Further Math with Ali Hashir

Motion of a Projectile

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/31, 9231/32 • May/June 2020 • Question 1

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Mark scheme.

For greatest height, $T = \frac{u}{2g}$.	**B1**
At $t = \frac{2T}{3}$, $v_v \uparrow = u - \frac{2Tg}{3} = \frac{u}{6}$.	**M1**
$v_h \rightarrow = \frac{u\sqrt{3}}{2}$.	**A1**
Speed $= \sqrt{v_v^2 + v_h^2} = \sqrt{\frac{u^2}{36} + \frac{3u^2}{4}}$.	**M1**
$= \frac{\sqrt{7}}{3}u$.	**A1**

Total: 5

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Q2

9231/33 • May/June 2020 • Question 6

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Mark scheme.

Greatest height $= \dfrac{(u\sin\theta)^2}{2g}$.	**M1A1**
At $\frac{3}{4}$ greatest height, $\rightarrow v\cos\alpha = u\cos\theta$.	**M1**
At $\frac{3}{4}$ greatest height, $\uparrow (v\sin\alpha)^2 = (u\sin\theta)^2 - 2g.\dfrac{3}{4}\dfrac{(u\sin\theta)^2}{2g}$.	**M1**
$v\sin\alpha = \dfrac{1}{2}u\sin\theta$.	**A1**
So $\tan\alpha = \dfrac{1}{2}\tan\theta$ (AG). **(b)**	**A1**
$\tan\alpha = \dfrac{1}{2}.\dfrac{4}{3} = \dfrac{2}{3}$; $\uparrow \dfrac{1}{2}u\sin\theta = u\sin\theta - gt$.	**M1**
$t = \dfrac{2u}{5g}$.	**A1**
$\rightarrow t = d/u\cos\theta$.	**M1**
$= \dfrac{6u^2}{25g}$.	**A1**
	Total: 10

Q3

9231/31, 9231/33 • October/November 2020 • Question 5

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Mark scheme.

$x = u\cos\alpha\, t$; $y = u\sin\alpha\, t - \frac{1}{2}gt^2$ (both)	**B1**
Eliminate t	**M1**
$y = x\tan\alpha - \dfrac{gx^2}{2u^2}\sec^2\alpha$ (AG) (b)	**A1**
Greatest height $= \dfrac{(u\sin\alpha)^2}{2g} = \dfrac{u^2}{4g}$ (accept alternative methods e.g. differentiate and equate to 0)	**M1A1**
$t = u\sin 45°/g$ so $d = u\cos 45°.u\sin 45°/g = \dfrac{u^2}{2g}$ (AG) (c)	**A1**
Use greatest-height coordinates in trajectory equation: $\dfrac{u^2}{4g} = \dfrac{u^2}{2g}\tan\alpha - \dfrac{gu^4}{2u^2\cdot 4g^2}\sec^2\alpha$	**M1**
Use $\sec^2\alpha = 1 + \tan^2\alpha$	**M1**
Obtain three-term quadratic: $\tan^2\alpha - 4\tan\alpha + 3 = 0$	**M1**
$\tan\alpha = 1, 3$ so $\alpha = 71.6°$ (both solutions needed)	**A1**
	Total: 10

Q4

9231/32 • October/November 2020 • Question 5

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Mark scheme.

Quote trajectory equation from MF19 and use $\cos\theta = 1/\sec\theta$: $y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$ (AG; allow derived from first principles) (b)	**B1**
Substitute $(10, 16)$ into result (a)	**M1**
$u^2 = 625$ $(u = 25)$	**A1**
Use equation again: $30 = 18\tan\theta - \frac{10\times 324}{2\times 625}(1+\tan^2\theta)$	**M1**
Three-term quadratic: $2.592(\tan\theta)^2 - 18\tan\theta + 32.592 = 0$	**A1**
Discriminant $= 324 - 4\times 2.592\times 32.592 = -13.91$	**M1**
As this is less than 0, no real solutions for θ (CWO)	**A1**
	Total: 7

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Q5

9231/31, 9231/32 • May/June 2021 • Question 7

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Mark scheme.

At greatest height $0 = 100\sin\theta - gt$.	**M1**
$t = 8$.	**A1**
Therefore times at height H are $t = 3$ (and $t = 13$).	**B1**
Substitute into $H = 100\sin\theta\, t - \frac{1}{2}gt^2$.	**M1**
$H = 195$. (b)	**A1**
Time to required point $= 15$ s.	**B1**
$v_{\uparrow} = 100\sin\theta - 10\times 15 = -70$; $v_{\rightarrow} = 100\cos\theta = 60$ (both components).	**B1**
Magnitude $= 92.2$.	**B1**
Angle below horizontal $= \tan^{-1}(70/60) = 49.4°$.	**B1**
	Total: 9

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Q6

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Mark scheme.

Set $y = 0$ in trajectory equation: $R\tan\theta - g\dfrac{R^2}{2u^2(\cos\theta)^2} = 0$.	**M1**
$R = \dfrac{2u^2\sin\theta\cos\theta}{g}$ only (any equivalent single-term expression). (b)	**A1**
$x = \frac{1}{2}R$ substituted into trajectory equation (or use SUVAT).	**M1**
$H = \dfrac{u^2(\sin\theta)^2}{2g}$. (c)	**A1**
Use $R = \dfrac{4H}{\sqrt{3}}$ and simplify: $\tan\theta = \sqrt{3}$, $\theta = 60°$ (AG). (d)	**B1**
Differentiate trajectory equation with respect to x: $\dfrac{dy}{dx} = \tan\theta - \dfrac{gx}{u^2(\cos\theta)^2}$.	**M1**
Use $\dfrac{dy}{dx} = \pm 1$ as limiting case.	**M1**
$x = \sqrt{3}+1$, $x = \sqrt{3}-1$.	**A1**
$\sqrt{3}-1 < x < \sqrt{3}+1$ (strict inequality, exact values).	**A1**
	Total: 9

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Q7

9231/31, 9231/33 • October/November 2021 • Question 5

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Mark scheme.

At A: $\uparrow u\sin\theta - 8g$, $\quad \rightarrow u\cos\theta$ (both).	**M1**
$\tan\alpha = \dfrac{u\sin\theta - 8g}{u\cos\theta}$.	**A1**
At B: $\uparrow u\sin\theta - 32g$, $\quad \rightarrow u\cos\theta$ (both).	**M1**
$\tan\beta = \dfrac{u\sin\theta - 32g}{u\cos\theta}$.	**A1**
Perpendicular directions, so $\tan\alpha \times \tan\beta = -1$: $\dfrac{u\sin\theta - 8g}{u\cos\theta} \times \dfrac{u\sin\theta - 32g}{u\cos\theta} = -1$.	**B1**
Simplify to a quadratic in u: $u^2 - 320u + 25600 = 0$.	**M1**
$u = 160$.	**A1**
	Total: 7

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Q8

9231/32 • October/November 2021 • Question 1

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Mark scheme.

(a)

Velocity: $\rightarrow u\cos\alpha$.	**B1**
$\uparrow u\sin\alpha - gT$ (allow 10 for g; must be T).	**B1**

(b)

$\dfrac{u\cos\alpha}{u\sin\alpha - gT} = -\dfrac{\sin\alpha}{\cos\alpha}$ OE (allow missing minus sign on RHS for M1; FT from (a)).	**M1 FT**
$T = \dfrac{u}{g\sin\alpha}$.	**A1**

(c)

$\sin\alpha < 1$ giving $T > \dfrac{u}{g}$ (AG).	**B1**

Total: 5

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Q9

9231/31, 9231/32 • May/June 2022 • Question 7

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Mark scheme.

For Q: $x = u\cos\beta\, T$.	**B1**
For P: $x = \dfrac{35}{2}\cos\alpha\,(T+1)$.	**B1**
Equate and attempt to rearrange.	**M1**
$\dfrac{35}{2}\times\dfrac{3}{5}(T+1) = u\times\dfrac{2}{\sqrt{5}}T \Rightarrow 4uT = 21\sqrt{5}(T+1)$ (AG, shown convincingly).	**A1**
(b)	
Vertical motion: for Q, $y = u\sin\beta\, T - \frac{1}{2}gT^2$; for P, $y = \dfrac{35}{2}\sin\alpha\,(T+1) - \frac{1}{2}g(T+1)^2$ (M1 for both expressions, one correct).	**M1 A1**
Equate and attempt to solve: $14(T+1) - \frac{1}{2}g\left(T^2+2T+1-T^2\right) = \dfrac{21}{4}(T+1)$.	**M1**
$16T + 36 = 21T + 21 \Rightarrow 15 = 5T \Rightarrow T = 3$. (c)	**A1**
$x = 42$.	**B1**
$\lvert y\rvert = 24$.	**M1**
$y = -24$ (or 24 m below O), correct sign or in words.	**A1**
Total: 11	

Q10

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Mark scheme.

Components of velocity at $t = 2$: $\rightarrow 25\cos\theta$, $\uparrow 25\sin\theta - 2g$.	**B1**
Expression for speed (or square of speed): Speed $= \sqrt{(25\cos\theta)^2 + (25\sin\theta - 2g)^2}$.	**M1 A1**
$(25\cos\theta)^2 + (25\sin\theta - 2g)^2 = 15^2 \Rightarrow 625 - 100g\sin\theta + 4g^2 = 225$.	**M1**
$\sin\theta = \dfrac{800}{1000} = \dfrac{4}{5}$. (b)	**A1**
Time of flight $= \dfrac{2 \times 25\sin\theta}{g} = 4$ s.	**B1**
Range $= \dfrac{2 \times 25\sin\theta}{g} \times 25\cos\theta$ (any equivalent method).	**M1**
Range $= 60$ m (CWO).	**A1**
	Total: 8

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Q11

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Mark scheme.

$0 = V\sin 75° \, t - \frac{1}{2}gt^2$	**M1**
$t = \dfrac{2V}{g}\sin 75°$ (AG) **(b)**	**A1**
Vertical component of velocity is unchanged. **(c)**	**B1**
Horizontally to wall: $15 = V\cos 75° \, t$, so $t = \dfrac{15}{V\cos 75°}$.	**B1**
Speed after rebound $= eV\cos 75°$.	**B1**
Time back to O: $T = \dfrac{15}{eV\cos 75°}$, $t = \frac{3}{5}T$.	**M1**
Vertically for whole flight: $t + T = \dfrac{2V}{g}\sin 75°$.	**M1**
$\dfrac{15}{V\cos 75°} + \dfrac{15}{eV\cos 75°} = \dfrac{2V}{g}\sin 75°$	**A1**
$V^2\cos 75°\sin 75° = 20g$	**M1**
Multiply by 2: $V^2\sin 150° = 40g$, so $V^2 = 80g$.	**A1**
	Total: 10

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Q12

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Mark scheme.

$x = u\cos\theta\, t$	**B1**
$y = u\sin\theta\, t - \frac{1}{2}gt^2$	**B1**
Eliminate t: $y = u\sin\theta \times \dfrac{x}{u\cos\theta} - \frac{1}{2}g\left(\dfrac{x}{u\cos\theta}\right)^2$	**M1**
$y = x\tan\theta - \dfrac{gx^2}{2u^2}(1+\tan^2\theta)$ (AG, must show intermediate line) **(b)**	**A1**
Substitute $x = 30$, $y = 20$, $\tan\theta = \frac{4}{3}$: $20 = 30 \times \frac{4}{3} - 10 \times \dfrac{30^2}{2u^2}\left(1 + \left(\frac{4}{3}\right)^2\right)$	**M1**
$u^2 = 625$, $[u = 25]$	**A1**
Substitute back: $18\tan^2\theta - 75\tan\theta + 68 = 0$	**M1**
One solution is $\frac{4}{3}$: $(3\tan\theta - 4)(6\tan\theta - 17) = 0$	**M1**
$\tan\theta = \dfrac{17}{6}$	**A1**
	Total: 9

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Q13

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Mark scheme.

$H = 80\sin^2\theta$ or $\dfrac{800\sin^2\theta}{g}$.	**B1**
$T = 4\sin\theta$ or $\dfrac{40\sin\theta}{g}$. (b)	**B1**
Between $t = T$ and $t = 3$: $\dfrac{1}{4}H = \dfrac{1}{2} \times 10 \times (3 - T)^2$. No extra terms.	**M1 A1**
Substitute results from part (a): $\dfrac{1}{4} \times 80\sin^2\theta = 5(3 - 4\sin\theta)^2 \Rightarrow 4\sin^2\theta - 8\sin\theta + 3 = 0$.	**M1**
$\sin\theta = \dfrac{1}{2}$, $\theta = 30°$. Single answer, NFWW. (c)	**A1**
When $t = 3$, speeds $\rightarrow 40\cos\theta$ and $\uparrow 40\sin\theta - 10 \times 3$.	**B1**
Square and add to find square of speed: $v^2 = (20\sqrt{3})^2 + (-10)^2$. Must be numerical.	**M1**
$v^2 = 1300$, $v = 10\sqrt{13}$ [$= 36.1$].	**A1**
	Total: 9

Q14

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Mark scheme.

For aircraft, $d = 5T$.	**B1**
For ball, $\uparrow 4 = u\sin\theta\, T - \frac{1}{2} \times 10 \times T^2$ (to point of collision).	**B1**
For ball, $\rightarrow u\cos\theta\, T = d + 8 = 5T + 8$.	**B1**
Eliminate u: $4 = \frac{u4}{5}T - \frac{1}{2} \times 10 \times T^2 \Rightarrow u = \frac{5(4+5T^2)}{4T}$ and $u = \frac{5(5T+8)}{3T}$; equate to give $3(4+5T^2) = 4(5T+8)$ (dependent on LHS of second B1 being 4, expression involving only T).	**M1**
$3T^2 - 4T - 4 = 0$ (dependent on previous M1; obtain and solve 3-term quadratic).	**DM1**
$T = 2$ (single correct answer). (b)	**A1**
$\tan^{-1}\left(\frac{u\sin\theta - 10T}{u\cos\theta}\right)$ (OE, accept 'tan = ...').	**M1**
$\tan^{-1}\frac{8}{9}$ (OE).	**A1**
Direction is 41.6° below the horizontal (CAO).	**A1**
	Total: 9

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Q15

9231/31, 9231/33 • October/November 2023 • Question 5

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Mark scheme.

Use equation of trajectory: $y = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta$: $\frac{3}{8}a = 3a \times \frac{1}{3} - g \times \frac{(3a)^2}{2u^2}\left(1+\frac{1}{9}\right)$	M1	**(a)**
$\frac{5}{8}a = \frac{5ga}{u^2}$, so $u^2 = 8ag$ (AG)	A1	
For P, time of flight $T = \frac{2u\sin\theta}{g}$	B1	**(b)**
For P, range $= Tu\cos\theta$	B1	
For Q, time of flight $= \frac{1}{2}T$, so range $= \frac{1}{2}TV\cos\alpha$; equate: $V\cos\alpha = 2u\cos\theta$ (1)	B1	
For Q vertically: $0 = V\sin\alpha \times \frac{1}{2}T - \frac{1}{2}g\left(\frac{1}{2}T\right)^2$, so $T = \frac{4V\sin\alpha}{g}$	M1	
Equate with result for P: $\frac{4V\sin\alpha}{g} = \frac{2u\sin\theta}{g}$, so $V\sin\alpha = \frac{1}{2}u\sin\theta$ (2)	A1	
From (1) and (2): $V^2 = u^2\left(4(\cos\theta)^2 + \frac{1}{4}(\sin\theta)^2\right)$	M1	
$V^2 = u^2 \times \left(4 \times \frac{9}{10} + \frac{1}{4} \times \frac{1}{10}\right) = \frac{145}{40}u^2$, so $V = \sqrt{29ag}$	A1	
		Total: 9

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Q16

9231/32 • October/November 2023 • Question 6

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Mark scheme.

$x = u\cos\alpha\, t, \quad y = u\sin\alpha\, t - \frac{1}{2}gt^2$	B1	**(a)**
Eliminate t: $y = u\sin\alpha \times \dfrac{x}{u\cos\alpha} - \frac{1}{2}g\left(\dfrac{x}{u\cos\alpha}\right)^2$	M1	
$y = x\tan\alpha - \dfrac{gx^2}{2u^2}\sec^2\alpha$ (AG)	A1	
$\frac{4}{5}h = 32\tan\alpha - \dfrac{g(32)^2}{2(40)^2}\sec^2\alpha \quad \left[= 32\tan\alpha - \dfrac{16}{5}\sec^2\alpha\right]$	B1	**(b)**
$h = 32\tan\alpha - \dfrac{g(32)^2}{2(40\sqrt{2})^2}\sec^2\alpha \quad \left[= 32\tan\alpha - \dfrac{8}{5}\sec^2\alpha\right]$	B1	
Equate expressions for h: $32\tan\alpha - \dfrac{8}{5}\sec^2\alpha = \dfrac{5}{4}\left(32\tan\alpha - \dfrac{16}{5}\sec^2\alpha\right)$, giving $3t^2 - 10t + 3 = 0$ where $t = \tan\alpha$	M1	
$t = 3,\ \frac{1}{3}$	A1	
Using t to find h: for using their value of t to work out one value of h	M1	
$h = 80,\ h = \dfrac{80}{9}$	A1	
		Total: 9

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Q17

9231/31, 9231/32 • May/June 2024 • Question 3

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Mark scheme.

$u\sin 60° - 5g$, and $u\cos 60°$, or $u\sin 60° - 15g$, and $u\cos 60°$.	**B1**
If θ is direction of velocity at $t = 5$, $\tan\theta = \dfrac{u\sin 60° - 5g}{u\cos 60°}$ (accept equivalent for $t = 15$).	**M1***
For perpendicular directions, $\dfrac{u\sin 60° - 5g}{u\cos 60°} \times \dfrac{u\sin 60° - 15g}{u\cos 60°} = -1$ (multiply two expressions involving relevant velocities and equate to -1).	**M1dep**
Simplify: $\dfrac{3}{4}u^2 + 75g^2 - 10\sqrt{3}ug + \dfrac{1}{4}u^2 = 0$, i.e. $u^2 - 100\sqrt{3}u + 7500 = 0$ (simplify to quadratic in u, may see g).	**M1**
$u = 5\sqrt{3}g$ (OE, accept $50\sqrt{3}$ or 86.6).	**A1**
	Total: 5

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Q18

9231/33 • May/June 2024 • Question 6

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Mark scheme.

B1: Components of velocity are $u\cos\theta$, $u\sin\theta - gt$.	**(a)**
Square and add components of velocity and equate to $\left(\frac{3}{4}u\right)^2$: $(u\cos\theta)^2 + (u\sin\theta - gt)^2 = \left(\frac{3}{4}u\right)^2$.	**M1**
$\frac{7}{16}u^2 - 100u\sin\theta + 2500 = 0$ (AG; at least one correct line of working seen).	**A1**
B1: Let α be angle of direction of motion with horizontal at $t = 5$: $\tan\alpha = \frac{u\sin\theta - 5g}{u\cos\theta}$.	**(b)**
$\tan\alpha\tan\theta = -1$, so $\tan\theta\left(\frac{u\sin\theta - 5g}{u\cos\theta}\right) = -1$ (must be -1 not $+1$; FT their expression for $\tan\alpha$).	**M1**
$u = 50\sin\theta$.	**A1**
Use in result from part (a) to form equation in u or $\sin\theta$.	**M1**
$u^2 = 1600$, $u = 40$ and $\sin\theta = \frac{4}{5}$ (both).	**A1**
	Total: 8

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Q19

9231/31, 9231/33 • October/November 2024 • Question 1

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Mark scheme.

Use equation of trajectory with point $(56, H)$ or $\left(84, \frac{1}{2}H\right)$: $H = 112 - \frac{5g}{2u^2} \times 56^2$ or $\frac{1}{2}H = 168 - \frac{5g}{2u^2} \times 84^2$ (one equation with one error).	**M1**
Both equations correct.	**A1**
Eliminate to find u or H.	**M1**
$u = 35$.	**A1**
$H = 48$.	**A1**
	Total: 5

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Q20

9231/31, 9231/33 • October/November 2024 • Question 7

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Mark scheme.

(a)

When P strikes plane, velocity is $u\cos\theta$; before impact components parallel to plane $\frac{3}{5}u\cos\alpha$, perpendicular $\frac{3}{5}u\sin\alpha$ (using $\frac{3}{5}u$).	**M1**
After impact: components $\frac{3}{5}u\cos\alpha$ (parallel), $\frac{3}{5}eu\sin\alpha$ (perpendicular).	**A1**
Since velocity is vertical after impact, $\tan\alpha = \frac{3}{5}u\cos\alpha / \frac{3}{5}eu\sin\alpha$.	**M1**
$\tan\alpha = 1/(e\tan\alpha)$, so $e\tan^2\alpha = 1$ (AG).	**A1**

(b)

Greatest height of P before impact: $H = \dfrac{(u\sin\theta)^2}{2g} = \dfrac{8u^2}{25g}$.	**M1A1**
After impact, vertical speed of P is $\frac{3}{5}u\sqrt{(\cos\alpha)^2 + e^2(\sin\alpha)^2}$.	**M1**
Use $V^2 = U^2 + 2as$ to greatest height equal to $\frac{3}{16}H$: $\dfrac{9}{25}u^2\left((\cos\alpha)^2 + e^2(\sin\alpha)^2\right) = 2g \times \frac{3}{16}H$.	**M1**
Use part (a): $\tan\alpha = \dfrac{1}{\sqrt{e}}$, $\cos\alpha = \sqrt{\dfrac{e}{1+e}}$, $\sin\alpha = \sqrt{\dfrac{1}{1+e}}$; substitute to find e.	**M1**
$3e^2 + 2e - 1 = 0$, $e = \dfrac{1}{3}$.	**A1**
	Total: 10

Q21

9231/32 • October/November 2024 • Question 5

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Mark scheme.

(a)

Use correct equation of trajectory: $\frac{4}{5}a = 3a \times \frac{1}{3} - \frac{g}{2u^2} \times (3a)^2 \times \left(1 + \frac{1}{9}\right)$.	**M1**
$\frac{4}{5}a = a - \frac{5ga^2}{u^2}$, $\frac{5ga}{u^2} = \frac{1}{5}$, $u^2 = 25ga$ (AG).	**A1**

(b)

Time of flight for P: $T = \frac{2u\sin\theta}{g} = \sqrt{\frac{10a}{g}}$; for Q, time of flight $\frac{1}{2}T$; both have range R.	**B1**
From motion of P: $R = \frac{2}{g} \times 25ag \times \frac{3}{10} = 15a$.	**B1**
For Q: $R = v\cos\alpha \times \frac{1}{2}T$, so $v\cos\alpha = \frac{2R}{T}$.	**M1**
$0 = v\sin\alpha \times \frac{1}{2}T - \frac{1}{2}g\left(\frac{1}{2}T\right)^2$, so $v\sin\alpha = \frac{1}{4}gT$.	**M1**
Square and add: $v^2 = \left(\frac{2R}{T}\right)^2 + \left(\frac{1}{4}gT\right)^2 = 90ag + \frac{5}{8}ag$.	**M1**
$v^2 = \frac{725}{8}ag$.	**A1**

Total: 8

Q22

9231/31, 9231/32 • May/June 2025 • Question 2

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Mark scheme.

(a)

Use trajectory equation to set up quadratic in $t = \tan\theta$ using $(35, 10)$: $10 = 35t - \frac{g}{2 \times 24^2}35^2(1 + t^2)$.	**B1**
Form three-term quadratic in t and equate to 0: $6125t^2 - 20160t + 11885 = 0$ (OE $1225t^2 - 4032t + 2377 = 0$).	**M1**
$\tan\theta = 0.769$ leading to $\theta = 37.6°$.	**A1**
$\tan\theta = 2.52$ leading to $\theta = 68.4°$.	**A1**

(b)

Complete method to find the range x using the larger of the two values found in (a).	**M1**
$x = 39.5$ (accept $39.4 \leq x \leq 39.5$).	**A1**

Total: 6

Q23

9231/33 • May/June 2025 • Question 7

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Mark scheme.

(a)

$v_y = U\sin 45 - gT$; $v_x = U\cos 45$.	**B1**

(b)

SOI: $\dfrac{v_y}{v_x} = -\tan 60°$.	**B1**
Substitute trig ratios to obtain linear equation in U, T: $U - \sqrt{2}gT = -\sqrt{3}U$.	**M1**
$T = \dfrac{U}{\sqrt{2}g}(1+\sqrt{3}) = \dfrac{U}{2g}(\sqrt{2}+\sqrt{6})$ (AG, shown convincingly).	**A1**

(c)

$D = \dfrac{U}{\sqrt{2}} \times T$.	**B1**
$H = \left\lvert \dfrac{U}{\sqrt{2}} \times T - \frac{1}{2}gT^2 \right\rvert$ (with or without modulus).	**B1**
Obtain numerical value via division/ratio: $\left\lvert \dfrac{1-\sqrt{3}}{2} \right\rvert$.	**M1**
$H : D = 1 : 1+\sqrt{3}$ (OE, e.g. $\sqrt{3}-1 : 2$, CWO).	**A1**

(d)

$w_y = \pm\dfrac{2\sqrt{3}}{3\sqrt{2}}U \left[= \pm\dfrac{\sqrt{2}}{\sqrt{3}}U\right]$ (OE, accept unsimplified, allow negative sign).	**B1**
$w^2 = w_y^2 + \left(\dfrac{U}{\sqrt{2}}\right)^2 \left[= \frac{2}{3}U^2 + \frac{1}{2}U^2\right]$.	**M1**
$w = U\sqrt{\frac{2}{3}+\frac{1}{2}} = U\sqrt{\frac{7}{6}}$ (CWO).	**A1**

Total: 11

Q24

9231/34 • May/June 2025 • Question 3

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Mark scheme.

(a)

$x = (u\cos\alpha)t$ (explicitly stated).	**B1**
$y = (u\sin\alpha)t - \frac{1}{2}gt^2$ (explicitly stated).	**B1**
Eliminate t: $y = u\sin\alpha\left(\frac{x}{u\cos\alpha}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$.	**M1**
$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha$ (AG, shown convincingly, dependent on all 3 previous marks).	**A1**

(b)

$8 = 64\tan\alpha - \frac{128}{5}(1+\tan^2\alpha)$ (may be unsimplified, single trig function).	**M1**
Obtain 3-term quadratic: $16\tan^2\alpha - 40\tan\alpha + 21 = 0$.	**M1**
$\tan\alpha = \frac{3}{4}, \frac{7}{4}$ (both answers required).	**A1**

Total: 7

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Q25

9231/32 • October/November 2025 • Question 4

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Mark scheme.

Horizontal velocities are the same: $U_q \cos\beta = U\cos\alpha$.	**B1**
Apply vertical equation of motion on both particles and combine: $d = (U\sin\alpha)T - \frac{1}{2}gT^2 + (U_q\sin\beta)T + \frac{1}{2}gT^2$.	**M1**
Substitute U_q and simplify: $d = (U\sin\alpha)T + \left(\dfrac{U\cos\alpha}{\cos\beta}\sin\beta\right)T$.	**M1**
AG, shown convincingly: $d = UT(\sin\alpha + \cos\alpha\tan\beta)$. **(b)**	**A1**
Correct equation for T: $U\sin\alpha = gT$.	**B1**
Substitute their expression for T into result from (a) or complete method for d in terms of U, g only.	**M1**
$d = \dfrac{U^2}{g}$.	**A1**
	Total: 7

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Q26

9231/31, 9231/33 • October/November 2025 • Question 7

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Mark scheme.

Valid attempt at differentiating: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \tan\theta - \dfrac{2gx}{2u^2\cos^2\theta}$.	**M1**
$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -1$.	**B1**
Form equation with -1 and attempt to find a value for x_A: $-1 = \frac{4}{3} - \frac{2}{45}x_A$.	**M1**
$x_A = \frac{105}{2}$ (52.5).	**A1**
$y_A = \frac{35}{4}$ (8.75). [SC B1 for $\frac{35}{4}$ OE from setting $\mathrm{d}y/\mathrm{d}x = +1$.] **(b)**	**A1**
$u_x = u\cos\theta = 15 = v_x$.	**B1**
$v_y = -v_x = -15$.	**B1**
Speed: $\sqrt{v_x^2 + v_y^2} = 15\sqrt{2}$ ($\mathrm{m\,s^{-1}}$). **(c)**	**B1 FT**
Correct application of NEL.	**M1**
$w = ev = \frac{5}{3}\sqrt{2}$.	**A1**
Substitute values into trajectory equation and solve to find x (or find time of flight after striking the barrier first).	**M1**
$x = \frac{5}{2}$ (metres).	**A1**
	Total: 12

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Q27

9231/34 • October/November 2025 • Question 2

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Mark scheme.

Use trajectory equation from MF19: $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$; substitute $\tan\theta = 2$, $\cos^2\theta = \frac{1}{5}$, $g = 10$ and simplify. AG, shown convincingly: $y = 2x - \dfrac{25x^2}{u^2}$.	**B1**
(b)	
Substitute correctly and solve equation from part (a): $12 = 2\times 8 - \dfrac{25\times 8^2}{u^2}$.	**M1**
$u^2 = 400$ $[u = 20]$.	**A1**
Solve the equation from part (a) with $y = 0$: $0 = 2D - \dfrac{25D^2}{400}$ leading to $[D = 0,]$ $D = 32$.	**M1**
Solve the equation from part (a) with $y = 7$: $7 = 2D - \dfrac{25D^2}{400}$ leading to $[D = 4,]$ $D = 28$.	**M1**
$28 \le D \le 32$ only (accept strict inequalities).	**A1**
	Total: 6

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Written by Ali Hashir.

Questions available in the companion questions booklet.