Further Math with Ali Hashir

Motion of a Projectile

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/31, 9231/32 • May/June 2020 • Question 1

A particle P is projected with speed u at an angle of $30°$ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The particle reaches its greatest height at time T after projection.

Find, in terms of u, the speed of P at time $\frac{2}{3}T$ after projection. [5]

Q2

9231/33 • May/June 2020 • Question 6

A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The direction of motion of P makes an angle α above the horizontal when P first reaches three-quarters of its greatest height.

(a) Show that $\tan\alpha = \frac{1}{2}\tan\theta$. [6]

(b) Given that $\tan\theta = \frac{4}{3}$, find the horizontal distance travelled by P when it first reaches three-quarters of its greatest height. Give your answer in terms of u and g. [4]

Q3

9231/31, 9231/33 • October/November 2020 • Question 5

A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha.$$

[3]

The point Q is the highest point on the trajectory of P in the case where $\alpha = 45°$.

(b) Show that the x-coordinate of Q is $\dfrac{u^2}{2g}$. [3]

(c) Find the other value of α for which P would pass through the point Q. [4]

Q4

9231/32 • October/November 2020 • Question 5

A particle P is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle of θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time $t\,\mathrm{s}$ are denoted by $x\,\mathrm{m}$ and $y\,\mathrm{m}$ respectively.

(a) Starting from the equation of the trajectory given in the List of Formulae (MF19), show that

$$y = x\tan\theta - \frac{gx^2}{2u^2}\left(1+\tan^2\theta\right).$$

[1]

When $\theta = \tan^{-1}2$, P passes through the point with coordinates $(10, 16)$.

(b) Show that there is no value of θ for which P can pass through the point with coordinates $(18, 30)$. [6]

Q5

9231/31, 9231/32 • May/June 2021 • Question 7

A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is $100\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. The two times at which P's height above the plane is H m differ by 10 s.

(a) Find the value of H. [5]

(b) Find the magnitude and direction of the velocity of P one second before it strikes the plane. [4]

Q6

9231/33 • May/June 2021 • Question 7

A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Use the equation of the trajectory given in the List of formulae (MF19), together with the condition $y = 0$, to establish an expression for the range R in terms of u, θ and g. [2]

(b) Deduce an expression for the maximum height H, in terms of u, θ and g. [2]

It is given that $R = \dfrac{4H}{\sqrt{3}}$.

(c) Show that $\theta = 60°$. [1]

It is given also that $u = \sqrt{40}\,\text{m}\,\text{s}^{-1}$.

(d) Find, by differentiating the equation of the trajectory or otherwise, the set of values of x for which the direction of motion makes an angle of less than 45° with the horizontal. [4]

Q7

9231/31, 9231/33 • October/November 2021 • Question 5

A particle P is projected from a point O on a horizontal plane and moves freely under gravity. Its initial speed is $u\,\text{m s}^{-1}$ and its angle of projection is $\sin^{-1}\left(\frac{4}{5}\right)$ above the horizontal. At time 8 s after projection, P is at the point A. At time 32 s after projection, P is at the point B. The direction of motion of P at B is perpendicular to its direction of motion at A.

Find the value of u. [7]

Q8

9231/32 • October/November 2021 • Question 1

A particle is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane. The particle moves freely under gravity.

(a) Write down the horizontal and vertical components of the velocity of the particle at time T after projection. [2]

At time T after projection, the direction of motion of the particle is perpendicular to the direction of projection.

(b) Express T in terms of u, g and α. [2]

(c) Deduce that $T > \frac{u}{g}$. [1]

Q9

9231/31, 9231/32 • May/June 2022 • Question 7

Particles P and Q are projected in the same vertical plane from a point O at the top of a cliff. The height of the cliff exceeds 50m. Both particles move freely under gravity. Particle P is projected with speed $\frac{35}{2}$ m s^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Particle Q is projected with speed u m s^{-1} at an angle β above the horizontal, where $\tan\beta = \frac{1}{2}$. Particle Q is projected one second after the projection of particle P. The particles collide T s after the projection of particle Q.

(a) Write down expressions, in terms of T, for the horizontal displacements of P and Q from O when they collide and hence show that $4uT = 21\sqrt{5}(T+1)$. [4]

(b) Find the value of T. [4]

(c) Find the horizontal and vertical displacements of the particles from O when they collide. [3]

Q10

9231/33 • May/June 2022 • Question 3

A particle P is projected with speed $25\,\text{m s}^{-1}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. After 2s the speed of P is $15\,\text{m s}^{-1}$.

(a) Find the value of $\sin\theta$. [5]

(b) Find the range of the flight. [3]

Q11

9231/31, 9231/33 • October/November 2022 • Question 7

A particle P is projected with speed $V\,\mathrm{m\,s^{-1}}$ at an angle $75°$ above the horizontal from a point O on a horizontal plane. It then moves freely under gravity.

(a) Show that the total time of flight, in seconds, is $\dfrac{2V}{g}\sin 75°$. [2]

A smooth vertical barrier is now inserted with its lower end on the plane at a distance 15 m from O. The particle is projected as before but now strikes the barrier, rebounds and returns to O. The coefficient of restitution between the barrier and the particle is $\dfrac{3}{5}$.

(b) Explain why the total time of flight is unchanged. [1]

(c) Find an expression for V in terms of g. [7]

Q12

9231/32 • October/November 2022 • Question 5

A particle P is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle of θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time $t\,\mathrm{s}$ are denoted by $x\,\mathrm{m}$ and $y\,\mathrm{m}$ respectively.

(a) Show that the equation of the trajectory is given by

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta).$$

[4]

In the subsequent motion P passes through the point with coordinates $(30, 20)$.

(b) Given that one possible value of $\tan\theta$ is $\frac{4}{3}$, find the other possible value of $\tan\theta$. [5]

Q13

9231/31, 9231/32 • May/June 2023 • Question 7

At time ts, a particle P is projected with speed $40\,\text{m s}^{-1}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The greatest height achieved by P during its flight is Hm and the corresponding time is Ts.

(a) Obtain expressions for H and T in terms of θ. [2]

During the time between $t = T$ and $t = 3$, P descends a distance $\frac{1}{4}H$.

(b) Find the value of θ. [4]

(c) Find the speed of P when $t = 3$. [3]

Q14

9231/33 • May/June 2023 • Question 7

The points O and P are on a horizontal plane, a distance 8m apart. A ball is thrown from O with speed $u\,\text{m s}^{-1}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At the same instant, a model aircraft is launched with speed $5\,\text{m s}^{-1}$ parallel to the horizontal plane from a point 4m vertically above P. The model aircraft moves in the same vertical plane as the ball and in the same horizontal direction as the ball. The model aircraft moves horizontally with a constant speed of $5\,\text{m s}^{-1}$. After Ts, the ball and the model aircraft collide.

(a) Find the value of T. [6]

(b) Find the direction in which the ball is moving immediately before the collision. [3]

Q15

9231/31, 9231/33 • October/November 2023 • Question 5

A particle P is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. During its flight P passes through the point which is a horizontal distance $3a$ from O and a vertical distance $\frac{3}{8}a$ above the horizontal plane. It is given that $\tan\theta = \frac{1}{3}$.

(a) Show that $u^2 = 8ag$. [2]

A particle Q is projected with speed $V\,\mathrm{m\,s^{-1}}$ at an angle α above the horizontal from O at the instant when P is at its highest point. Particles P and Q both land at the same point on the horizontal plane at the same time.

(b) Find V in terms of a and g. [7]

Q16

9231/32 • October/November 2023 • Question 6

A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$

[3]

During its flight, P must clear an obstacle of height h m that is at a horizontal distance of 32 m from the point of projection. When $u = 40\sqrt{2}\,\text{m s}^{-1}$, P just clears the obstacle. When $u = 40\,\text{m s}^{-1}$, P only achieves 80% of the height required to clear the obstacle.

(b) Find the two possible values of h. [6]

Q17

9231/31, 9231/32 • May/June 2024 • Question 3

At time $t = 0$ seconds, a particle P is projected with speed u m s^{-1} at an angle $60°$ above the horizontal from a point O. In the subsequent motion P moves freely under gravity. The direction of motion of P when $t = 5$ is perpendicular to its direction of motion when $t = 15$.

Find the value of u. [5]

Q18

9231/33 • May/June 2024 • Question 6

A particle P is projected with speed u m s^{-1} at an angle θ above the horizontal from a point O and moves freely under gravity. After 5 seconds the speed of P is $\frac{3}{4}u$.

(a) Show that $\frac{7}{16}u^2 - 100u\sin\theta + 2500 = 0$. [3]

(b) It is given that the velocity of P after 5 seconds is perpendicular to the initial velocity.

Find, in either order, the value of u and the value of $\sin\theta$. [5]

Q19

9231/31, 9231/33 • October/November 2024 • Question 1

A particle P is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle $\tan^{-1} 2$ above the horizontal from a point O on a horizontal plane and moves freely under gravity. When P has travelled a distance 56 m horizontally from O, it is at a vertical height H m above the plane. When P has travelled a distance 84 m horizontally from O, it is at a vertical height $\frac{1}{2}H$ m above the plane.

Find, in either order, the value of u and the value of H. [5]

Q20

9231/31, 9231/33 • October/November 2024 • Question 7

A particle P is projected with speed u at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ above the horizontal from a point O on a horizontal plane and moves freely under gravity. When P is moving horizontally, it strikes a smooth inclined plane at the point A. This plane is inclined to the horizontal at an angle α, and the line of greatest slope through A lies in the vertical plane through O and A.

As a result of the impact, P moves vertically upwards. The coefficient of restitution between P and the inclined plane is e.

(a) Show that $e\tan^2\alpha = 1$. [4]

In its subsequent motion, the greatest height reached by P above A is $\frac{3}{16}$ of the vertical height of A above the horizontal plane.

(b) Find the value of e. [6]

Q21

9231/32 • October/November 2024 • Question 5

A particle P is projected from a point O on horizontal ground with speed u at an angle θ above the horizontal, where $\tan\theta = \frac{1}{3}$. The particle P moves freely under gravity and passes through the point with coordinates $\left(3a, \frac{4}{5}a\right)$ relative to horizontal and vertical axes through O in the plane of the motion.

(a) Use the equation of the trajectory to show that $u^2 = 25ag$. [2]

At the instant when P is moving horizontally, a particle Q is projected from O with speed V at an angle α above the horizontal. The particles P and Q reach the ground at the same point and at the same time.

(b) Express V^2 in the form kag, where k is a rational number. [6]

Q22

9231/31, 9231/32 • May/June 2025 • Question 2

A particle P is projected with speed $24\,\text{m}\,\text{s}^{-1}$ at an angle $\theta°$ above the horizontal from a point O on a horizontal plane, and moves freely under gravity. At a horizontal distance 35 m from O, there is a vertical wall of height 10 m which is perpendicular to the vertical plane of motion of P.

(a) Determine the two values of θ for which P just clears the wall. [4]

(b) Given that P clears the wall, find the minimum distance from point O where P can land. [2]

Q23

9231/33 • May/June 2025 • Question 7

A particle P is projected from a point O with speed U at an angle $45°$ above the horizontal and moves freely under gravity.

(a) State the vertical and horizontal components of velocity at time t. [1]

At time T, particle P is moving at an angle of $60°$ below the horizontal.

(b) Show that $T = \frac{U}{2g}(\sqrt{2} + \sqrt{6})$. [3]

At time T, the particle strikes a smooth horizontal plane at a point which is a horizontal distance D from O and a vertical distance H below O.

(c) Find the ratio $H : D$. [4]

After striking the horizontal plane, P rebounds with speed w. The coefficient of restitution between P and the plane is $\frac{2}{3}$.

(d) Find w in terms of U. [3]

Q24

9231/34 • May/June 2025 • Question 3

A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at any subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form $y = x\tan\alpha - \dfrac{gx^2}{2u^2}\sec^2\alpha$. [4]

It is given that $u = 20\sqrt{2}\,\text{m s}^{-1}$ and that the particle P passes through the point where $x = 64\,\text{m}$ and $y = 8\,\text{m}$.

(b) Find the possible values of $\tan\alpha$. [3]

Q25

9231/32 • October/November 2025 • Question 4

A particle Q is initially positioned at a distance d vertically above a particle P. Particle P is projected with speed U at an angle α above the horizontal. At the same time, Q is projected at an angle β below the horizontal. Both particles move freely under gravity. The particles collide at time T after the projections.

(a) Show that $d = UT(\sin\alpha + \cos\alpha\tan\beta)$. [4]

The particles collide when P is at its maximum height.

(b) Given that $\alpha = 30°$ and $\beta = 60°$, find d in terms of U and g. [3]

Q26

9231/31, 9231/33 • October/November 2025 • Question 7

A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is $25\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At point A, the direction of motion of P makes an angle of $45°$ with the downward vertical through A.

(a) By differentiating the equation of the trajectory or otherwise, find the coordinates of A. [5]

At point A, the particle strikes a fixed smooth barrier, rebounds, and lands on the horizontal plane. The barrier is inclined at an angle of $45°$ to the horizontal.

(b) Find the speed of P immediately before it collides with the barrier. [3]

(c) Given that the coefficient of restitution between the barrier and the particle is $\frac{1}{9}$, find the horizontal distance travelled by P after it strikes the barrier. [4]

Q27

9231/34 • October/November 2025 • Question 2

A particle P is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle θ, where $\tan\theta = 2$, above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a time $t\,\mathrm{s}$ are denoted by $x\,\mathrm{m}$ and $y\,\mathrm{m}$ respectively.

(a) Use the equation of the trajectory given in the list of formulae (MF 19) to show that

$$y = 2x - \frac{25x^2}{u^2}.$$

[1]

(b) In the subsequent motion, P passes through the point with coordinates $(8, 12)$. The particle then hits a fixed vertical barrier $7\,\mathrm{m}$ high that is at a horizontal distance of $D\,\mathrm{m}$ from the point of projection.

Find the set of possible values of D. [5]

Answers available in the companion mark scheme booklet.