases}$$

[5]

Find

(i) the median value of Y, [2]

(ii) the expected value of Y. [2]

Q4

9231/21, 9231/22 • May/June 2016 • Question 8

The random variable X has probability density function f given by

$$f(x) = \begin{cases} 2e^{-2x} & x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [2]

(ii) Find the median value of X. [3]

The random variable Y is defined by $Y = e^X$.

(iii) Find the probability density function of Y. [4]

Q5

9231/23 • May/June 2016 • Question 7

The lifetime, in months, of a particular type of light bulb is a random variable T and its probability density function, f, is given by

$$f(t) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where λ is a positive constant. The manager of a shopping mall uses this type of light bulb and he notes that, out of 1000 new light bulbs, 280 were still working after 12 months.

(i) Find an estimate for the value of λ. [3]

(ii) Hence estimate the mean of T. [1]

An improved version of the light bulb is produced. The lifetime of the improved version also has a negative exponential distribution, with a mean 25% larger than the original mean.

(iii) What percentage of the improved version of the light bulb fail within the first 12 months? [4]

Q6

9231/21, 9231/22, 9231/23 • October/November 2016 • Question 5

The distance, X km, completed by a new car before any mechanical fault occurs has distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 1 - \mathrm{e}^{-ax} & x \geqslant 0, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a positive constant. The mean value of X is 10 000. Find

(i) the value of a, [1]

(ii) the probability that a new car completes less than 15 000 km before any mechanical fault occurs. [2]

The probability that a new car completes at least d km before any mechanical fault occurs is 0.75.

(iii) Find the value of d. [3]

Q7

9231/21, 9231/22, 9231/23 • October/November 2016 • Question 7

The random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{6}x & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

The random variable Y is defined by $Y = X^3$. Find

(ii) the probability density function of Y, [2]

(iii) the value of k for which $\mathrm{P}(Y \geqslant k) = \frac{7}{12}$. [3]

Q8

9231/21, 9231/22 • May/June 2017 • Question 8

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{4}(x-1) & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

The random variable Y is defined by $Y = (X-1)^3$.

(ii) Find the probability density function of Y. [4]

(iii) Find the median value of Y. [3]

Q9

9231/23 • May/June 2017 • Question 9

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} 0 & x < 0, \\ a\mathrm{e}^{-x\ln 2} & x \geqslant 0, \end{cases}$$

where a is a positive constant.

(i) Find the value of a. [2]

(ii) State the value of $\mathrm{E}(X)$. [1]

(iii) Find the interquartile range of X. [4]

The variable Y is related to X by $Y = 2^X$.

(iv) Find the probability density function of Y. [5]

Q10

9231/21, 9231/22, 9231/23 • October/November 2017 • Question 7

The random variable X has probability density function f given by

$$f(x) = \begin{cases} 0.2e^{-0.2x} & x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [2]

(ii) Find $P(X > 2)$. [2]

(iii) Find the median of X. [3]

Q11

9231/21, 9231/22 • May/June 2018 • Question 6

The continuous random variable X has distribution function given by

$$F(x) = \begin{cases} 1 - e^{-0.4x} & x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $P(X > 2)$. [2]

(ii) Find the interquartile range of X. [4]

Q12

9231/23 • May/June 2018 • Question 9

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{20}\left(3 - \frac{1}{\sqrt{x}}\right) & 1 \leq x \leq 9, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable Y is defined by $Y = \sqrt{X}$.

(i) Show that the probability density function of Y is given by

$$g(y) = \begin{cases} \frac{1}{10}(3y - 1) & 1 \leq y \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$

[7]

(ii) Find the mean value of Y. [2]

Q13

9231/21, 9231/23 • October/November 2018 • Question 6

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{80}\left(3\sqrt{x} - \dfrac{8}{\sqrt{x}}\right) & 4 \leqslant x \leqslant 16, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

The random variable Y is defined by $Y = \sqrt{X}$.

(ii) Find the probability density function of Y. [3]

Q14

9231/21, 9231/23 • October/November 2018 • Question 7

The random variable T is the lifetime, in hours, of a particular type of battery. It is given that T has a negative exponential distribution with mean 500 hours.

(i) Write down the probability density function of T. [1]

(ii) Find the probability that a randomly chosen battery of this type has a lifetime of more than 750 hours. [3]

(iii) Find the median value of T. [3]

Q15

9231/22 • October/November 2018 • Question 7

The continuous random variable X has distribution function given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{90}(x^2 + x^4) & 0 \leqslant x \leqslant 3, \\ 1 & x > 3. \end{cases}$$

The random variable Y is defined by $Y = X^2$.

(i) Find the probability density function of Y. [4]

(ii) Find the mean value of Y. [2]

Q16

9231/21, 9231/22 • May/June 2019 • Question 7

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{3}{4x^2} + \dfrac{1}{4} & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

(ii) Find the exact value of the interquartile range of X. [5]

Q17

9231/23 • May/June 2019 • Question 6

The random variable T is the lifetime, in hours, of a randomly chosen battery of a particular type. It is given that T has a negative exponential distribution with mean 400 hours.

(i) Write down the probability density function of T. [1]

(ii) Find the probability that a battery of this type has a lifetime that is less than 500 hours. [2]

(iii) Find the median of the distribution. [3]

Q18

9231/21, 9231/22, 9231/23 • October/November 2019 • Question 7

The time, T days, before an electrical component develops a fault has distribution function F given by

$$F(t) = \begin{cases} 1 - \mathrm{e}^{-at} & t \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a positive constant. The mean value of T is 200.

(i) Write down the value of a. [1]

(ii) Find the probability that an electrical component of this type develops a fault in less than 150 days. [2]

A piece of equipment contains n of these components, which develop faults independently of each other. The probability that, after 150 days, at least one of the n components has not developed a fault is greater than 0.99.

(iii) Find the smallest possible value of n. [4]

Q19

9231/21, 9231/22, 9231/23 • October/November 2019 • Question 10

The random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{30}\left(\frac{8}{x^2} + 3x^2 - 14\right) & 2 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

The random variable Y is defined by $Y = X^2$.

(ii) Find the probability density function of Y. [4]

(iii) Find the value of y such that $\mathrm{P}(Y < y) = 0.8$. [3]

Q20

9231/41, 9231/42 • May/June 2020 • Question 3

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{3}{16}(2-\sqrt{x}) & 0 \le x < 1, \\ \frac{3}{16\sqrt{x}} & 1 \le x \le 9, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $\mathrm{E}(X)$. [3]

The random variable Y is such that $Y = \sqrt{X}$.

(b) Find the probability density function of Y. [5]

Q21

9231/43 • May/June 2020 • Question 3

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{5}x & 0 \leq x < 2, \\ \frac{2}{15}(5-x) & 2 \leq x \leq 5, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

(b) Find the median value of X. [2]

(c) Find $\mathrm{E}(X^2)$. [2]

(d) Find $\mathrm{P}(1 \leq X \leq 3)$. [2]

Q22

9231/41, 9231/43 • October/November 2020 • Question 6

The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0, \\ \dfrac{1}{60}(16x - x^2) & 0 \leqslant x \leqslant 6, \\ 1 & x > 6. \end{cases}$$

(a) Find the interquartile range of X. [4]

(b) Find $\mathrm{E}(X^3)$. [4]

The random variable Y is such that $Y = \sqrt{X}$.

(c) Find the probability density function of Y. [3]

Q23

9231/42 • October/November 2020 • Question 4

The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 2, \\ \frac{1}{60}x^2 - \frac{1}{15} & 2 \leqslant x \leqslant 8, \\ 1 & x > 8. \end{cases}$$

(a) Find $\mathrm{P}(3 \leqslant X \leqslant 6)$. [1]

(b) Find $\mathrm{E}(\sqrt{X})$. [3]

(c) Find $\mathrm{Var}(\sqrt{X})$. [2]

(d) The random variable Y is defined by $Y = X^3$. Find the probability density function of Y. [3]

Q24

9231/41, 9231/42 • May/June 2021 • Question 3

The continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{81}x^2 & 0 \leqslant x \leqslant 9, \\ 1 & x > 9. \end{cases}$$

(a) Find $\mathrm{E}\left(\sqrt{X}\right)$. [3]

(b) Find $\mathrm{Var}\left(\sqrt{X}\right)$. [2]

(c) The random variable Y is given by $Y^3 = X$. Find the probability density function of Y. [3]

Q25

9231/43 • May/June 2021 • Question 6

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{8} & 0 \leqslant x < 1, \\ \frac{1}{28}(8 - x) & 1 \leqslant x \leqslant 8, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

(b) Find the value of the constant a such that $\mathrm{P}(X \leqslant a) = \frac{5}{7}$. [3]

The random variable Y is given by $Y = \sqrt[3]{X}$.

(c) Find the probability density function of Y. [5]

Q26

9231/41, 9231/43 • October/November 2021 • Question 2

The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < -1, \\ \frac{1}{2}(1+x)^2 & -1 \leqslant x \leqslant 0, \\ 1 - \frac{1}{2}(1-x)^2 & 0 < x \leqslant 1, \\ 1 & x > 1. \end{cases}$$

(a) Find the probability density function of X. [2]

(b) Find $\mathrm{P}\left(-\frac{1}{2} \leqslant X \leqslant \frac{1}{2}\right)$. [2]

(c) Find $\mathrm{E}(X^2)$. [2]

(d) Find $\mathrm{Var}(X^2)$. [2]

Q27

9231/42 • October/November 2021 • Question 3

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} a + \frac{1}{5}x & 0 \leqslant x < 1, \\ 2a - \frac{1}{5}x & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise}, \end{cases}$$

where a is a constant.

(a) Find the value of a. [3]

(b) Find $\mathrm{E}(X^2)$. [2]

(c) Find the cumulative distribution function of X. [3]

Q28

9231/41, 9231/42 • May/June 2022 • Question 3

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} kx(4-x) & 0 \leqslant x < 2, \\ k(6-x) & 2 \leqslant x \leqslant 6, \\ 0 & \text{otherwise}, \end{cases}$$

where k is a constant.

(a) Show that $k = \dfrac{3}{40}$. [1]

(b) Given that $\mathrm{E}(X) = 2.5$, find $\mathrm{Var}(X)$. [3]

(c) Find the median value of X. [4]

Q29

9231/43 • May/June 2022 • Question 4

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{3}{8}\left(1 + \frac{1}{x^2}\right) & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $\mathrm{E}\left(\sqrt{X}\right)$. [3]

The random variable Y is given by $Y = X^2$.

(b) Find the probability density function of Y. [4]

(c) Find the 40th percentile of Y. [3]

Q30

9231/41, 9231/43 • October/November 2022 • Question 5

The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0, \\ 1 - \dfrac{1}{144}(12 - x)^2 & 0 \leq x \leq 12, \\ 1 & x > 12. \end{cases}$$

(a) Find the upper quartile of X. [2]

(b) Find $\mathrm{Var}(X^2)$. [5]

The random variable Y is given by $Y = \sqrt{X}$.

(c) Find the probability density function of Y. [3]

Q31

9231/42 • October/November 2022 • Question 4

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} k & 0 \leq x < 1, \\ kx & 1 \leq x \leq 2, \\ 0 & \text{otherwise}, \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{2}{5}$. [1]

(b) Find the interquartile range of X. [5]

(c) Find $\mathrm{Var}(X)$. [4]

Q32

9231/41, 9231/42 • May/June 2023 • Question 6

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{3}{28}\left(\mathrm{e}^{\frac{1}{2}x} + 4\mathrm{e}^{-\frac{1}{2}x}\right) & 0 \leqslant x \leqslant 2\ln 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

The random variable Y is defined by $Y = \mathrm{e}^{\frac{1}{2}(X)}$.

(b) Find the probability density function of Y. [3]

(c) Find the 30th percentile of Y. [3]

(d) Find $\mathrm{E}(Y^4)$. [2]

Q33

9231/43 • May/June 2023 • Question 1

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{6}\left(x^{-\frac{1}{3}} - x^{-\frac{2}{3}}\right) & 1 \leqslant x \leqslant 27, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

The random variable Y is defined by $Y = X^{\frac{1}{3}}$.

(b) Find the probability density function of Y. [3]

(c) Find the exact value of the median of Y. [2]

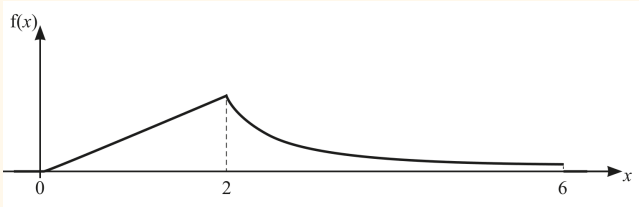
Q34

9231/41, 9231/43 • October/November 2023 • Question 4

As shown in the diagram, the continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} mx & 0 \leqslant x \leqslant 2, \\ \dfrac{k}{x^2} + c & 2 \leqslant x \leqslant 6, \\ 0 & \text{otherwise,} \end{cases}$$

where m, k and c are constants.



(a) Given that $\mathrm{P}(X \leqslant 2) = \frac{1}{3}$, show that $m = \frac{1}{6}$ and find the values of k and c. [4]

(b) Find the exact numerical value of the interquartile range of X. [5]

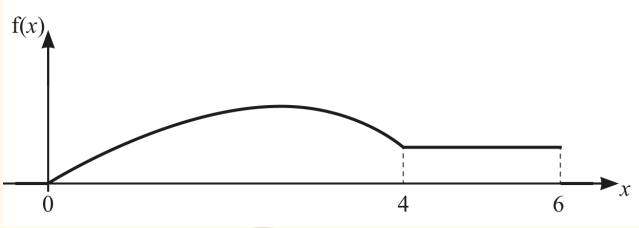
Q35

9231/42 • October/November 2023 • Question 4

The diagram shows the continuous random variable X with probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{128}(4ax - bx^3) & 0 \leqslant x \leqslant 4, \\ c & 4 \leqslant x \leqslant 6, \\ 0 & \text{otherwise}, \end{cases}$$

where a, b and c are constants.



The upper quartile of X is equal to 4.

(a) Show that $c = \frac{1}{8}$ and find the values of a and b. [4]

(b) Find the exact value of the median of X. [3]

(c) Find $\mathrm{E}(\sqrt{X})$, giving your answer correct to 2 decimal places. [3]

Q36

9231/41, 9231/42 • May/June 2024 • Question 7

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{x}{4}(4 - x^2) & 0 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $\mathrm{Var}\left(\sqrt{X}\right)$. [4]

The continuous random variable Y is defined by $Y = X^2$.

(b) Find the probability density function of Y. [4]

(c) Find the exact value of the median of Y. [2]

Q37

9231/43 • May/June 2024 • Question 5

The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 2, \\ \dfrac{(x-2)^2}{12} & 2 \le x < 4, \\ 1 - \dfrac{(8-x)^2}{24} & 4 \le x \le 8, \\ 1 & x > 8. \end{cases}$$

(a) Sketch the graph of the probability density function of X. [3]

(b) Find $E(X)$. [3]

(c) Find the exact value of the interquartile range of X. [4]

Q38

9231/41, 9231/43 • October/November 2024 • Question 4

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} kx^3 & 0 \le x < 1, \\ k(5-x) & 1 \le x \le 5, \\ 0 & \text{otherwise}, \end{cases}$$

where k is a constant.

(a) Sketch the graph of f. [1]

(b) Show that $k = \dfrac{4}{33}$. [2]

(c) Find the cumulative distribution function of X. [3]

(d) Find the median value of X. [4]

Q39

9231/42 • October/November 2024 • Question 4

The random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{21}(x-1)^2 & 2 \le x \le 5, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

The random variable Y is defined by $Y = (X-1)^4$.

(b) Find the probability density function of Y. [3]

(c) Find the median value of Y. [2]

(d) Find $\mathrm{E}(Y)$. [2]

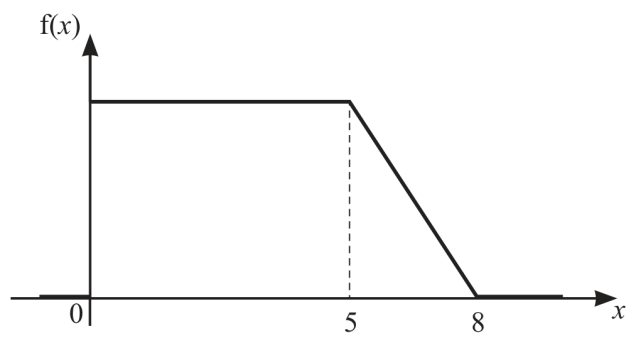
Q40

9231/41, 9231/42 • May/June 2025 • Question 2

As shown in the diagram, the continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} a & 0 \leqslant x \leqslant 5, \\ b - cx & 5 \leqslant x \leqslant 8, \\ 0 & \text{otherwise}, \end{cases}$$

where a, b and c are constants.



(a) Show that $a = \frac{2}{13}$ and find the values of b and c. [3]

(b) Find the mean of X. [3]

(c) Find the median of X. [2]

The random variable Y is defined by $Y = X^2$.

(d) Find the cumulative distribution function for Y. [4]

Q41

9231/43 • May/June 2025 • Question 3

A continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} kx & 0 \leqslant x < 1, \\ k(8-x) & 1 \leqslant x \leqslant 8, \\ 0 & \text{otherwise}, \end{cases}$$

where k is a constant.

(a) Show that $k = \dfrac{1}{25}$. [2]

(b) Find the median value of X. [3]

The random variable Y is defined by $Y = \sqrt[3]{X}$.

(c) Find the probability density function of Y. [5]

Q42

9231/44 • May/June 2025 • Question 4

The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} kx & 0 \leqslant x < 1, \\ kx^2 & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $k = \dfrac{6}{17}$. [2]

(b) Find the cumulative distribution function of X. [3]

(c) Find the median value of X. [2]

(d) Find $\mathrm{E}\left(\dfrac{1}{X}\right)$. [2]

Q43

9231/41, 9231/43 • October/November 2025 • Question 5

A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{16}\sqrt{x} & 0 \leq x < 4, \\ \frac{1}{k\sqrt{x}} & 4 \leq x \leq 9, \\ 0 & \text{otherwise}, \end{cases}$$

where k is a constant.

(a) Show that $k = 3$. [2]

(b) Find the median value of X. [3]

The random variable Y is defined by $Y = \sqrt{X}$.

(c) Find the probability density function of Y. [5]

Q44

9231/42 • October/November 2025 • Question 4

A continuous random variable X has cumulative distribution function F given by

$$F(x) = \begin{cases} 0 & x < 1, \\ \frac{1}{5}x + a & 1 \le x < 4, \\ \frac{1}{50}x^2 + b & 4 \le x \le 6, \\ 1 & x > 6, \end{cases}$$

where a and b are constants.

(a) Find the value of a and the value of b. [2]

(b) Find the probability density function of X. [2]

(c) Given that $\mathrm{E}(X) = \frac{529}{150}$, find $\mathrm{Var}(X)$. [3]

(d) Find the 10th and 90th percentiles of X. [3]

Q45

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The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{4}{9}(x+1) & 0 \leq x \leq 1, \\ (x-2)^2 & 1 < x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

(b) Find the exact value of the median of X. [2]

The random variable Y is defined by $Y = \sqrt{X}$.

(c) Find the cumulative distribution function of Y. [3]

(d) Determine whether the median of Y is greater than, or less than, the median of X. [2]

Answers available in the companion mark scheme booklet.