



Further Math with Ali Hashir

Chi-Squared Tests

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at vectorspacepk@gmail.com.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/23 • May/June 2015 • Question 6

The reliability of the broadband connection received from two suppliers, A and B , is classified as good, fair or poor by a random sample of householders. The information collected is summarised in the following table.

		Reliability		
		Good	Fair	Poor
Supplier	A	65	63	33
	B	51	44	44

Test, at the 5% significance level, whether reliability is independent of supplier. [8]

Q2

9231/21, 9231/22, 9231/23 • October/November 2015 • Question 8

The number of goals scored by a certain football team was recorded for each of 100 matches, and the results are summarised in the following table.

Number of goals	0	1	2	3	4	5	6 or more
Frequency	12	16	31	25	13	3	0

Fit a Poisson distribution to the data, and test its goodness of fit at the 5% significance level. [10]

Q3

9231/21, 9231/22 • May/June 2016 • Question 9

Applicants for a national teacher training course are required to pass a mathematics test. Each year, the applicants are tested in groups of 6 and the number of successful applicants in each group is recorded. The overall proportion of successful applicants has remained constant over the years and is equal to 60% of the applicants. The results from 150 randomly chosen groups are shown in the following table.

Number of successful applicants	0	1	2	3	4	5	6
Number of groups	1	3	25	51	38	30	2

Test, at the 5% significance level, the goodness of fit of the distribution $B(6, 0.6)$ for the number of successful applicants in a group. [10]

Q4

9231/23 • May/June 2016 • Question 9

A company produces four different flavours of ice cream: mint, strawberry, chocolate and fudge. Customers were asked which of the four flavours they preferred. The responses from a random sample of 80 male customers and 70 female customers are given in the following table.

	Mint	Strawberry	Chocolate	Fudge
Male	25	12	30	13
Female	16	22	25	7

Test, at the 10% significance level, whether there is a difference between ice cream flavour preferences of male and female customers. [9]

Q5

9231/21, 9231/22, 9231/23 • October/November 2016 • Question 9

The number of visitors arriving at an art exhibition is recorded for each 10-minute period of time during the ten hours that it is open on a particular day. The results are as follows.

Number of visitors in a 10-minute period	0	1	2	3	4	5	6	7	8	≥ 9
Number of 10-minute periods	2	2	12	8	11	13	4	7	1	0

(i) Calculate the mean and variance for this sample and explain whether your answers support a suggestion that a Poisson distribution might be a suitable model for the number of visitors in a 10-minute period. [3]

(ii) Use an appropriate Poisson distribution to find the two expected frequencies missing from the following table. [2]

Number of visitors in a 10-minute period	0	1	2	3	4	5	6	7	8	≥ 9
Expected number of 10-minute periods	1.10		8.79		11.72	9.38	6.25	3.57	1.79	1.28

(iii) Test, at the 10% significance level, the goodness of fit of this Poisson distribution to the data. [8]

Q6

9231/23 • May/June 2017 • Question 10

Roberto owns a small hotel and offers accommodation to guests. Over a period of 100 nights, the numbers of rooms, x , that are occupied each night at Roberto's hotel and the corresponding frequencies are shown in the following table.

Number of rooms occupied (x)	0	1	2	3	4	5	6	≥ 7
Number of nights	4	9	18	26	20	16	7	0

- (i) Show that the mean number of rooms that are occupied each night is 3.25. [1]

The following table shows most of the corresponding expected frequencies, correct to 2 decimal places, using a Poisson distribution with mean 3.25.

Number of rooms occupied (x)	0	1	2	3	4	5	6	≥ 7
Observed frequency	4	9	18	26	20	16	7	0
Expected frequency	3.88	12.60	20.48	22.18	18.02	11.72		

- (ii) Show how the expected value of 22.18, for $x = 3$, is obtained and find the expected values for $x = 6$ and for $x \geq 7$. [4]

- (iii) Use a goodness-of-fit test at the 5% significance level to determine whether the Poisson distribution is a suitable model for the number of rooms occupied each night at Roberto's hotel. [7]

Q7

9231/21, 9231/22, 9231/23 • October/November 2017 • Question 8

Members of a Statistics club are voting to elect a new president of the club. Members must choose to vote either by post or by text or by email. The method of voting chosen by a random sample of 60 male members and 40 female members is given in the following table.

		Method of voting		
		Post	Text	Email
Gender	Male	10	12	38
	Female	5	21	14

Test, at the 1% significance level, whether there is an association between method of voting and gender. [8]

Q8

9231/21, 9231/22 • May/June 2018 • Question 8

A manufacturer produces three types of car: hatchbacks, saloons and estates. Each type of car is available in one of three colours: silver, blue and red. The manufacturer wants to know whether the popularity of the colour of the car is related to the type of car. A random sample of 300 cars chosen by customers gives the information summarised in the following table.

		Colour of car		
		Silver	Blue	Red
Type of car	Hatchback	53	36	41
	Saloon	29	40	31
	Estate	28	24	18

Test at the 10% significance level whether the colour of car chosen by customers is independent of the type of car. [8]

Q9

9231/22 • October/November 2018 • Question 10

The number of accidents, x , that occur each day on a motorway are recorded over a period of 40 days. The results are shown in the following table.

Number of accidents	0	1	2	3	4	5	6	≥ 7
Observed frequency	3	5	8	10	5	7	2	0

- (i) Show that the mean number of accidents each day is 2.95 and calculate the variance for this sample. Explain why these values suggest that a Poisson distribution might fit the data. [3]

A Poisson distribution with mean 2.95, as found from the data, is used to calculate the expected frequencies, correct to 2 decimal places. The results are shown in the following table.

Number of accidents	0	1	2	3	4	5	6	≥ 7
Observed frequency	3	5	8	10	5	7	2	0
Expected frequency	2.09	6.18	9.11	8.96	6.61	3.90	1.92	1.23

- (ii) Show how the expected frequency of 6.61 for $x = 4$ is obtained. [2]
- (iii) Test at the 5% significance level the goodness of fit of this Poisson distribution to the data. [7]

Q10

9231/21, 9231/22 • May/June 2019 • Question 9

A random sample of 50 observations of the continuous random variable X was taken and the values are summarised in the following table.

Interval	$0 \leq x < 0.8$	$0.8 \leq x < 1.6$	$1.6 \leq x < 2.4$	$2.4 \leq x < 3.2$	$3.2 \leq x < 4$
Observed frequency	18	16	8	6	2

It is required to test the goodness of fit of the distribution with probability density function f given by

$$f(x) = \begin{cases} \frac{3}{16}(4-x)^{1/2} & 0 \leq x < 4, \\ 0 & \text{otherwise.} \end{cases}$$

The relevant expected frequencies, correct to 2 decimal places, are given in the following table.

Interval	$0 \leq x < 0.8$	$0.8 \leq x < 1.6$	$1.6 \leq x < 2.4$	$2.4 \leq x < 3.2$	$3.2 \leq x < 4$
Expected frequency	14.22	12.54	10.59	8.18	4.47

- (i) Show how the expected frequency for $1.6 \leq x < 2.4$ is obtained. [3]
- (ii) Carry out a goodness of fit test at the 5% significance level. [7]

Q11

9231/23 • May/June 2019 • Question 8

Two salesmen, A and B , work at a company that arranges different types of holidays: self-catering, hotel and cruise. The table shows, for a random sample of 150 holidays, the number of each type arranged by each salesman.

		Type of holiday		
		Self-catering	Hotel	Cruise
Salesman	A	25	38	21
	B	28	21	17

Test at the 10% significance level whether the type of holiday arranged is independent of the salesman. [8]

Q12

9231/41, 9231/42 • May/June 2020 • Question 1

Two randomly selected groups of students, with similar ranges of abilities, take the same examination in different rooms. One group of 140 students takes the examination with background music playing. The other group of 210 students takes the examination in silence. Each student is awarded a grade for their performance in the examination and the numbers from each group gaining each grade are shown in the following table.

	Grade awarded		
	A	B	C
Background music	49	51	40
Silence	93	68	49

Test at the 10% significance level whether grades awarded are independent of whether background music is playing during the examination. [6]

Q13

9231/43 • May/June 2020 • Question 1

Young children are learning to read using two different reading schemes, A and B . The standards achieved are measured against the national average standard achieved and classified as above average, average or below average. For two randomly chosen groups of young children, the numbers in each category are shown in the table.

	Standard achieved		
	Above average	Average	Below average
Scheme A	31	35	22
Scheme B	19	50	43

Test at the 5% significance level whether standard achieved is independent of the reading scheme used. [6]

Q14

9231/41, 9231/43 • October/November 2020 • Question 3

Apples are sold in bags of 5. Based on her previous experience, Freya claims that the probability of any apple weighing more than 100 grams is 0.35, independently of other apples in the bag.

The apples in a random sample of 150 bags are checked and the number, x , in each bag weighing more than 100 grams is recorded. The results are shown in the following table.

x	0	1	2	3	4	5
Frequency	12	39	46	37	12	4

Carry out a goodness of fit test at the 5% significance level and hence comment on Freya's claim. [7]

Q15

9231/42 • October/November 2020 • Question 3

A random sample of 200 observations of the continuous random variable X was taken and the values are summarised in the following table.

Interval	$0 \leq x < 0.5$	$0.5 \leq x < 1$	$1 \leq x < 1.5$	$1.5 \leq x < 2$	$2 \leq x < 2.5$	$2.5 \leq x < 3$
Observed frequency	5	23	40	41	46	45

It is required to test the goodness of fit of the distribution with probability density function f given by

$$f(x) = \begin{cases} \frac{1}{9}x(4-x) & 0 \leq x \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$

Most of the relevant expected frequencies, correct to 2 decimal places, are given in the following table.

Interval	$0 \leq x < 0.5$	$0.5 \leq x < 1$	$1 \leq x < 1.5$	$1.5 \leq x < 2$	$2 \leq x < 2.5$	$2.5 \leq x < 3$
Expected frequency	p	q	37.96	43.52	43.52	37.96

(a) Show that $p = 10.19$ and find the value of q . [3]

(b) Carry out a goodness of fit test, at the 5% significance level, to test whether f is a satisfactory model for the data. [4]

Q16

9231/41, 9231/42 • May/June 2021 • Question 2

A driving school employs four instructors to prepare people for their driving test. The allocation of people to instructors is random. For each of the instructors, the following table gives the number of people who passed and the number who failed their driving test last year.

	Instructor <i>A</i>	Instructor <i>B</i>	Instructor <i>C</i>	Instructor <i>D</i>	Total
Pass	72	42	52	68	234
Fail	33	34	41	58	166
Total	105	76	93	126	400

Test at the 10% significance level whether success in the driving test is independent of the instructor. [7]

Q17

9231/43 • May/June 2021 • Question 5

Chai packs china mugs into cardboard boxes. Chai's manager suspects that breakages occur at random times and that the number of breakages may follow a Poisson distribution. He takes a small sample of observations and finds that the number of breakages in a one-hour period has a mean of 2.4 and a standard deviation of 1.5.

- (a) Explain how this information tends to support the manager's suspicion. [2]

The manager now takes a larger sample and claims that the numbers of breakages in a one-hour period follow a Poisson distribution. The numbers of breakages in a random sample of 180 one-hour periods are summarised in the following table.

Number of breakages	0	1	2	3	4	5	6	7 or more
Frequency	21	33	46	31	23	16	10	0

The mean number of breakages calculated from this sample is 2.5.

- (b) Use the data from this larger sample to carry out a goodness of fit test, at the 10% significance level, to test the claim. [8]

Q18

9231/41, 9231/43 • October/November 2021 • Question 3

A supermarket sells pears in packs of 8. Some of the pears in a pack may not be ripe, and the supermarket manager claims that the number of unripe pears in a pack can be modelled by the distribution $B(8, 0.15)$.

A random sample of 150 packs was selected and the number of unripe pears in each pack was recorded. The following table shows the observed frequencies together with some of the expected frequencies using the manager's binomial distribution.

Number of unripe pears per pack	0	1	2	3	4	5	≥ 6
Observed frequency	35	48	43	15	6	3	0
Expected frequency	40.874	p	35.641	12.579	2.775	0.392	q

(a) Find the values of p and q . [2]

(b) Carry out a goodness of fit test, at the 5% significance level, to test whether the manager's claim is justified. [6]

Q19

9231/42 • October/November 2021 • Question 2

It is claimed that the heights of a particular age group of boys follow a normal distribution with mean 125 cm and standard deviation 12 cm. Observations for a randomly chosen group of 60 boys in this age group are summarised in the following table. The table also gives the expected frequencies, correct to 2 decimal places, based on the normal distribution with mean 125 cm and standard deviation 12 cm.

Height, x cm	$x < 100$	$100 \leq x < 110$	$110 \leq x < 120$	$120 \leq x < 130$	$130 \leq x < 140$	$x \geq 140$
Observed frequency	0	3	15	23	11	8
Expected frequency	1.12	5.22	13.97	19.38	13.97	6.34

- (a) Show how the expected frequency for $130 \leq x < 140$ is obtained. [2]
- (b) Carry out a goodness of fit test, at the 5% significance level, to determine whether the claim is supported by the data. [6]

Q20

9231/41, 9231/42 • May/June 2022 • Question 4

A scientist is investigating the numbers of a particular type of butterfly in a certain region. He claims that the numbers of these butterflies found per square metre can be modelled by a Poisson distribution with mean 2.5. He takes a random sample of 120 areas, each of one square metre, and counts the number of these butterflies in each of these areas. The following table shows the observed frequencies together with some of the expected frequencies using the scientist's Poisson distribution.

Number per square metre	0	1	2	3	4	5	6	≥ 7
Observed frequency	12	20	36	32	13	6	1	0
Expected frequency	9.85	24.63	30.78	25.65	p	8.02	3.34	q

- (a) Find the values of p and q , correct to 2 decimal places. [2]
- (b) Carry out a goodness of fit test, at the 10% significance level, to test the scientist's claim. [6]

Q21

9231/43 • May/June 2022 • Question 2

A scientist is investigating the size of shells at various beach locations. She selects four beach locations and takes a random sample of shells from each of these beaches. She classifies each shell as large or small. Her results are summarised in the following table.

		Beach location				
		<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	Total
Size of shell	Large	68	69	96	81	314
	Small	28	55	64	39	186
	Total	96	124	160	120	500

Test, at the 10% significance level, whether the size of shell is independent of the beach location. [7]

Q22

9231/41, 9231/43 • October/November 2022 • Question 2

An organisation runs courses to train students to become engineers. These students are taught in groups of 8. The director of the organisation claims that on average 60% of the students in a group achieve a pass. A random sample of 150 groups of 8 students is chosen. The following table shows the observed frequencies together with some of the expected frequencies using the appropriate binomial distribution.

Number of passes per group	0	1	2	3	4	5	6	7	8
Observed frequency	0	0	8	24	45	36	26	10	1
Expected frequency	p	1.180	6.193	18.579	34.836	q	r	13.437	2.519

- (a) Find the values of p , q and r giving your answers correct to 3 decimal places. [2]
- (b) Carry out a goodness of fit test, at the 10% significance level, to test whether there is evidence to reject the director's claim. [6]

Q23

9231/42 • October/November 2022 • Question 2

In the colleges in three regions of a particular country, students are given individual targets to achieve. Their performance is measured against their individual target and graded as 'above target', 'on target' or 'below target'. For a random sample of students from each of the three regions, the observed frequencies are summarised in the following table.

		Region			Total
		A	B	C	
Performance	Above target	62	41	44	147
	On target	102	94	95	291
	Below target	56	45	61	162
Total		220	180	200	600

Test, at the 10% significance level, whether performance is independent of region. [7]

Q24

9231/41, 9231/42 • May/June 2023 • Question 3

A random sample of 50 values of the continuous random variable X was taken. These values are summarised in the following table.

Interval	$1 \leq x < 1.5$	$1.5 \leq x < 2$	$2 \leq x < 2.5$	$2.5 \leq x < 3$	$3 \leq x < 3.5$	$3.5 \leq x \leq 4$
Observed frequency	3	3	8	11	13	12

It is required to test the goodness of fit of the distribution with probability density function f given by

$$f(x) = \begin{cases} \frac{1}{24} \left(\frac{4}{x^2} + x^2 \right) & 1 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

The expected frequencies, correct to 4 decimal places, are given in the following table.

Interval	$1 \leq x < 1.5$	$1.5 \leq x < 2$	$2 \leq x < 2.5$	$2.5 \leq x < 3$	$3 \leq x < 3.5$	$3.5 \leq x \leq 4$
Expected frequency	4.4271	a	6.1285	8.4549	b	14.9678

(a) Show that $a = 4.6007$ and find the value of b . [3]

(b) Carry out a goodness of fit test, at the 10% significance level, to test whether f is a satisfactory model for the data. [6]

Q25

9231/43 • May/June 2023 • Question 6

A scientist is investigating whether the ability to remember depends on age. A random sample of 150 students in different age groups is chosen. Each student is shown a set of 20 objects for thirty seconds and then asked to list as many as they can remember. The students are graded *A* or *B* according to how many objects they remembered correctly: grade *A* for 16 or more correct and grade *B* for fewer than 16 correct. The results are shown in the table.

	Age of students		
	11–12 years	13–14 years	15–16 years
Grade <i>A</i>	25	16	19
Grade <i>B</i>	28	45	17

(a) Carry out a χ^2 -test at the 2.5% significance level to test whether grade is independent of age of student. [7]

The scientist decides instead to use three grades: grade *A* for 16 or more correct, grade *B* for 10 to 15 correct and grade *C* for fewer than 10 correct. The results are shown in the following table.

	Age of students		
	11–12 years	13–14 years	15–16 years
Grade <i>A</i>	25	16	19
Grade <i>B</i>	12	27	11
Grade <i>C</i>	16	18	6

With this second set of data, the test statistic is calculated as 10.91.

(b) Complete the χ^2 -test at the 2.5% significance level for this second set of data. [2]

(c) State, with a reason, whether you would prefer to use the result from part (a) or part (b) to investigate whether the ability to remember depends on age. [1]

Q26

9231/41, 9231/43 • October/November 2023 • Question 2

A town council has published its plans for redeveloping the town centre and residents are being asked whether they approve or disapprove. A random sample of 250 responses has been selected from residents in the four main streets in the town: North, East, South and West Streets. The results are shown in the table.

	North Street	East Street	South Street	West Street
Approve	33	54	42	26
Disapprove	19	39	28	9

Test, at the 5% significance level, whether the opinions of the residents are independent of the streets on which they live. [7]

Q27

9231/42 • October/November 2023 • Question 2

The number of breakdowns on a particular section of road is recorded each day over a period of 90 days. It is suggested that the number of breakdowns follows a Poisson distribution with mean 3.5. The data is summarised in the table, together with some of the expected frequencies resulting from the suggested Poisson distribution.

Number of breakdowns per day	0	1	2	3	4	5	6	7	8 or more
Observed frequency	0	5	13	17	21	16	9	5	4
Expected frequency	2.718	9.512	16.646	–	16.993	11.895	–	3.469	2.407

(The expected frequencies for 3 and 6 breakdowns per day are missing from the table and are to be found in part (a).)

(a) Complete the table. [2]

(b) Carry out a goodness of fit test, at the 10% significance level, to determine whether or not $Po(3.5)$ is a good fit to the data. [6]

Q28

9231/41, 9231/42 • May/June 2024 • Question 5

Two companies, P and Q , produce a certain type of paint brush. An independent examiner rates the quality of the brushes produced as poor, satisfactory or good. He takes a random sample of brushes from each company. The examiner's ratings are summarised in the table.

Company	Poor	Satisfactory	Good
P	18	43	64
Q	22	22	31

- (a) Test, at the 5% significance level, whether quality of brushes is independent of company. [7]
- (b) Compare the quality of the brushes produced by the two companies. [1]

Q29

9231/43 • May/June 2024 • Question 3

There are three bus companies in a city. The council is investigating whether the buses reliably arrive at their destination on time. The results from random samples of buses from each company are summarised in the following table.

		Bus company			
		<i>A</i>	<i>B</i>	<i>C</i>	Total
Arrival	Early	22	22	10	54
	On time	30	52	42	124
	Late	28	26	18	72
	Total	80	100	70	250

Test, at the 5% significance level, whether the reliability of buses is independent of bus company. [7]

Q30

9231/41, 9231/43 • October/November 2024 • Question 3

A statistician believes that the number of telephone calls received by an advice centre in a 10-minute interval can be modelled by the Poisson distribution $Po(1.9)$. The number of calls received in a randomly chosen 10-minute interval was recorded on each of 100 days. The results are summarised in the table, together with some of the expected frequencies corresponding to the distribution $Po(1.9)$.

Number of calls	0	1	2	3	4	5	6 or more
Observed frequency	10	18	35	21	11	4	1
Expected frequency	14.957	28.418	26.997				1.322

- (a) Complete the table. [2]
- (b) Carry out a goodness of fit test, at the 10% significance level, to determine whether the statistician's belief is reasonable. [6]

Q31

9231/42 • October/November 2024 • Question 3

Rosie sows 5 seeds in each of 150 plant pots. The number of seeds that germinate is recorded for each pot. The results are summarised in the following table.

Number of seeds that germinate	0	1	2	3	4	5
Number of pots	12	40	43	35	16	4

Rosie suggests that the number of seeds that germinate follows the binomial distribution $B(5, p)$.

- (a) Use Rosie's results to show that $p = 0.42$. [1]
- (b) Carry out a goodness of fit test, at the 10% significance level, to test whether the distribution $B(5, 0.42)$ is a good fit for the data. [9]

Q32

9231/41, 9231/42 • May/June 2025 • Question 3

Eggs in a supermarket are sold in boxes of six. A supermarket manager wishes to model the number of broken eggs in the boxes sold in the store. A random sample of 2000 boxes is taken and the number of broken eggs recorded. The observed frequencies are shown in the table below.

Number of broken eggs	0	1	2	3	4	5	6
Observed frequency	1844	143	11	0	1	0	1

- (a) Use the data to estimate the probability that an egg is broken. Give your answer correct to 4 significant figures. [1]

It is decided to carry out a goodness of fit test at the 0.5% significance level to determine whether a binomial distribution fits the data.

The observed frequencies and the expected frequencies are given in the following table.

Number of broken eggs	0	1	2	3	4	5	6
Observed frequency	1844	143	11	0	1	0	1
Expected frequency	1831.3	a	6.016	0.119	0.001	0.000	0.000

- (b) Show that $a = 162.6$ correct to 1 decimal place. [1]
- (c) Carry out a goodness of fit test at the 0.5% level of significance to test whether a binomial distribution is a satisfactory model for the data. [5]
- (d) Give a reason why a binomial distribution may not be a suitable model in this situation. [1]

Q33

9231/43 • May/June 2025 • Question 1

A person's eye colour may be categorised as “brown”, “blue” or “other”. A scientist claims that these eye colours are uniformly distributed and hence are equally likely to occur in the population. A survey of 120 people from this population found that 38 people had brown eyes, 52 people had blue eyes and 30 people had eyes which were neither brown nor blue.

Use the data to carry out a goodness of fit test at the 5% significance level to test the scientist's claim. [6]

1 0 1
Further Math with Ali Hashir
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Q34

9231/44 • May/June 2025 • Question 3

A shop selling electrical goods has a team of three salespeople: Avril, Ben and Charlie. The manager wishes to investigate whether the salespeople are equally successful at selling particular types of items. The following table gives a record of a random sample of 250 sales of laptops, cameras and televisions, with the number sold by each of the three salespeople.

	Laptop	Camera	Television	Total
Avril	31	40	24	95
Ben	23	45	29	97
Charlie	21	25	12	58
Total	75	110	65	250

Test, at the 10% significance level, whether there is independence between the type of item sold and the salesperson. [7]

Q35

9231/41, 9231/43 • October/November 2025 • Question 2

The manager of a car park claims that the number of cars entering the car park follows a Poisson distribution with mean 2.8. The numbers of cars entering the car park are recorded on a working day during successive 5-minute periods. The following table contains the observed frequencies, together with most of the expected frequencies and their contributions to the χ^2 -test statistic.

Number of cars	0	1	2	3	4	5	≥ 6
Observed frequency	2	15	31	29	13	3	7
Expected frequency	6.081	17.03	23.84	p	15.57	8.721	6.511
χ^2 -test statistic	2.739	0.241	2.152	q	0.425	3.753	0.037

- (a) Find the value of p and the value of q . [2]
- (b) Carry out a goodness of fit test at the 5% significance level to investigate the manager's claim. [4]

Q36

9231/42 • October/November 2025 • Question 3

A traffic expert claims that the number of breakdowns occurring each day on a busy section of a motorway follows a Poisson distribution with mean 0.7. The number of breakdowns each day over a 200-day period was recorded. The following table contains the observed frequencies together with some of the expected frequencies using the expert's distribution.

Number of breakdowns per day	0	1	2	3	4	≥ 5
Observed frequency	88	73	26	7	3	3
Expected frequency	99.317	m	24.333	5.678	0.994	n

- (a) Find the value of m and the value of n , correct to 3 decimal places. [2]
- (b) Carry out a goodness of fit test at the 5% significance level to investigate the expert's claim. [6]

Q37

9231/44 • October/November 2025 • Question 5

A driving instructor believes that the performance (pass or fail) of students when taking a driving test is associated with their age. The following table summarises the number of students who pass and who fail, and the ages in years of the students taking the test, over a period of three years.

	age of student			total
	under 20	20–40	over 40	
pass	34	41	6	81
fail	16	39	9	64
total	50	80	15	145

Test, at the 10% significance level, whether performance is independent of age of student. [7]

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Written by Ali Hashir.

Answers available in the companion mark scheme booklet.

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