Further Math with Ali Hashir

Continuous Random Variables

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/21, 9231/22 • May/June 2015 • Question 9

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Mark scheme.

(i)	
Relate $\mathrm{P}(X > x)$ to number of flaws (AEF): $\mathrm{P}(X > x) = \mathrm{P}(\text{zero flaws in } x \text{ m})$.	**B1**
Relate this to Poisson distribution (AEF): $= \mathrm{P}_0(0.8x) = e^{-0.8x}$ (AG).	**B1**
(ii)	
Find P(number of flaws $\geqslant 1$): $1 - \mathrm{P}_0(0.8 \times 4) = 1 - e^{-3.2} = 1 - 0.0408 = 0.959$ (M0 if "$1-$" omitted).	**M1 A1**
(iii)(a)	
Find or state distribution function $F(x)$: $F(x) = \mathrm{P}(X \leqslant x) = 1 - \mathrm{P}(X > x) = 1 - e^{-0.8x}$.	**B1**
(iii)(b)	
Find or state pdf $f(x)$: $f(x) = dF/dx = 0.8e^{-0.8x}$ (S.R. deduct A1 if (a),(b) interchanged).	**M1 A1**
(iii)(c)	
Formulate equation for either quartile value Q: $F(Q) = 1 - e^{-0.8Q} = \frac{1}{4}$ or $\frac{3}{4}$.	**M1**
Find lower quartile $Q_1 = 1.2\ln(4/3)$ [$= 0.360$].	**A1**
Find upper quartile $Q_3 = 1.2\ln 4$ [$= 1.733$].	**A1**
Find interquartile range: $Q_3 - Q_1$ [$= 1.2\ln 3$] $= 1.37$.	**A1**
	Total: 11

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Q2

9231/23 • May/June 2015 • Question 9

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Mark scheme.

Find a by equating $\int_2^\infty f(x)\,dx$ to 1: $\left[-ae^{-(x-2)}\right]_2^\infty = a = 1$ (AG).	**B1**
Find or state distribution function $F(x)$ for $x \geqslant 2$: $F(x) = \int f(x)\,dx = -ae^{-(x-2)} + c$, $F(2) = 0$ so $F(x) = 1 - e^{-(x-2)}$.	**M1 A1**
State $F(x)$ for $x < 2$: $F(x) = 0$ for $x < 2$.	**A1**
Find median m from $F(m) = \frac{1}{2}$: $1 - e^{-(m-2)} = \frac{1}{2}$, $e^{m-2} = 2$, $m = 2 + \ln 2$ or $2 - \ln \frac{1}{2}$ or 2.69.	**M1 A1**
(i)	
Find or state $G(y)$ from $Y = e^X$ for $x \geqslant 2$: $G(y) = \mathrm{P}(Y < y) = \mathrm{P}(e^X < y) = \mathrm{P}(X < \ln y) = F(\ln y) = 1 - e^{-(\ln y - 2)} = 1 - e^2/y$.	**M1 A1**
Find $g(y)$ by differentiation (allow $e^2 = 7.39$): $g(y) = e^2/y^2$.	**A1**
State corresponding range of y: for $y \geqslant e^2$ ($g(y) = 0$ for $y < e^2$).	**A1**
(ii)	
Find $\mathrm{P}(Y > 10)$: $\mathrm{P}(Y > 10) = 1 - G(10) = e^2/10 = 0.739$ (M0 for $G(10)$ or $1 - g(10)$).	**M1 A1**
	Total: 12

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Q3

9231/21, 9231/22, 9231/23 • October/November 2015 • Question 7

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Mark scheme.

$F(x) = (x^3 - 1)/63$ for $1 \le x \le 4$.	**B1**
$G(y) = P(Y < y) = P(X^2 < y) = P(X < y^{1/2}) = F(y^{1/2}) = (y^{3/2} - 1)/63$.	**M1A1**
$g(y) = y^{1/2}/42$ (A.G.).	**A1**
Range $1 \le y \le 16$ (A.G.).	**B1**
Median: $(m^{3/2} - 1)/63 = \frac{1}{2}$, giving $m = 32.5^{2/3} = 10.2$.	**M1A1**
$E(Y) = \int y\, g(y)\, dy = [y^{5/2}]_1^{16}/105 = 1023/105 = 341/35 = 9.74$.	**M1A1**
	Total: 9

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Q4

9231/21, 9231/22 • May/June 2016 • Question 8

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Mark scheme.

(i)	
Find or state distribution function $F(x)$ for $x \geq 0$: $F(x) = \int f(x)\,dx = -e^{-2x} + c$	**M1**
Use $F(0) = 0$ to find $F(x)$: $F(x) = 0\ (x < 0),\ 1 - e^{-2x}\ (x \geq 0)$	**A1**
(ii)	
Find median value m from $F(m) = \frac{1}{2}$: $1 - e^{-2m} = \frac{1}{2}$, $e^{2m} = 2$	**M1**
$m = \frac{1}{2}\ln 2$ or 0.347	**M1 A1**
(iii)	
Find or state $G(y)$ from $Y = e^X$ for $x \geq 0$: $G(y) = P(Y < y) = P(e^X < y) = P(X < \ln y) = F(\ln y) = 1 - e^{-2\ln y}$ $[= 1 - 1/y^2]$	**M1 A1**
Find $g(y)$ by differentiation: $g(y) = 2/y^3$	**A1**
State corresponding range of y: for $y \geq 1$ $[g(y) = 0$ for $y < 1]$	**A1**
	Total: 9

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Q5

9231/23 • May/June 2016 • Question 7

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Mark scheme.

(i)	
Find $F(t)$ by integration: $F(t) = 1 - e^{-\lambda t}$	**B1**
Estimate λ by equating $1 - F(12)$ to 280/1000: $e^{-12\lambda} = 0.28$, $\lambda = (-\ln 0.28)/12 = 0.106$	**M1 A1**
(ii)	
Estimate mean μ of T: $\mu = 1/\lambda = 9.43$	**B1**
(iii)	
Relate new exponential parameter λ' to λ: $\lambda' = 1/(1.25\mu) = \lambda/1.25$ or 0.8λ [$= 0.08486$ or 11.78^{-1}]	**B1**
Find $P(t \leq 12)$: $P(t \leq 12) = 1 - e^{-12\lambda'} = 1 - e^{-1.018}$ or $1 - 0.28^{0.8}$	**M1**
Find percentage: $= 63.9\%$	**A1 A1**
	Total: 8

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Q6

9231/21, 9231/22, 9231/23 • October/November 2016 • Question 5

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Mark scheme.

(i)	
Find a from mean: $a = 1/10\,000$ or 10^{-4}	**B1**
(ii)	
Find $\mathrm{P}(X < 15\,000)$: $1 - \mathrm{e}^{-15\,000a} = 1 - \mathrm{e}^{-1.5} = 0.777$	**M1 A1**
(iii)	
Formulate condition for d: $1-(1-\mathrm{e}^{-ad}) = 0.75$ (M0 for $1-\mathrm{e}^{-ad} = 0.75$, giving $d = 13\,900$)	**M1**
Rearrange and take logs: $d = -(\ln 0.75)/a$	**A1**
$= 2877$ or 2880	**A1**
	Total: 6

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Q7

9231/21, 9231/22, 9231/23 • October/November 2016 • Question 7

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Mark scheme.

(i)	
Find or state distribution function $F(x)$ for $2 \leqslant x \leqslant 4$: $F(x) = \int f(x)\,dx = x^2/12 + c$	**M1**
Use $F(2) = 0$ or $F(4) = 1$ to find $F(x)$: $F(x) = x^2/12 - \frac{1}{3}$ $[2 \leqslant x \leqslant 4]$	**A1**
0 $(x < 2)$, 1 $(x > 4)$	**A1**
(ii)	
Find $G(y)$ from $Y = X^3$: $G(y) = P(Y < y) = P(X^3 < y) = P(X < y^{1/3}) = F(y^{1/3}) = y^{2/3}/12 - \frac{1}{3}$	**M1**
Find $g(y)$ by differentiation: $g(y) = (1/18)y^{-1/3}$ for $8 \leqslant y \leqslant 64$, 0 otherwise	**A1**
(iii)	
Formulate condition for k: $7/12 = 1 - G(k) = 1 - k^{2/3}/12 + \frac{1}{3}$	**M1**
$k^{2/3} = 9$	**A1**
$k = 27$	**A1**
	Total: 8

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Q8

9231/21, 9231/22 • May/June 2017 • Question 8

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Mark scheme.

$F(x) = x^2/8 - x/4\,[+c]$.	**M1**
$= x^2/8 - x/4$ or $\{(x-1)^2 - 1\}/8$ (AEF).	**A1**
$F(x) = 0$ $(x < 2)$, $F(x) = 1$ $(x > 4)$. **(ii)**	**A1**
$G(y) = (1 + y^{1/3})^2/8 - (1 + y^{1/3})/4$ or $(y^{2/3} - 1)/8$.	**M1,A1**
$g(y) = (1/12)y^{-1/3}$.	**A1**
for $1 \leqslant y \leqslant 27$. **(iii)**	**A1**
$(m^{2/3} - 1)/8 = \frac{1}{2}$.	**M1**
$m = \sqrt{125}$ or $5\sqrt{5}$ or 11.2.	**M1,A1**
	Total: 10

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Q9

9231/23 • May/June 2017 • Question 9

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Mark scheme.

State a or find a by equating $\int_0^\infty \mathrm{f}(x)\,dx$ to 1: $(a/\ln 2)[-\mathrm{e}^{-x\ln 2}]_0^\infty = a/\ln 2$, so $a = \ln 2$ or 0.693. **(ii)**	**M1,A1**
State or find $\mathrm{E}(X)$: $\mathrm{E}(X) = 1/\ln 2$ or 1.44. **(iii)**	**B1**
Formulate equation for either quartile value Q: $F(Q) = 1 - \mathrm{e}^{-Q\ln 2} = \frac{1}{4}$ or $\frac{3}{4}$.	**M1**
Find lower quartile: $Q_1 = (\ln(4/3))/\ln 2\,[= 0.415]$ (AEF).	**A1**
Find upper quartile: $Q_3 = (\ln 4)/\ln 2\,[= 2]$ (AEF).	**A1**
Find interquartile range (allow $Q_1 - Q_3$): $Q_3 - Q_1\,[= (\ln 3)/\ln 2] = 1.58$ [or 1.59]. **(iv)**	**A1**
EITHER: find or state $G(y) = \mathrm{P}(Y < y) = \mathrm{P}(2^X < y) = \mathrm{P}(X < (\ln y)/(\ln 2)) = F((\ln y)/(\ln 2))$ or $F(\log_2 y)$ (AEF) (allow $<$ or $\leqslant$).	**M1,A1**
$= 1 - \mathrm{e}^{-\ln y}$ or $1 - 1/y$.	**A1**
OR: use $x = (\ln y)/(\ln 2)$ to find both $f(x)$ and dx/dy.	**M1**
$f(x) = (\ln 2)\mathrm{e}^{-x\ln 2} = (\ln 2)\mathrm{e}^{-\ln y} = (1/y)\ln 2$.	**A1**
and $dx/dy = 1/(y\ln 2)$.	**A1**
Find $g(y)$ in simplified form: $g(y) = 1/y^2$.	**A1**
State corresponding range: for $y \geqslant 1$ [$g(y) = 0$ otherwise].	**A1**
	Total: 12

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Q10

9231/21, 9231/22, 9231/23 • October/November 2017 • Question 7

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Mark scheme.

(i)	
State, or integrate and use $F(0) = 0$ or $F(x) \to 1$ as $x \to \infty$: $F(x) = \int f(x)\,dx = -e^{-0.2x} + c = 1 - e^{-0.2x}$ $(x \geq 0)$.	**M1**
and $F(x) = 0$ $(x < 0)$.	**A1**
(ii)	
$P(X > 2) = 1 - F(2) = e^{-0.4} = 0.670$.	**M1, A1**
(iii)	
Find median value m from $F(m) = \frac{1}{2}$: $1 - e^{-0.2m} = \frac{1}{2}$, $e^{0.2m} = 2$.	**M1**
$m = 5 \ln 2 = 3.47$.	**M1, A1**
	Total: 7

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Q11

9231/21, 9231/22 • May/June 2018 • Question 6

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Mark scheme.

$P(X > 2) = 1 - F(2) = \exp(-0.8) = 0.449$ (ii)	**M1 A1**
Formulate equation for either quartile value Q: $F(Q) = 1 - \exp(-0.4x) = \frac{1}{4}$ or $\frac{3}{4}$	**M1**
$Q_1 = (\ln 4/3)/0.4 = 0.7192$	**A1**
$Q_3 = (\ln 4)/0.4 = 3.466$	**A1**
$Q_3 - Q_1 = (\ln 3)/0.4 = 2.75$ (FT on Q_1, Q_3)	**A1**
	Total: 6

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Q12

9231/23 • May/June 2018 • Question 9

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Mark scheme.

Find or state distribution function $F(x) = \int f(x)\,dx = \frac{1}{20}(3x - 2\sqrt{x} + c)$; with $c = -1$: $F(x) = \frac{1}{20}(3x - 2\sqrt{x} - 1)$	**M1**
Equivalent form $\frac{3}{20}x - \frac{1}{10}\sqrt{x} - \frac{1}{20}$	**A1**
Find or state $G(y) = P(Y < y) = P(\sqrt{X} < y) = P(X < y^2) = F(y^2) = \frac{1}{20}(3y^2 - 2y - 1)$	**M1**
Equivalent form $\frac{3}{20}y^2 - \frac{1}{10}y - \frac{1}{20}$ (verify $g(y)$, differentiation may be implied)	**A2**
$g(y) = G'(y) = \frac{1}{10}(3y - 1)$ (AG), for $1 \le y \le 3$, $g(y) = 0$ otherwise (ii)	**M1 A1**
Find mean of Y from $\int y\,g(y)\,dy$: $E(Y) = \frac{1}{10}\int(3y^2 - y)\,dy$	**M1**
$= \frac{1}{10}[y^3 - \frac{1}{2}y^2]_1^3 = \frac{11}{5} = 2.2$	**A1**
	Total: 9

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Q13

9231/21, 9231/23 • October/November 2018 • Question 6

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Mark scheme.

(i)	
$\mathrm{F}(x) = \int \mathrm{f}(x)\,\mathrm{d}x = (1/80)(2x^{3/2} - 16x^{1/2})[+c]$ (find or state distribution function for $4 \leqslant x \leqslant 16$ using F(4)=0 or F(16)=1 to find c if necessary)	**M1**
$= (1/80)(2x^{3/2} - 16x^{1/2} + 16)$ or $(1/40)(x^{3/2} - 8x^{1/2} + 8)$ or $x^{3/2}/40 - x^{1/2}/5 + 1/5$ (AEF) – state $\mathrm{F}(x)$ for other values of x	**A1**
$\mathrm{F}(x) = 0$ $(x < 4)$, $\mathrm{F}(x) = 1$ $(x > 16)$	**A1**
EITHER:	**(ii)**
$\mathrm{G}(y)[= \mathrm{P}(Y < y) = \mathrm{P}(\sqrt{X} < y) = \mathrm{P}(X < y^2)] = \mathrm{F}(y^2) = (1/40)(y^3 - 8y + 8)$ (AEF) – find or state $\mathrm{G}(y)$ for $2 \leqslant y \leqslant 4$ from $Y = \sqrt{X}$ (allow $<$ or $\leqslant$ throughout; FT on constant term)	**M1A1**
$\mathrm{g}(y)[= \mathrm{G}'(y)] = (1/40)(3y^2 - 8)$ or $(3/40)y^2 - 1/5$ [for $2 \leqslant y \leqslant 4$, $g(y) = 0$ otherwise] – find $g(y)$ in simplified form for $2 \leqslant y \leqslant 4$	**A1**
	Total: 6

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Q14

9231/21, 9231/23 • October/November 2018 • Question 7

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Mark scheme.

(i)	
$\mathrm{f}(t) = (1/500)\exp(-t/500)$ [0 otherwise] (AEF) – state pdf of T for $t \geqslant 0$	**B1**
(ii)	
$\mathrm{F}(t) = 1 - \exp(-t/500)$ (find or imply F(t))	**M1**
$\mathrm{P}(T > 750) = 1 - \mathrm{F}(750) = \exp(-750/500) = 0.223$ (M0 for F(750))	**M1A1**
(iii)	
$1 - \exp(-m/500) = \frac{1}{2}$ or $\exp(-m/500) = \frac{1}{2}$ (find median value m of T from $\mathrm{F}(m) = \frac{1}{2}$)	**M1**
$m = 500\ln 2 = 347$ [hours]	**M1A1**
	Total: 7

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Q15

9231/22 • October/November 2018 • Question 7

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Mark scheme.

EITHER:	**(i)**
$G(y)[= P(Y < y) = P(X^2 < y) = P(X < y^{1/2}) = F(y^{1/2})] = (1/90)(y + y^2)$ (find or state $G(y)$ for $0 \leqslant x \leqslant 3$ from $Y = X^2$, allow $<$ or $\leqslant$ throughout)	**M1A1**
$g(y)[= G'(y)] = (1/90)(1 + 2y)$ (find $g(y)$ in simplified form)	**A1**
for $0 \leqslant y \leqslant 9$ $[g(y) = 0$ otherwise$]$ (state corresponding range of y at any stage)	**A1**
(ii)	
$E(Y) = (1/90)\int(y + 2y^2)\,dy$ (find mean of Y from $\int y\,g(y)\,dy$)	**M1**
$= (1/90)[\frac{1}{2}y^2 + \frac{2}{3}y^3]_0^9 = 117/20$ or 5.85	**A1**
	Total: 6

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Q16

9231/21, 9231/22 • May/June 2019 • Question 7

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Mark scheme.

Find or state distribution function $F(x)$ for $1 \leqslant x \leqslant 3$: $F(x) = \int f(x)\,\mathrm{d}x = -3/(4x) + x/4\,[+c]$	**M1**
Using $F(1) = 0$ or $F(3) = 1$ to find c if necessary: $= -3/(4x) + x/4 + 1/2$ or $\frac{1}{4}(-3/x + x + 2)$	**A1**
State $F(x)$ for other values of x: $F(x) = 0$ $(x \leqslant 1)$, $F(x) = 1$ $(x \geqslant 3)$ (ii)	**A1**
Formulate equation for either quartile value Q: $\int_1^Q f(x)\,\mathrm{d}x = -3/4Q + Q/4 + 1/2 = 1/4$ (or $3/4$)	**M1**
$Q^2 \pm Q - 3 = 0$	**A1**
Find lower quartile Q_1 and upper quartile Q_3: $Q_1 = \frac{1}{2}(-1 + \sqrt{13})$, $Q_3 = \frac{1}{2}(1 + \sqrt{13})$	**A1 A1**
Find interquartile range: $Q_3 - Q_1 = 1$	**A1**
	Total: 8

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Q17

9231/23 • May/June 2019 • Question 6

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Mark scheme.

State probability density function f(t) for $t \geqslant 0$: $f(t) = (1/400)\exp(-t/400)$ (ii)	**B1**
Find $P(T < 500)$: $= \int_0^{500} f(t)\,dt = [\exp(-t/400)]_0^{500} = 1 - e^{-5/4} = 0.7135$ or 0.713 (iii)	**M1 A1**
Find median value m from $F(m) = 1/2$: $1 - e^{-m/400}$ or $e^{-m/400} = 1/2$, $e^{m/400} = 2$	**M1**
$m = 400 \ln 2$ or 277	**M1 A1**
	Total: 6

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Q18

9231/21, 9231/22, 9231/23 • October/November 2019 • Question 7

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Mark scheme.

(i)	
State a or find a by equating mean value to $1/a$: $a = 1/200 = 0.005$.	**B1**
(ii)	
Find $\mathrm{P}(T < 150)$: $p = \mathrm{P}(T < 150) = F(150) = 1 - \mathrm{e}^{-150a}$.	**M1**
$p = 1 - \mathrm{e}^{-0.75} = 0.528$.	**A1**
(iii)	
Formulate condition for n: $1 - p^n > 0.99$.	**M1**
$0.01 > (1 - \mathrm{e}^{-0.75})^n$ or $0.01 > 0.528^n$.	**A1**
Rearrange and take logs to give bound: $n > \log 0.01/\log 0.528$.	**M1**
Find $n_{\min}$: $n > 7.20$ (or 7.21) so $n_{\min} = 8$.	**A1**
	Total: 7

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Q19

9231/21, 9231/22, 9231/23 • October/November 2019 • Question 10

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Mark scheme.

(i)	
Find or state distribution function $F(x)$ for $2 \le x \le 4$: $F(x) = \int f(x)\,dx = \frac{1}{30}(-8/x + x^3 - 14x) + c$.	**M1**
Using $F(2) = 0$ or $F(4) = 1$ to find c (AEF): $F(x) = \frac{1}{30}(-8/x + x^3 - 14x + 24)$.	**M1**
State $F(x)$ for other values of x: $F(x) = 0$ $(x \le 2)$, $F(x) = 1$ $(x \ge 4)$.	**A1**
(ii)	
Find or state $G(y)$ for $2 \le x \le 4$ from $Y = X^2$: $G(y) = \mathrm{P}(Y < y) = \mathrm{P}(X^2 < y) = \mathrm{P}(X < \sqrt{y}) = F(\sqrt{y})$.	**M1**
$G(y) = \frac{1}{30}(-8y^{-1/2} + y^{3/2} - 14y^{1/2} + 24)$.	**A1**
Find $g(y)$ (AEF): $g(y) = G'(y) = \frac{1}{30}(4y^{-3/2} + \frac{3}{2}y^{1/2} - 7y^{-1/2})$.	**A1**
State corresponding range: for $4 \le y \le 16$ [$g(y) = 0$ otherwise].	**A1**
(iii)	
Set $G(y) = 0.8$: $\frac{1}{30}(-8y^{-1/2} + y^{3/2} - 14y^{1/2} + 24) = 0.8$.	**M1**
Rearrange to give quadratic in y and solve: $-8 + y^2 - 14y = 0$, $y = 7 + \sqrt{57}$ or 14.55 (rejecting $7 - \sqrt{57}$; allow 14.6).	**M1 A1**
	Total: 10

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Q20

9231/41, 9231/42 • May/June 2020 • Question 3

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Mark scheme.

(a)

$\mathrm{E}(X) = \int_0^1 \frac{3}{16}x(2-\sqrt{x})\,dx + \int_1^9 \frac{3x}{16\sqrt{x}}\,dx$	**M1**
$\frac{3}{16}\left[x^2 - \frac{2}{5}x^{5/2}\right]_0^1 + \frac{3}{16}\left[\frac{2}{3}x^{3/2}\right]_1^9$	**A1**
$= \frac{269}{80}$	**A1**

(b)

$\mathrm{F}(x) = \begin{cases} \frac{3}{8}\left(x - \frac{1}{3}x^{3/2}\right), & 0 \le x < 1 \\ \frac{3}{8}x^{1/2} - \frac{1}{8}, & 1 \le x \le 9 \end{cases}$	**M1A1**
$\mathrm{G}(y) = \begin{cases} \frac{3}{8}\left(y^2 - \frac{1}{3}y^3\right), & 0 \le y < 1 \\ \frac{3}{8}y - \frac{1}{8}, & 1 \le y \le 3 \end{cases}$ $(\mathrm{G}(y) = 0$ for $y \le 0$, $= 1$ for $y \ge 3)$	**M1A1**
$\mathrm{g}(y) = \begin{cases} \frac{3}{8}(2y - y^2), & 0 \le y < 1 \\ \frac{3}{8}, & 1 \le y \le 3 \\ 0, & \text{otherwise} \end{cases}$	**A1**

Total: 8

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Q21

9231/43 • May/June 2020 • Question 3

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Mark scheme.

(a)	
$\mathrm{F}(x) = \begin{cases} \frac{x^2}{10}, & 0 \le x < 2 \\ \frac{1}{15}(10x - x^2) - \frac{2}{3}, & 2 \le x \le 5 \end{cases}$	**M1A1**
$= 0$ for $x < 0$ and $= 1$ for $x > 5$	**A1**
(b)	
$\mathrm{F}(m) = \frac{1}{2} \Rightarrow 2m^2 - 20m + 35 = 0$	**M1**
$m = 5 - \frac{1}{2}\sqrt{30} = 2.26$	**A1**
(c)	
$\mathrm{E}(X^2) = \int_0^2 x^2 \cdot \frac{x}{5}\, dx + \int_2^5 \frac{2}{15}(5-x)x^2\, dx = \left[\frac{x^4}{20}\right] + \left[\frac{2}{15}\left(\frac{5}{3}x^3 - \frac{x^4}{4}\right)\right]$	**M1**
$= 6.5$	**A1**
(d)	
$\mathrm{F}(3) - \mathrm{F}(1) = \frac{11}{15} - \frac{1}{10}$	**M1**
$= \frac{19}{30}$	**A1**
	Total: 9

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Q22

9231/41, 9231/43 • October/November 2020 • Question 6

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Mark scheme.

UQ: $\mathrm{F}(u) = 0.75 \Rightarrow u^2 - 16u + 45 = 0$	M1	**(a)**
$u = 8 - \sqrt{19}$ $(= 3.64)$ – value of UQ	**A1**	
LQ: $l^2 - 16l + 15 = 0$, $l = 1$ – value of LQ	**A1**	
FT IQR $=$ UQ $-$ LQ $= 7 - \sqrt{19} = 2.64$	**A1**	
B1 $\mathrm{f}(x) = \frac{1}{30}(8 - x)$, $0 \leqslant x \leqslant 6$ (may be implied)		**(b)**
Attempt integration $\frac{1}{30}\int_0^6 (8x^3 - x^4)\,\mathrm{d}x = \left[2x^4 - \frac{1}{5}x^5\right]_0^6$ (correct integrated expression, with limits)	**M1 A1**	
$= 34.56$	**A1**	
M1 $\mathrm{G}(y) = \frac{1}{60}(16y^2 - y^4)$ – CDF for Y		**(c)**
Differentiate to find PDF for Y: $\mathrm{g}(y) = \frac{1}{60}(32y - 4y^3)$ for $0 \leqslant y \leqslant \sqrt{6}$	**M1 A1**	
		Total: 11

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Q23

9231/42 • October/November 2020 • Question 4

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Mark scheme.

B1 $F(6) - F(3) = \frac{9}{20}$ or 0.45	**(a)**
B1 $f(x) = \frac{1}{30}x,\ 2 \leqslant x \leqslant 8$ (may be implied)	**(b)**
$E(\sqrt{X}) = \int_2^8 \frac{1}{30}x^{3/2}\,dx = \left[\frac{1}{75}x^{5/2}\right]$ – integrated	**M1**
$= 2.34$ (2.338...); answer of 2.34 without working scores B1B1 (2/3)	**A1**
M1 $\mathrm{Var}(\sqrt{X}) = \left[\int_2^8 \frac{1}{30}x^2\,dx = \left[\frac{1}{90}x^3\right]_2^8\right] - E(\sqrt{X})^2$	**(c)**
$= 5.6 - 2.338^2 = 0.133$ or 0.134	**A1**
M1 $G(y) = \frac{1}{60}y^{2/3} - \frac{1}{15}$ – CDF for Y	**(d)**
$g(y) = \frac{1}{90}y^{-1/3}$ for $8 \leqslant y \leqslant 512$ (0 otherwise) – differentiate; correct $g(y)$ and range	**M1 A1**
	Total: 9

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Q24

9231/41, 9231/42 • May/June 2021 • Question 3

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Mark scheme.

(a)

$\int_0^9 \frac{2}{81}x^{1.5}\,dx$ (using $f(x) = \frac{2}{81}x$, correct expression as integrand)	**B1**
$\frac{2}{81} \times \frac{2}{5}x^{2.5}$ (integrate, FT only on their pdf expression)	**M1**
$\frac{12}{5} = 2.4$	**A1**

(b)

$\int_0^9 \frac{2}{81}x^2\,dx - (\text{their } a)^2$	**M1**
$\frac{2}{81} \times \frac{9^3}{3} - 2.4^2 = \frac{6}{25}$	**A1**

(c)

$G(y) = \frac{1}{81}y^6$	**M1**
$g(y) = \frac{2}{27}y^5$	**M1**
Fully correct including "for $0 \leqslant y \leqslant \sqrt[3]{9}$ and 0 otherwise" (condone 2.08)	**A1**

Total: 8

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Q25

9231/43 • May/June 2021 • Question 6

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Mark scheme.

(a)	
Integrate both parts (all powers of x increase by 1, allow M1 if no constant term)	**M1**
$F(x) = \begin{cases} \frac{1}{8}x & 0 \leqslant x < 1, \\ \frac{1}{28}\left(8x - \frac{1}{2}x^2\right) - \frac{1}{7} & 1 \leqslant x \leqslant 8. \end{cases}$ (both parts correct, with domains; any equivalent form, e.g. $\frac{2}{7}x - \frac{1}{56}x^2 - \frac{1}{7}$)	**A1**
$F(x) = 0$ for $x < 0$ and $F(x) = 1$ for $x > 8$	**A1**
(b)	
$F(a) = \frac{5}{7}$ leading to $\frac{1}{28}\left(8a - \frac{1}{2}a^2\right) - \frac{1}{7} = \frac{5}{7}$ (correct method and attempt to simplify, ft their part (a))	**M1**
$a^2 - 16a + 48 = 0$, $a = 4$ $[a = 12]$ (obtain quadratic and solve)	**DM1**
$a = 4$ (correct single answer, WWW)	**A1**
(c)	
Substitute $x = y^3$ into both parts of their $F(x)$ from part (a)	**M1**
$G(y) = \begin{cases} \frac{1}{8}y^3 & 0 \leqslant y < 1, \\ \frac{1}{56}(16y^3 - y^6) - \frac{1}{7} & 1 \leqslant y \leqslant 2. \end{cases}$ (correct functions in terms of y, any equivalent form)	**A1**
$0 \leqslant y < 1$ and $1 \leqslant y \leqslant 2$ seen in CDF or PDF (condone $\leqslant$ instead of $<$)	**A1**
Differentiate their $G(y)$, all powers of y decrease by 1	**M1**
$g(y) = \begin{cases} \frac{3}{8}y^2 & 0 \leqslant y < 1, \\ \frac{3}{28}(8y^2 - y^5) & 1 \leqslant y < 2, \\ 0 & \text{otherwise.} \end{cases}$ (must include 'otherwise')	**A1**
	Total: 11

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Q26

9231/41, 9231/43 • October/November 2021 • Question 2

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Mark scheme.

Differentiation attempted.	**M1**
All correct, including 0 otherwise.	**A1**

$\boxed{\text{f}(x) = 1 + x \text{ for } -1 \leqslant x \leqslant 0;\ \text{f}(x) = 1 - x \text{ for } 0 < x \leqslant 1;\ 0 \text{ otherwise.}}$

(b)

Can do by integration with correct limits: $\text{F}\left(\frac{1}{2}\right) - \text{F}\left(-\frac{1}{2}\right) = \frac{7}{8} - \frac{1}{8}$.	**M1**
$\frac{3}{4}$.	**A1**

$\boxed{\frac{3}{4}}$

(c)

$\int_{-1}^{0} x^2(1+x)\,dx + \int_{0}^{1} x^2(1-x)\,dx$.	**M1**
$\frac{1}{12} + \frac{1}{12} = \frac{1}{6}$.	**A1**

$\boxed{\frac{1}{6}}$

(d)

Complete method: $\int_{-1}^{0} x^4(1+x)\,dx + \int_{0}^{1} x^4(1-x)\,dx - \left(\frac{1}{6}\right)^2$, with their mean squared explicit.	**M1**
$\frac{1}{15} - \frac{1}{36} = \frac{7}{180}$.	**A1**

Total: 8

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Q27

9231/42 • October/November 2021 • Question 3

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Mark scheme.

Total probability = 1: correct expression integrated and equated to 1, powers of x correct.	***M1**
Limits used and attempt to solve: $3a - \frac{1}{5} = 1$.	**DM1**
$a = \frac{2}{5}$.	**A1**

$\boxed{a = \frac{2}{5}}$

(b)

Correct expressions integrated, FT their a.	**M1**
$\frac{11}{60} + \frac{67}{60} = \frac{13}{10}$.	**A1**

$\boxed{\frac{13}{10}}$

(c)

Integration of their pdf.	**M1**
Middle 2 parts correct.	**A1**
First and last parts correct, and all domains correct with no gaps.	**A1**

Total: 8

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Q28

9231/41, 9231/42 • May/June 2022 • Question 3

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Mark scheme.

$k\left[2x^2 - \frac{1}{3}x^3\right]_0^2 + k\left[6x - \frac{1}{2}x^2\right]_2^6 = 1 \Rightarrow k\left[8 - \frac{8}{3} + 36 - 18 - 12 + 2\right] = \frac{40}{3}k = 1 \Rightarrow k = \frac{3}{40}$ (integrate and equate to 1, no errors seen). **(b)**	**B1**
$E(X^2) = k\int_0^2 (4x^3 - x^4)\,dx + k\int_2^6 (6x^2 - x^3)\,dx$ (attempt at $E(X^2)$, correct limits, integrated).	**M1***
$= k\left[x^4 - \frac{1}{5}x^5\right] + k\left[2x^3 - \frac{1}{4}x^4\right] = k[9.6 + 96] = 7.92.$	
$\mathrm{Var}(X) = E(X^2) - E^2(X) = 7.92 - 2.5^2$ (use of correct formula, with their $E(X^2)$).	**depM1**
1.67. **(c)**	**A1**
$\int_0^2 kx(4-x)\,dx + \int_2^m k(6-x)\,dx = \frac{1}{2}$ (correct expression and attempt to integrate).	**M1***
$\int_0^2 kx(4-x)\,dx = \frac{2}{5}$ (may be implied).	**B1**
$\frac{2}{5} + k\left[6x - \frac{1}{2}x^2\right] = \frac{1}{2} \Rightarrow 6m - \frac{1}{2}m^2 - 12 + 2 = \frac{1}{10k} \Rightarrow m^2 - 12m + \frac{68}{3} = 0$ (first integral and $\frac{1}{2}$ may be seen combined as $\frac{1}{10}$; obtain quadratic equation).	**depM1**
$m = 6 \pm \frac{2}{3}\sqrt{30}$; $m = 6 - \frac{2}{3}\sqrt{30}$ $(= 2.35)$ (single answer).	**A1**
	Total: 8

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Q29

9231/43 • May/June 2022 • Question 4

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Mark scheme.

$\mathrm{E}(\sqrt{X}) = \int \sqrt{x}\,\frac{3}{8}\left(1+\frac{1}{x^2}\right)dx = \int_1^3 \frac{3}{8}\left(x^{0.5}+x^{-1.5}\right)dx.$	**M1**
$\frac{3}{8}\left[\frac{2}{3}x^{1.5} - 2x^{-0.5}\right].$	**A1**
$\frac{3}{8}\left(2\sqrt{3} - \frac{2}{\sqrt{3}} - \frac{2}{3} + 2\right) = \frac{1}{2}(1+\sqrt{3})\ (= 1.37)$. **(b)**	**A1**
$F(x) = \frac{3}{8}\left(x - \frac{1}{x}\right)$ (integrate).	**M1**
$G(y) = \frac{3}{8}\left(\sqrt{y} - \frac{1}{\sqrt{y}}\right)$ (change variable throughout in their CDF).	**M1**
$g(y) = \frac{3}{16}\left(y^{-0.5}+y^{-1.5}\right)$, $(1 \leqslant y \leqslant 9)$ (differentiate to find $g(y)$).	**A1**
0 otherwise ($1 \leqslant y \leqslant 9$ and 0 otherwise, dependent on second M1). **(c)**	**A1**
$F(p) = 0.4$ so $\frac{3}{8}\left(p - \frac{1}{p}\right) = 0.4$ (correct method).	**M1***
$15p^2 - 16p - 15 = 0 \Rightarrow (5p+3)(3p-5) = 0,\ p = \frac{5}{3}$ (obtain quadratic and solve).	**depM1**
40th percentile of Y is $\frac{25}{9}$.	**A1**
	Total: 10

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Q30

9231/41, 9231/43 • October/November 2022 • Question 5

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Mark scheme.

$\mathrm{F}(U) = 0.75$, so $1 - \frac{1}{144}(12-U)^2 = \frac{3}{4}$	**M1**
$(12-U)^2 = 36$, $12 - U = \pm 6$, $U = 6$ only **(b)**	**A1**
$f(x) = \frac{1}{72}(12-x)$ (differentiate given CDF)	**B1**
$\mathrm{E}(X^2) = \int_0^{12} \frac{1}{72}(12x^2 - x^3)\,dx = \frac{1}{72}\left(4x^3 - \frac{1}{4}x^4\right)$, $[= 24]$ (integrate $x^2 f(x)$)	**M1**
$\mathrm{E}(X^4) = \int_0^{12} \frac{1}{72}(12x^4 - x^5)\,dx = \frac{1}{72}\left(\frac{12}{5}x^5 - \frac{1}{6}x^6\right)$, $[= 1382.4]$ (integrate $x^4 f(x)$)	**M1**
$\mathrm{Var}(X^2) = \mathrm{E}(X^4) - \mathrm{E}^2(X^2)$, $[= 1382.4 - 24^2]$ using correct formula	**M1**
$1382.4 - 576 = 806(.4)$ **(c)**	**A1**
Change the variable: $\mathrm{G}(y) = 1 - \frac{1}{144}(12 - y^2)^2$ (domain allowed unchanged at this stage)	**M1**
Differentiate their $\mathrm{G}(y)$: $\frac{1}{36}(12y - y^3)$	**M1**
$g(y) = \begin{cases} \frac{1}{36}(12y - y^3) & 0 \le y \le \sqrt{12} \\ 0 & \text{otherwise} \end{cases}$	**A1**
	Total: 10

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Q31

9231/42 • October/November 2022 • Question 4

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Mark scheme.

$\int_0^1 k\,dx + \int_1^2 kx\,dx = [kx] + \left[\frac{k}{2}x^2\right]$, $k + \frac{3k}{2} = 1$, $k = \frac{2}{5}$ (integration and equation seen) **(b)**	**B1**
$F(x) = \begin{cases} kx & 0 \le x < 1 \\ \frac{k}{2}x^2 + \frac{1}{5} & 1 \le x \le 2 \end{cases}$ (CDF or integration with $\frac{1}{5}x^2 + \frac{1}{5}$ seen)	**M1**
UQ: $\frac{k}{2}x^2 + \frac{1}{5} = \frac{3}{4}$ (method for UQ)	**M1**
$x^2 + 1 = 5 \times \frac{3}{4}$, $x^2 = \frac{11}{4}$, $x = \frac{1}{2}\sqrt{11}$	**A1**
LQ: $kx = \frac{1}{4}$, $x = \frac{5}{8}$	**B1**
IQR $= \frac{1}{2}\sqrt{11} - \frac{5}{8} = 1.03(3)$ **(c)**	**A1**
$E(X) = \int_0^1 kx\,dx + \int_1^2 kx^2\,dx = \left[\frac{k}{2}x^2\right] + \left[\frac{k}{3}x^3\right] = \frac{17}{15}$ (with correct limits)	**M1**
$E(X^2) = \int_0^1 kx^2\,dx + \int_1^2 kx^3\,dx = \left[\frac{k}{3}x^3\right] + \left[\frac{k}{4}x^4\right] = \frac{49}{30}$ (with correct limits)	**M1**
$\mathrm{Var}(X) = \frac{49}{30} - \left(\frac{17}{15}\right)^2$ (using correct formula with numerical values)	**M1**
$\frac{157}{450} (= 0.349)$	**A1**
	Total: 10

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Q32

9231/41, 9231/42 • May/June 2023 • Question 6

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Mark scheme.

$\mathrm{F}(x) = \frac{3}{28}\left(2\mathrm{e}^{\frac{1}{2}x} - 8\mathrm{e}^{-\frac{1}{2}x}\right)(+c)$. $+c$ not required.	**M1**
$\mathrm{F}(x) = \frac{3}{28}\left(2\mathrm{e}^{\frac{1}{2}x} - 8\mathrm{e}^{-\frac{1}{2}x}\right) + \frac{9}{14}$.	**A1**
Correct ranges including $0 \leqslant x \leqslant 2\ln 3$ associated with their $\mathrm{F}(x)$; $\mathrm{F}(x) = 0$ for $x < 0$ and 1 for $x > 2\ln 3$. No gaps in range. **(b)**	**B1**
$\mathrm{G}(y) = \frac{3}{28}\left(2y - \frac{8}{y} + 6\right)$. y substituted into their F.	**M1**
$1 \leqslant y \leqslant 3$. Seen anywhere, condone $1 \leqslant y \leqslant \mathrm{e}^{\ln 3}$.	**B1**
$\mathrm{g}(y) = \frac{3}{28}\left(2 + \frac{8}{y^2}\right)$. Correct expression, not containing logs. **(c)**	**A1**
$\frac{3}{28}\left(2y - \frac{8}{y} + 6\right) = \frac{3}{10}$. Their $\mathrm{G}(y) = \frac{3}{10}$.	**M1**
$5y^2 + 8y - 20 = 0$. Obtain from $\mathrm{G}(y)$ and solve quadratic equation to find y.	**M1**
$y = 1.35$. Single correct answer. **(d)**	**A1**
$\mathrm{E}(Y^4) = \int_1^3 \frac{3}{28}\left(2 + \frac{8}{y^2}\right) y^4 \,\mathrm{d}y = \int_1^3 \frac{3}{28}\left(2y^4 + 8y^2\right) \mathrm{d}y = \frac{3}{28}\left[\frac{2}{5}y^5 + \frac{8}{3}y^3\right]$. Integrate $y^4 \times$ their $\mathrm{g}(y)$. Limits must be 1 and 3.	**M1**
$17.8\ (= \frac{89}{5})$.	**A1**
	Total: 11

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Q33

9231/43 • May/June 2023 • Question 1

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Mark scheme.

$\mathrm{F}(x) = \frac{1}{6}\left[\frac{3}{2}x^{\frac{2}{3}} - 3x^{\frac{1}{3}}\right] + c$. Integrate $\mathrm{f}(x)$, $+c$ not required, at least one power correct.	**M1**
$\frac{1}{4}x^{\frac{2}{3}} - \frac{1}{2}x^{\frac{1}{3}} + \frac{1}{4}$. AEF.	**A1**
Correct ranges including $1 \leqslant x \leqslant 27$ associated with their $\mathrm{F}(x)$; $\mathrm{F}(x) = 0$ for $x < 1$ and $\mathrm{F}(x) = 1$ for $x > 27$. No gaps. **(b)**	**A1**
$\mathrm{G}(y) = \frac{1}{4}\left(y^2 - 2y + 1\right)$. Using $y = x^{\frac{1}{3}}$ in their $\mathrm{F}(x)$.	**M1**
$\mathrm{g}(y) = \frac{1}{2}(y-1)$. Correct, AEF, 0 otherwise not required.	**A1**
For $1 \leqslant y \leqslant 3$. Seen anywhere, correct variable. **(c)**	**B1**
$\frac{1}{4}\left(y^2 - 2y + 1\right) = \frac{1}{2}$, $(y-1)^2 = 2$ OE. Equate their $\mathrm{G}(y)$ to $\frac{1}{2}$ and solve to find y.	**M1**
$y = 1 + \sqrt{2}$. Only.	**A1**
	Total: 8

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Q34

9231/41, 9231/43 • October/November 2023 • Question 4

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Mark scheme.

$\mathrm{P}(X \leqslant 2) = \frac{1}{3}$, so $\left[\frac{mx^2}{2}\right]_0^2 = \frac{1}{3} \Rightarrow m\left(\frac{4}{2}\right) = \frac{1}{3} \Rightarrow m = \frac{1}{6}$ (AG, reasoning required).	**B1**
$\int_2^6 \left(\frac{k}{x^2} + c\right) \mathrm{d}x = \frac{2}{3}$, giving $4c + \frac{1}{3}k = \frac{2}{3}$ (or $k + 12c = 2$).	**M1**
Matching at $x = 2$: $2m = \frac{k}{4} + c$, i.e. $\frac{1}{3} = \frac{k}{4} + c$ (or $3k + 12c = 4$).	**M1**
Solve: $k = 1$, $c = \frac{1}{12}$. For part (b):	**A1**
LQ: $\int_0^L \frac{1}{6}x \,\mathrm{d}x = \frac{1}{4}$, giving $L = \sqrt{3}$.	**B1**
UQ: $\frac{1}{3} + \int_2^U \left(\frac{k}{x^2} + c\right) \mathrm{d}x = \frac{3}{4}$ (or use CDF).	**M1**
Attempt at integral with correct limits; simplify to quadratic $U^2 - U - 12 = 0$.	**M1**
$U = 4$.	**A1**
IQR $= 4 - \sqrt{3}$ (FT their UQ − their LQ, exact values only).	**A1**
	Total: 9

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Q35

9231/42 • October/November 2023 • Question 4

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Mark scheme.

UQ $= 4$ so $\int_4^6 c\,\mathrm{d}x = \frac{1}{4}$: $c[6-4] = \frac{1}{4}$, $c = \frac{1}{8}$ (AG).	**B1**
$\int_0^4 \frac{1}{128}(4ax - bx^3)\mathrm{d}x = \frac{3}{4}$, giving $32a - 64b = 96$.	**M1**
Equate at $x = 4$: $a - 4b = 1$.	**M1**
Solve: $a = 5$, $b = 1$. For part (b):	**A1**
$\int_0^m \frac{1}{128}(20x - x^3)\mathrm{d}x = \frac{1}{2}$ (or use CDF, could be in terms of a, b, c).	**M1***
Simplify to quartic $m^4 - 40m^2 + 256 = 0$.	**M1**
$m = 2\sqrt{2}$ (positive answer only). For part (c):	**A1**
$\mathrm{E}(\sqrt{X}) = \frac{1}{128}\int_0^4 \left(20x^{3/2} - x^{7/2}\right)\mathrm{d}x + \int_4^6 \frac{1}{8}x^{1/2}\,\mathrm{d}x$.	**M1**
Correct use of correct limits in the integral.	**M1**
$= \frac{4}{9} + \frac{1}{2}\sqrt{6} = 1.67$ (CAO).	**A1**
	Total: 10

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Q36

9231/41, 9231/42 • May/June 2024 • Question 7

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Mark scheme.

$\mathrm{f}(x) = \dfrac{x}{4}(4-x^2)$, $0 \le x \le 2$, 0 otherwise (given). [Working: $(E(X) =)$ $\int_0^2 \frac{1}{4}(4x^2-x^4)\,dx = \frac{1}{4}\left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right] = \frac{16}{15}$ $(= 1.067)$, shown but not separately marked.]	**B1**
$(E(\sqrt{X}) =)$ $\int_0^2 \frac{1}{4}(4x^{1.5}-x^{3.5})\,dx = \frac{1}{4}\left[\frac{8}{5}x^{2.5} - \frac{2}{9}x^{4.5}\right]_0^2 = \frac{32}{45}\sqrt{2} = 1.005(7)$. Integration, correct powers required, ignore limits.	**M1**
$\mathrm{Var}(\sqrt{X}) = E(X) - \left(E(\sqrt{X})\right)^2$ used. Correct formula used following attempt at integration of both terms.	**M1**
$\left[\frac{16}{15} - \left(\frac{32}{45}\sqrt{2}\right)^2\right] = 0.0553$ $\left(= \frac{112}{2025}\right)$. (b)	**A1**
$F(x) = \frac{1}{16}(8x^2 - x^4)$, $[0 \le x \le 2]$.	**B1**
$G(y) = \frac{1}{16}(8y - y^2)$, $[0 \le y \le 4]$. Change variable in their $F(x)$.	**M1**
$g(y) = \frac{1}{16}(8-2y)$, $[0 \le y \le 4]$. Differentiate their $G(y)$.	**M1**
$g(y) = \begin{cases} \frac{1}{16}(8-2y) & 0 \le y \le 4, \\ 0 & \text{otherwise.} \end{cases}$ Complete and correct. (c)	**A1**
$\frac{1}{16}(8m - m^2) = \frac{1}{2}$. Equate their $G(y)$ to $\frac{1}{2}$.	**M1**
$[m^2 - 8m + 8 = 0,]$ $m = 4 - 2\sqrt{2}$. Single answer in an exact form.	**A1**
	Total: 10

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Q37

9231/43 • May/June 2024 • Question 5

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Mark scheme.

Attempt at differentiating both parts of CDF to obtain linear expressions: $\dfrac{x-2}{6}$, $\dfrac{8-x}{12}$. May be implied by first A1.	**M1**
Correct shape (two line segments touch the x-axis and meet at a point above the x-axis).	**A1**
Correct shape with 2 and 8 and at least one of 4 or 1/3 correctly labelled. (b)	**A1**
$\int_2^4 \frac{1}{6}x(x-2)\,dx + \int_4^8 \frac{1}{12}x(8-x)\,dx$, FT their PDF, with correct limits.	**B1 FT**
$\frac{1}{6}\left[\frac{1}{3}x^3 - x^2\right] + \frac{1}{12}\left[4x^2 - \frac{1}{3}x^3\right]$. Attempt at integration with their PDF (not CDF), no limits needed. May be implied by correct answer.	**M1**
$\dfrac{14}{3}$ (accept 4.67 or any equivalent fraction). (c)	**A1**
LQ: $\frac{1}{12}(x-2)^2 = \frac{1}{4}$, $x = 2 + \sqrt{3}$ (allow 3.73).	**B1**
UQ: $1 - \frac{(8-x)^2}{24} = \frac{3}{4}$, $x = 8 - \sqrt{6}$ (allow 5.55).	**B1**
IQR = UQ − LQ, with values found for both quartiles.	**M1**
$6 - \sqrt{6} - \sqrt{3}$. Exact answer required.	**A1**
	Total: 10

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Q38

9231/41, 9231/43 • October/November 2024 • Question 4

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Mark scheme.

(a)	
Correct sketch (label 1 and 5 on x-axis).	**B1**
(b)	
$\frac{kx^4}{4}$ with limits 0 and 1 gives $\frac{k}{4}$; $k\left(5x - \frac{x^2}{2}\right)$ with limits 1 and 5 gives $8k$.	**B1**
$\frac{k}{4} + 8k = 1 \Rightarrow k = \frac{4}{33}$ (AG).	**B1**
(c)	
Integrate both parts.	**M1**
Middle two parts correct AEF: $\mathrm{F}(x) = \frac{1}{33}x^4$ for $0 \le x < 1$; $\frac{4}{33}\left(5x - \frac{1}{2}x^2 - \frac{17}{4}\right)$ for $1 \le x \le 5$.	**A1**
$\mathrm{F}(x) = 0$ for $x < 0$ and $\mathrm{F}(x) = 1$ for $x > 5$ correct.	**A1**
(d)	
$\int_0^1 kx^3\,dx + \int_1^m k(5-x)\,dx = \frac{1}{2}$ (or integrate m to 5, or use $\mathrm{F}(x) = 0.5$).	**M1***
Reduce to a quadratic in m.	**M1**
$4m^2 - 40m + 67 = 0$.	**DM1**
$m = \frac{1}{2}(10 - \sqrt{33}) = 2.13$.	**A1**
	Total: 10

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Q39

9231/42 • October/November 2024 • Question 4

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Mark scheme.

(a)

Integrate $\frac{1}{21}(x^3 - 3x^2 + 3x)[+c]$ oe.	**M1**
Correct F(x) (with $c = -\frac{2}{63}$ evaluated): $\mathrm{F}(x) = \frac{1}{63}\left[(x-1)^3 - 1\right]$ for $2 \le x \le 5$.	**A1**
$\mathrm{F}(x) = 0$ for $x < 2$ and $\mathrm{F}(x) = 1$ for $x > 5$, inequalities correct.	**A1**

(b)

Use transformation correctly on the function: $G(y) = \frac{1}{63}(y^{0.75} - 1)$.	**M1**
Differentiate and change variable in domain.	**M1**
$\mathrm{g}(y) = \frac{1}{84}y^{-0.25}$ for $1 \le y \le 256$, 0 otherwise – all correct and simplified.	**A1**

(c)

Equate their $G(y)$ to 0.5 and attempt to solve: $\frac{1}{63}(y^{0.75} - 1) = 0.5$.	**M1**
$y = 104$ (AWRT 104).	**A1**

(d)

$\int_1^{256} \frac{1}{84}y(y^{-0.25})\,dy = \int_1^{256} \frac{1}{84}(y^{0.75})\,dy = \left[\frac{1}{84} \times \frac{4}{7}y^{1.75}\right]$.	**M1**
$\frac{5461}{49} = 111$.	**A1**

Total: 10

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Q40

9231/41, 9231/42 • May/June 2025 • Question 2

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Mark scheme.

B1 $5a + \frac{3}{2}a = 1 \Rightarrow a = \frac{2}{13}$ (AG, shown convincingly).	**(a)**
M1 Attempt to solve for b and c using area $= 1$, eliminating b, c.	
A1 $b = \frac{16}{39}$, $c = \frac{2}{39}$.	
M1 Attempt to integrate $x\mathrm{f}(x)$ with their b, c, limits seen.	**(b)**
M1 Correct integrated expression with their b, c, including limits.	
A1 $\frac{43}{13} = 3.31$ (AWRT 3.31).	
M1 Uses $0 < m < 5$ and area $= 0.5$.	**(c)**
A1 $m = \frac{13}{4} = 3.25$.	
B1 FT Correct expressions for integrating both pieces of pdf, using their b, c.	**(d)**
B1 FT $k = -\frac{25}{39}$.	
M1 Both parts with $\sqrt{y}$ substituted (condone missing k), ignore limits.	
A1 FT $\mathrm{G}(y) = \begin{cases} 0 & y < 0 \\ \frac{2}{13}\sqrt{y} & 0 \leqslant y \leqslant 25 \\ \frac{16}{39}\sqrt{y} - \frac{1}{39}y - \frac{25}{39} & 25 < y \leqslant 64 \\ 1 & y > 64 \end{cases}$	

Total: 12

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Q41

9231/43 • May/June 2025 • Question 3

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Mark scheme.

M1 Linear equation in k formed following attempt to integrate with correct limits: $\int_0^1 kx\,dx + \int_1^8 k(8-x)\,dx = 1$.	**(a)**
A1 $25k = 1$, leading to $k = \frac{1}{25}$ (AG, shown convincingly).	
*M1 Use of definite integrals and attempt at integration to form equation with 0.5 and their k.	**(b)**
DM1 Quadratic equation in m: $m^2 - 16m + 39 = 0$.	
A1 $m = 3$ (reject other root). CAO.	
M1 $\mathrm{F}(x) = \begin{cases} \frac{1}{50}x^2 & [0 \leqslant x < 1] \\ \frac{8}{25}x - \frac{1}{50}x^2 - \frac{7}{25} & [1 \leqslant x \leqslant 8] \end{cases}$.	**(c)**
B1 For $-\frac{7}{25}$.	
M1 Both parts with y substituted in their $\mathrm{F}(x)$: $\mathrm{G}(y) = \begin{cases} \frac{1}{50}y^6 & [0 \leqslant y < 1] \\ \frac{8}{25}y^3 - \frac{1}{50}y^6 - \frac{7}{25} & [1 \leqslant y \leqslant 2] \end{cases}$.	
M1 Differentiation of their $\mathrm{G}(y)$.	
A1 $\mathrm{g}(y) = \begin{cases} \frac{3}{25}y^5 & 0 \leqslant y < 1 \\ \frac{24}{25}y^2 - \frac{3}{25}y^5 & 1 \leqslant y \leqslant 2 \\ 0 & \text{otherwise.} \end{cases}$	
	Total: 10

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Q42

9231/44 • May/June 2025 • Question 4

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Mark scheme.

M1 Linear equation in k formed following attempt to integrate (all powers correct) and correct limits: $\int_0^1 kx\,dx + \int_1^2 kx^2\,dx = 1$.	**(a)**
A1 $\frac{17}{6}k = 1$, leading to $k = \frac{6}{17}$ (AG, shown convincingly).	
M1 Attempt to integrate both parts, all powers correct, condone missing c.	**(b)**
A1 Both parts correct with $c = \frac{1}{17}$.	
A1 $\mathrm{F}(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{17}x^2 & 0 \leqslant x < 1 \\ \frac{2}{17}x^3 + \frac{1}{17} & 1 \leqslant x \leqslant 2 \\ 1 & x > 2 \end{cases}$	
M1 Use $1 \leqslant m \leqslant 2$ and their $\mathrm{F}(x)$ to form equation with 0.5: $\frac{2}{17}m^3 + \frac{1}{17} = \frac{1}{2}$.	**(c)**
A1 $m^3 = \frac{15}{4}$, $m = \sqrt[3]{15/4} = 1.55$.	
M1 Correct method and attempt to integrate both parts: $\frac{6}{17}\int_0^1 x \cdot \frac{1}{x}\,dx + \frac{6}{17}\int_1^2 x^2 \cdot \frac{1}{x}\,dx$.	**(d)**
A1 $\frac{6}{17}(1 + 2 - \frac{1}{2}) = \frac{15}{17} = 0.882$ (CWO).	

Total: 9

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Q43

9231/41, 9231/43 • October/November 2025 • Question 5

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Mark scheme.

Equation in terms of k formed following attempt to integrate with correct limits: $\frac{1}{16}\left[\frac{2}{3}x^{3/2}\right]_0^4 + \frac{1}{k}\left[2\sqrt{x}\right]_4^9 = 1$.	**M1**
$\frac{2}{k} = \frac{2}{3}$, $k = 3$ (AG, no errors seen). (b)	**A1**
Use of $4 < m < 9$ to form equation equal to 0.5: $\frac{1}{16}\int_0^4 \sqrt{x}\,dx + \frac{1}{3}\int_4^m x^{-1/2}\,dx = \frac{1}{2}$.	***M1**
Integrate and form an equation in $\sqrt{m}$: $\frac{1}{3} + \frac{2}{3}(\sqrt{m} - 2) = \frac{1}{2}$.	**DM1**
$m = \frac{81}{16}$ (AWRT 5.06). (c)	**A1**
Attempt to integrate both parts of f to find $\mathrm{F}(x)$, limits not required.	**M1**
Constant -1 obtained in expression for $4 \le x \le 9$.	**B1**
Change to y: $\mathrm{G}(y) = \mathrm{F}(y^2)$.	**M1**
Differentiate to obtain $\mathrm{g}(y)$.	**M1**
$\mathrm{g}(y) = \begin{cases} \frac{1}{8}y^2 & 0 \le y < 2, \\ \frac{2}{3} & 2 \le y \le 3, \\ 0 & \text{otherwise}, \end{cases}$ fully correct with correct domain covering all reals.	**A1**
	Total: 10

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Q44

9231/42 • October/November 2025 • Question 4

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Mark scheme.

$a = -\frac{1}{5}$.	**B1**
$b = \frac{7}{25}$. (b)	**B1**
Attempt at differentiating both parts.	**M1**
$\mathrm{f}(x) = \frac{1}{5}$ for $1 \le x < 4$, $\frac{1}{25}x$ for $4 \le x \le 6$, 0 otherwise; all correct, fully defined including domain. (c)	**A1**
Attempt to integrate $x^2\mathrm{f}(x)$ with correct limits (can be implied by $\frac{73}{5}$ or 14.6).	**M1**
Variance formula used with their $\mathrm{E}(X^2)$: $\mathrm{Var}(X) = \frac{73}{5} - \left(\frac{529}{150}\right)^2$.	**M1**
2.16. (d)	**A1**
Correct method to find at least one of the 10th or 90th percentile, FT their a and b: $\frac{1}{5}x - \frac{1}{5} = \frac{1}{10}$; $\frac{1}{50}x^2 + \frac{7}{25} = \frac{9}{10}$.	**M1**
10th percentile $= 1.5$.	**A1**
90th percentile $= \sqrt{31}$ (AWRT 5.57).	**A1**
	Total: 10

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Q45

9231/44 • October/November 2025 • Question 2

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Mark scheme.

Attempt to integrate both parts, condone missing c, ignore domains.	**M1**
Both parts correct with correct c, ignore domains.	**A1**
Correct domains, must cover all reals: $\mathrm{F}(x) = 0$ for $x < 0$; $\frac{2}{9}x^2 + \frac{4}{9}x$ for $0 \le x < 1$; $\frac{1}{3}(x-2)^3 + 1$ for $1 \le x \le 2$; 1 for $x > 2$. (b)	**B1**
Identifies correct interval for their piecewise $\mathrm{F}(x)$ to form equation $\mathrm{F}(m) = \frac{1}{2}$: $\frac{2}{9}m^2 + \frac{4}{9}m = \frac{1}{2}$.	**M1**
$m = \frac{1}{2}(-2 + \sqrt{13})$. (c)	**A1**
Substitute $x = y^2$ into both parts of their $\mathrm{F}(x)$, ignore domains.	**M1**
Correct functions in terms of y: $\mathrm{G}(y) = \frac{2}{9}y^4 + \frac{4}{9}y^2$ for $0 \le y < 1$; $\frac{1}{3}(y^2-2)^3 + 1$ for $1 \le y \le \sqrt{2}$.	**A1**
Correct domain in terms of y, condone missing 0 for $y < 0$ but must have 1 for $y > \sqrt{2}$. (d)	**B1**
Evaluates G(their m).	**M1**
$\mathrm{G}\left(\frac{1}{2}(-2+\sqrt{13})\right) = 0.379 < 0.5$, so the median of Y is greater than the median of X.	**A1**
	Total: 10

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