Further Math with Ali Hashir

Momentum

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/21, 9231/22 • May/June 2015 • Question 5

Three uniform small smooth spheres A, B and C have equal radii and masses $3m$, $2m$ and m respectively. The spheres are at rest in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between A and B is e and the coefficient of restitution between B and C is e'. Sphere A is projected directly towards B with speed u. Show that, after the collision between B and C, the speed of C is $\frac{2}{5}u(1+e)(1+e')$ and find the corresponding speed of B. [7]

After this collision between B and C it is found that each of the three spheres has the same momentum. Find the values of e and e'. [5]

Q2

9231/23 • May/June 2015 • Question 1

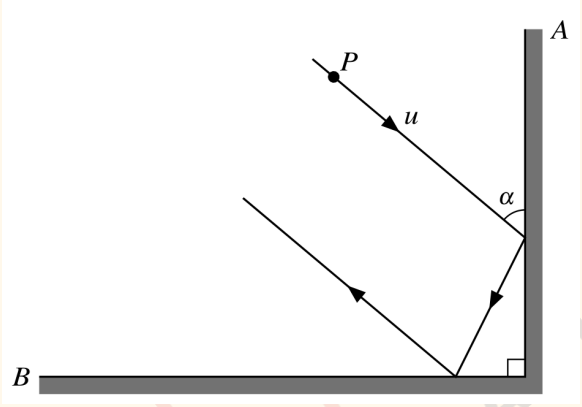
Two uniform small smooth spheres, A and B, of equal radii and masses 2 kg and 3 kg respectively, are at rest and not in contact on a smooth horizontal plane. Sphere A receives an impulse of magnitude 8 N s in the direction AB. The coefficient of restitution between the spheres is e. Find, in terms of e, the speeds of A and B after A collides with B. [5]

Given that the spheres move in opposite directions after the collision, show that $e > \frac{2}{3}$. [1]

Q3

9231/23 • May/June 2015 • Question 2

A uniform sphere P of mass m is at rest on a smooth horizontal table. The sphere is projected along the table with speed u and strikes a smooth vertical barrier A at an acute angle α. It then strikes another smooth vertical barrier B which is at right angles to A (see diagram). The coefficient of restitution between P and each of the barriers is e. Show that the final direction of motion of P makes an angle $\frac{1}{2}\pi - \alpha$ with the barrier B and find the total loss in kinetic energy as a result of the two impacts. [7]



Q4

9231/21, 9231/22, 9231/23 • October/November 2015 • Question 2

A small uniform sphere A, of mass $2m$, is moving with speed u on a smooth horizontal surface when it collides directly with a small uniform sphere B, of mass m, which is at rest. The spheres have equal radii and the coefficient of restitution between them is e. Find expressions for the speeds of A and B immediately after the collision. [4]

Subsequently B collides with a vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is 0.4. After B has collided with the wall, the speeds of A and B are equal. Find e. [2]

Initially B is at a distance d from the wall. Find the distance of B from the wall when it next collides with A. [4]

Q5

9231/21, 9231/22 • May/June 2016 • Question 1

A bullet of mass 0.01 kg is fired horizontally into a fixed vertical barrier which exerts a constant resisting force of magnitude 1000 N. The bullet enters the barrier with speed 320 m s^{-1} and emerges with speed 20 m s^{-1}. You may assume that the motion takes place in a horizontal straight line. Find

(i) the magnitude of the impulse that acts on the bullet, [2]

(ii) the thickness of the barrier, [2]

(iii) the time taken for the bullet to pass through the barrier. [1]

Q6

9231/21, 9231/22 • May/June 2016 • Question 2

A small smooth sphere A of mass m is moving with speed u on a smooth horizontal surface when it collides directly with an identical sphere B which is initially at rest on the surface. The coefficient of restitution between the spheres is e. Sphere B subsequently collides with a fixed vertical barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is $\frac{1}{2}$. Given that 80% of the initial kinetic energy is lost as a result of the two collisions, find the value of e. [8]

Q7

9231/23 • May/June 2016 • Question 5

Three small smooth spheres A, B and C, of masses $5m$, m and km respectively, are at rest on a smooth horizontal surface. The spheres lie in a straight line with B between A and C. Sphere A is projected directly towards B with speed u. The coefficient of restitution between any pair of the spheres A, B and C is $\frac{4}{5}$. Show that the speed of A after the first collision is $\frac{7}{10}u$ and find the speed of B. [3]

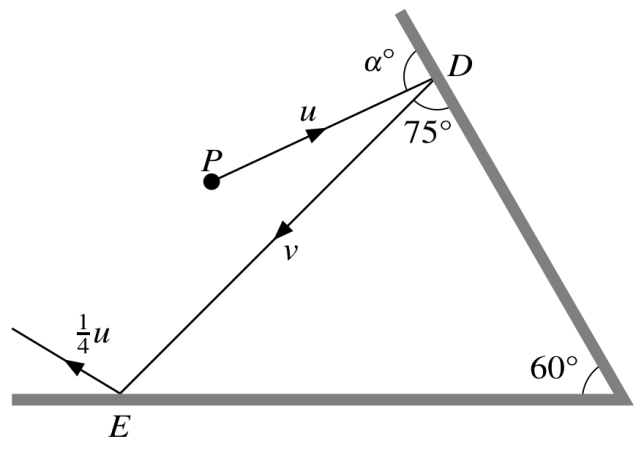
Given that the speeds of A and C are equal after the second collision,

(i) show that the value of k is $\frac{20}{7}$, [4]

(ii) find the percentage loss in the kinetic energy of the system as a result of the two collisions. [5]

Q8

9231/21, 9231/22, 9231/23 • October/November 2016 • Question 2



Two smooth vertical walls each with their base on a smooth horizontal surface intersect at an angle of 60°. A small smooth sphere P is moving on the horizontal surface with speed u when it collides with the first vertical wall at the point D. The angle between the direction of motion of P and the wall is $\alpha°$ before the collision and 75° after the collision. The speed of P after this collision is v and the coefficient of restitution between P and the first wall is e. Sphere P then collides with the second vertical wall at the point E. The speed of P after this second collision is $\frac{1}{4}u$ (see diagram). The coefficient of restitution between P and the second wall is $\frac{3}{4}$.

(i) By considering the collision at E, show that $v = \frac{\sqrt{2}}{5}u$. [5]

(ii) Find the value of α and the value of e. [5]

Q9

9231/21, 9231/22 • May/June 2017 • Question 1

A bullet of mass 0.08 kg is fired horizontally into a fixed vertical barrier. It enters the barrier horizontally with speed $300\,\mathrm{m\,s^{-1}}$ and emerges horizontally after 0.02 s. There is a constant horizontal resisting force of magnitude 1000 N. Find the speed with which the bullet emerges from the barrier. [3]

Q10

9231/21, 9231/22 • May/June 2017 • Question 3

Two uniform small smooth spheres A and B have equal radii and masses $3m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is e.

(i) Find, in terms of u and e, expressions for the velocities of A and B after the collision. [3]

Sphere B continues to move until it strikes a fixed smooth vertical barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is $\frac{3}{4}$. When the spheres subsequently collide, A is brought to rest.

(ii) Find the value of e. [7]

Q11

9231/23 • May/June 2017 • Question 3

Two uniform small smooth spheres A and B have equal radii and each has mass m. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is $\frac{2}{3}$. Sphere B is initially at a distance d from a fixed smooth vertical wall which is perpendicular to the direction of motion of A. The coefficient of restitution between B and the wall is $\frac{1}{3}$.

(i) Show that the speed of B after its collision with the wall is $\frac{5}{18}u$. [4]

(ii) Find the distance of B from the wall when it collides with A for the second time. [6]

Q12

9231/21, 9231/22, 9231/23 • October/November 2017 • Question 3

Three uniform small smooth spheres A, B and C have equal radii and masses m, km and m respectively, where k is a constant. The spheres are moving in the same direction along a straight line on a smooth horizontal surface, with B between A and C. The speeds of A, B and C are $2u$, u and $\frac{4}{3}u$ respectively. The coefficient of restitution between any pair of the spheres is $\frac{1}{2}$. After sphere A has collided with sphere B, sphere B collides with sphere C.

(i) Find an inequality satisfied by k. [5]

(ii) Given that $k = 2$, show that after B has collided with C there are no further collisions between any of the three spheres. [5]

Q13

9231/21, 9231/22 • May/June 2018 • Question 1

A bullet of mass m kg is fired horizontally into a fixed vertical block of material. It enters the block horizontally with speed $250\,\mathrm{m\,s^{-1}}$ and emerges horizontally with speed $70\,\mathrm{m\,s^{-1}}$ after 0.04 s. The block offers a constant horizontal resisting force of magnitude 450 N. Find the value of m. [3]

Q14

9231/21, 9231/22 • May/June 2018 • Question 3

Two identical uniform small spheres A and B, each of mass m, are moving towards each other in a straight line on a smooth horizontal surface. Their speeds are u and ku respectively, and they collide directly. The coefficient of restitution between the spheres is e. Sphere B is brought to rest by the collision.

(i) Show that $e = \dfrac{k-1}{k+1}$. [3]

Given that 60% of the total initial kinetic energy is lost in the collision,

(ii) find the values of k and e. [6]

Q15

9231/23 • May/June 2018 • Question 2

Two uniform small spheres A and B have equal radii and masses $4m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is e.

(i) Show that after the collision A moves with speed $\frac{1}{5}u(4-e)$ and find the speed of B. [4]

Sphere B continues to move until it collides with a fixed smooth vertical barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is $\frac{3}{4}e$. After this collision, the speeds of A and B are equal.

(ii) Find the value of e. [3]

The spheres A and B now collide directly again.

(iii) Determine whether sphere B collides with the barrier for a second time. [2]

Q16

9231/21, 9231/23 • October/November 2018 • Question 2

Two uniform small smooth spheres A and B have equal radii and masses $5m$ and $2m$ respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is moving towards it with speed $2u$. The coefficient of restitution between the spheres is e.

(i) Show that the speed of B after the collision is $\frac{1}{7}u(1+15e)$ and find an expression for the speed of A. [4]

In the collision, the speed of A is halved and its direction of motion is reversed.

(ii) Find the value of e. [2]

(iii) For this collision, find the ratio of the loss of kinetic energy of A to the loss of kinetic energy of B. [3]

Q17

9231/22 • October/November 2018 • Question 2

Two uniform small smooth spheres A and B have equal radii and masses $2m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is $\frac{2}{3}$.

(i) Find, in terms of u, the speeds of A and B after this collision. [4]

Sphere B is initially at a distance d from a fixed smooth vertical wall which is perpendicular to the direction of motion of A. The coefficient of restitution between B and the wall is $\frac{1}{2}$.

(ii) Find, in terms of d and u, the time that elapses between the first and second collisions between A and B. [5]

Q18

9231/21, 9231/22 • May/June 2019 • Question 3

Three uniform small spheres A, B and C have equal radii and masses $2m$, $4m$ and m respectively. The spheres are moving in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between each pair of spheres is e. Spheres A and B are moving towards each other with speeds $2u$ and u respectively. The first collision is between A and B.

(i) Find the velocities of A and B after this collision. [3]

Sphere C is moving towards B with speed $\frac{4}{3}u$ and now collides with it. As a result of this collision, B is brought to rest.

(ii) Find the value of e. [4]

(iii) Find the total kinetic energy lost by the three spheres as a result of the two collisions. [3]

Q19

9231/23 • May/June 2019 • Question 1

A bullet of mass 0.2 kg is fired into a fixed vertical barrier. It enters the barrier horizontally with speed $250\,\mathrm{m\,s^{-1}}$ and emerges horizontally after a time T seconds with speed $40\,\mathrm{m\,s^{-1}}$. There is a constant horizontal resisting force of magnitude 1200 N. Find T. [4]

Q20

9231/23 • May/June 2019 • Question 3

Three uniform small spheres A, B and C have equal radii and masses $3m$, m and m respectively. The spheres are at rest in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between each pair of spheres is e. Sphere A is projected directly towards B with speed u.

(i) Find, in terms of u and e, expressions for the speeds of A, B and C after the first two collisions. [6]

(ii) Given that A and C are moving with equal speeds after these two collisions, find the value of e. [3]

Q21

9231/21, 9231/22, 9231/23 • October/November 2019 • Question 3

Three uniform small spheres A, B and C have equal radii and masses $5m$, $5m$ and $3m$ respectively. The spheres are at rest on a smooth horizontal surface, in a straight line, with B between A and C. The coefficient of restitution between each pair of spheres is e. Sphere A is projected directly towards B with speed u.

(i) Show that the speed of A after its collision with B is $\frac{1}{2}u(1-e)$ and find the speed of B. [3]

Sphere B now collides with sphere C. Subsequently there are no further collisions between any of the spheres.

(ii) Find the set of possible values of e. [6]

Q22

9231/31, 9231/32 • May/June 2020 • Question 6

A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. The particle strikes a fixed vertical barrier. At the instant of impact the direction of motion of P makes an angle α with the barrier. The coefficient of restitution between P and the barrier is e. As a result of the impact, the direction of motion of P is turned through $90°$.

(a) Show that $\tan^2 \alpha = \dfrac{1}{e}$. [3]

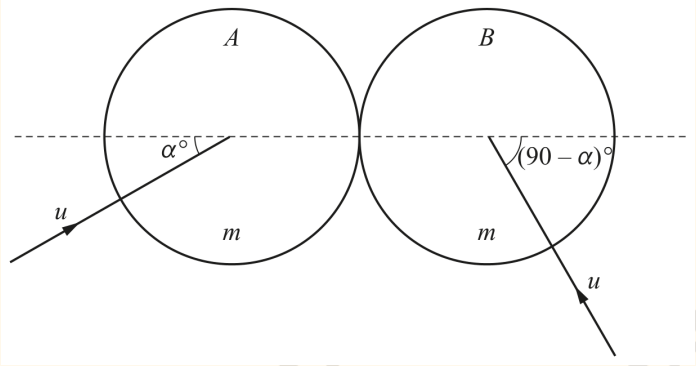
The particle P loses two-thirds of its kinetic energy in the impact.

(b) Find the value of α and the value of e. [5]

Q23

9231/33 • May/June 2020 • Question 5

Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion makes an angle of α° with the line of centres, and B's direction of motion is perpendicular to that of A (see diagram). The coefficient of restitution between the spheres is e.



Immediately after the collision, B moves in a direction at right angles to the line of centres.

(a) Show that $\tan\alpha = \dfrac{1+e}{1-e}$. [4]

(b) Given that $\tan\alpha = 2$, find the speed of A after the collision. [4]

Q24

9231/31, 9231/33 • October/November 2020 • Question 6

Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B. The coefficient of restitution between the spheres is e.

(a) Find, in terms of u and e, the velocities of A and B after the collision. [3]

Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B, where $\tan\theta = \frac{3}{4}$.

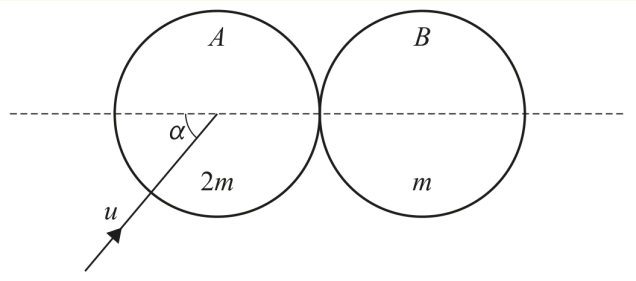
The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B.

(b) Find the possible values of e. [7]

Q25

9231/32 • October/November 2020 • Question 2

Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides with B. Immediately before the collision, the direction of motion of A makes an angle α with the line of centres of the spheres, where $\tan\alpha = \frac{4}{3}$ (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

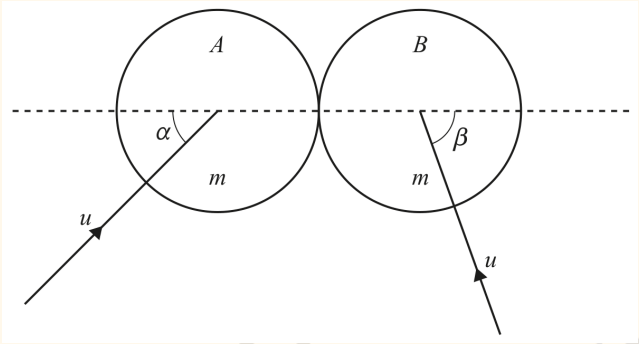


Find the speed of A after the collision. [5]

Q26

9231/31, 9231/32 • May/June 2021 • Question 6

Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision, A's direction of motion makes an angle α with the line of centres, and B's direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$ and $2\cos\beta = \cos\alpha$.



(a) Show that the direction of motion of A after the collision is perpendicular to the line of centres. [4]

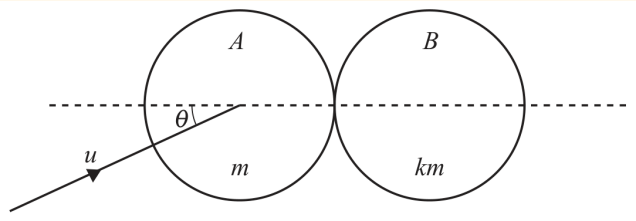
The total kinetic energy of the spheres after the collision is $\frac{3}{4}mu^2$.

(b) Find the value of α. [4]

Q27

9231/33 • May/June 2021 • Question 6

Two uniform smooth spheres A and B of equal radii have masses m and km respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides with sphere B which is at rest. Immediately before the collision, A's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.



(a) Show that the speed of B after the collision is $\dfrac{4u\cos\theta}{3(1+k)}$. [3]

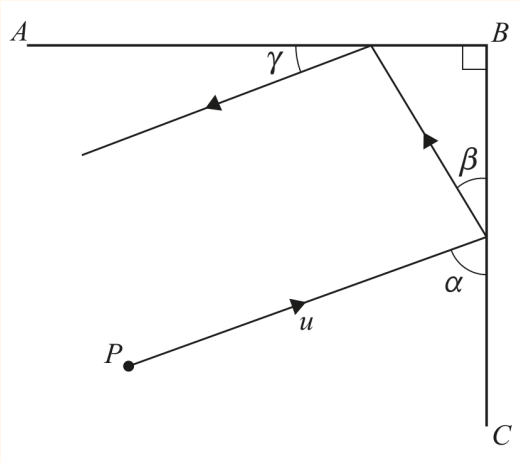
70% of the total kinetic energy of the spheres is lost as a result of the collision.

(b) Given that $\tan\theta = \frac{1}{3}$, find the value of k. [6]

Q28

9231/31, 9231/33 • October/November 2021 • Question 7

The smooth vertical walls AB and CB are at right angles to each other. A particle P is moving with speed u on a smooth horizontal floor and strikes the wall CB at an angle α. It rebounds at an angle β to the wall CB. The particle then strikes the wall AB and rebounds at an angle γ to that wall (see diagram). The coefficient of restitution between each wall and P is e.



(a) Show that $\tan\beta = e\tan\alpha$. [3]

(b) Express γ in terms of α and explain what this result means about the final direction of motion of P. [4]

As a result of the two impacts the particle loses $\frac{8}{9}$ of its initial kinetic energy.

(c) Given that $\alpha + \beta = 90°$, find the value of e and the value of $\tan\alpha$. [4]

Q29

9231/32 • October/November 2021 • Question 2

A light spring AB has natural length a and modulus of elasticity $5mg$. The end A of the spring is attached to a fixed point on a smooth horizontal surface. A particle P of mass m is attached to the end B of the spring. The spring and particle P are at rest on the surface.

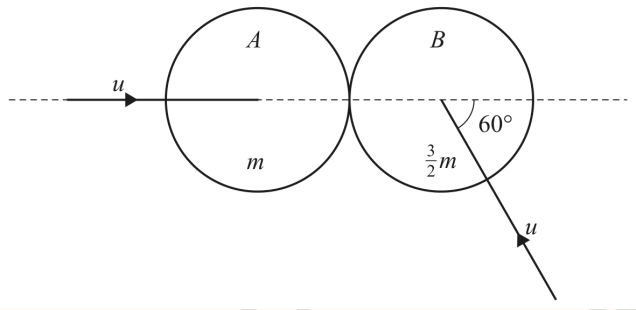
Another particle Q of mass km is moving with speed $\sqrt{4ga}$ along the horizontal surface towards P in the direction BA. The particles P and Q collide directly and coalesce. In the subsequent motion the greatest amount by which the spring is compressed is $\frac{1}{5}a$.

Find the value of k. [6]

Q30

9231/32 • October/November 2021 • Question 5

Two uniform smooth spheres A and B of equal radii have masses m and $\frac{3}{2}m$ respectively. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion is along the line of centres, and B's direction of motion makes an angle of $60°$ with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$.



(a) Find the angle through which the direction of motion of B is deflected by the collision. [6]

(b) Find the loss in the total kinetic energy of the system as a result of the collision. [3]

Q31

9231/31, 9231/32 • May/June 2022 • Question 6

Two uniform smooth spheres A and B of equal radii have masses m and km respectively. The two spheres are on a horizontal surface. Sphere A is travelling with speed u towards sphere B which is at rest. The spheres collide. Immediately before the collision, the direction of motion of A makes an angle α with the line of centres. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(a) Show that the speed of B after the collision is $\frac{3u\cos\alpha}{2(1+k)}$ and find also an expression for the speed of A along the line of centres after the collision, in terms of k, u and α. [4]

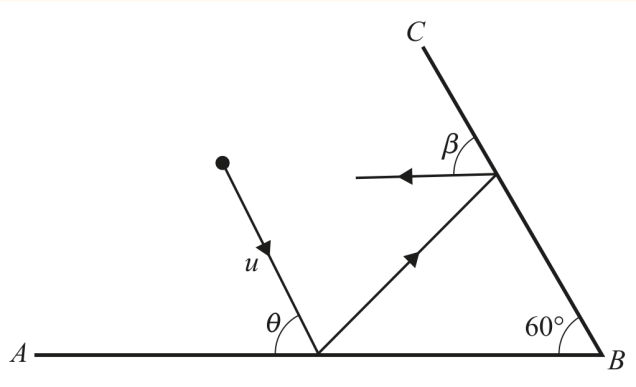
After the collision, the kinetic energy of A is equal to the kinetic energy of B.

(b) Given that $\tan\alpha = \frac{2}{3}$, find the possible values of k. [5]

Q32

9231/33 • May/June 2022 • Question 6

AB and BC are two fixed smooth vertical barriers on a smooth horizontal surface, with angle $ABC = 60°$. A particle of mass m is moving with speed u on the surface. The particle strikes AB at an angle θ with AB. It then strikes BC and rebounds at an angle β with BC (see diagram). The coefficient of restitution between the particle and each barrier is e and $\tan\theta = 2$.



The kinetic energy of the particle after the first collision is 40% of its kinetic energy before the first collision.

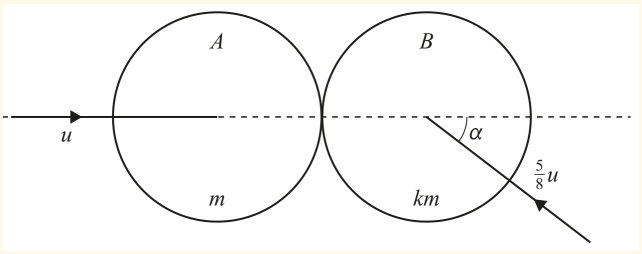
(a) Find the value of e. [4]

(b) Find the size of angle β. [4]

Q33

9231/31, 9231/33 • October/November 2022 • Question 6

Two uniform smooth spheres A and B of equal radii have masses m and km respectively. The two spheres are moving on a horizontal surface with speeds u and $\frac{5}{8}u$ respectively. Immediately before the spheres collide, A is travelling along the line of centres, and B's direction of motion makes an angle α with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$ and $\tan\alpha = \frac{3}{4}$.



After the collision, the direction of motion of B is perpendicular to the line of centres.

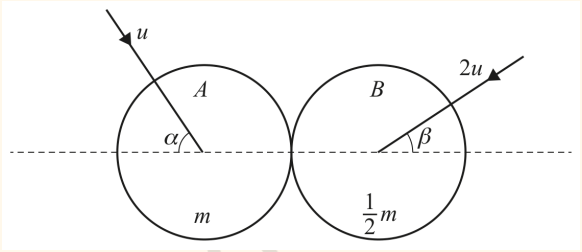
(a) Find the value of k. [4]

(b) Find the loss in the total kinetic energy as a result of the collision. [4]

Q34

9231/32 • October/November 2022 • Question 7

Two uniform smooth spheres A and B of equal radii have masses m and $\frac{1}{2}m$ respectively. The two spheres are moving on a horizontal surface when they collide. Immediately before the collision, sphere A is travelling with speed u and its direction of motion makes an angle α with the line of centres. Sphere B is travelling with speed $2u$ and its direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{5}{8}$ and $\alpha + \beta = 90°$.



(a) Find the component of the velocity of B parallel to the line of centres after the collision, giving your answer in terms of u and α. [4]

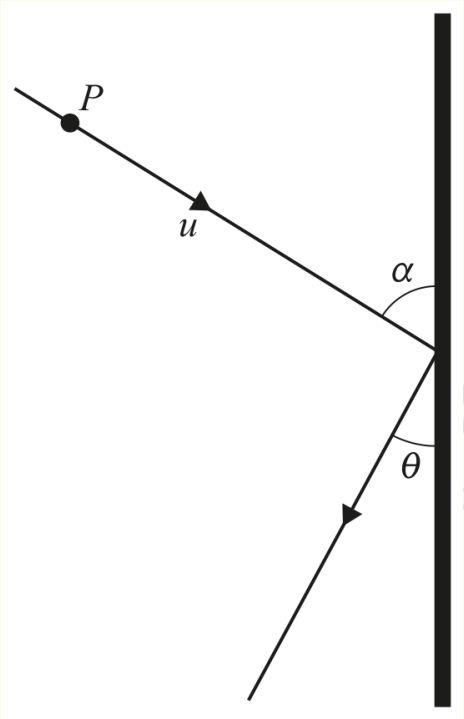
The direction of motion of B after the collision is parallel to the direction of motion of A before the collision.

(b) Find the value of $\tan\alpha$. [5]

Q35

9231/31, 9231/32 • May/June 2023 • Question 2

A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. It collides at an angle α with a fixed smooth vertical barrier. After the collision, P moves at an angle θ with the barrier, where $\tan\theta = \frac{1}{2}$ (see diagram). The coefficient of restitution between P and the barrier is e. The particle P loses 20% of its kinetic energy as a result of the collision.

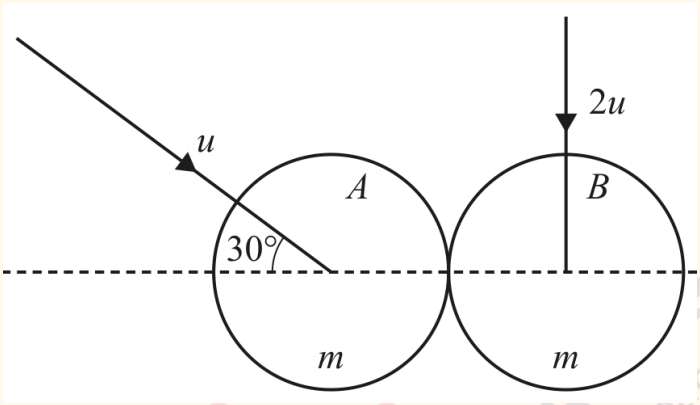


Find the value of e. [5]

Q36

9231/33 • May/June 2023 • Question 4

Two identical smooth uniform spheres A and B each have mass m. The two spheres are moving on a smooth horizontal surface when they collide with speeds u and $2u$ respectively. Immediately before the collision, A's direction of motion makes an angle of $30°$ with the line of centres, and B's direction of motion is perpendicular to the line of centres (see diagram). After the collision, A and B are moving in the same direction. The coefficient of restitution between the spheres is e.



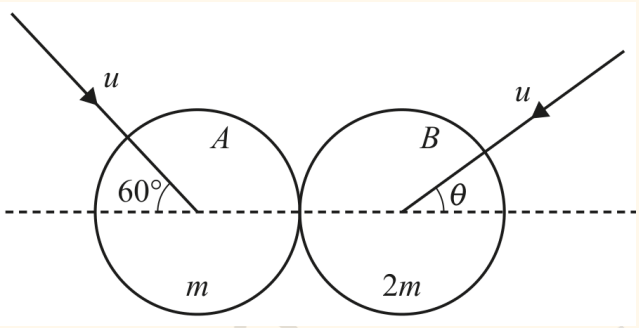
(a) Find the value of e. [5]

(b) Find the loss in the total kinetic energy of the spheres as a result of the collision. [3]

Q37

9231/31, 9231/33 • October/November 2023 • Question 1

Two uniform smooth spheres A and B of equal radii have masses m and $2m$ respectively. The two spheres are moving with equal speeds u on a smooth horizontal surface when they collide. Immediately before the collision, A's direction of motion makes an angle of $60°$ with the line of centres, and B's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is e.



After the collision, the component of the velocity of A along the line of centres is v and B moves perpendicular to the line of centres. Sphere A now has twice as much kinetic energy as sphere B.

(a) Show that $v = \frac{1}{2}u(4\cos\theta - 1)$. [1]

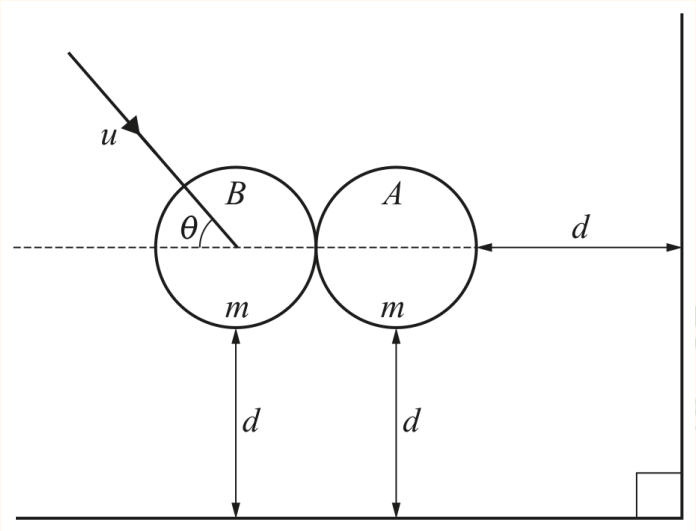
(b) Find the value of $\cos\theta$. [4]

(c) Find the value of e. [2]

Q38

9231/32 • October/November 2023 • Question 4

Two smooth vertical walls meet at right angles. The smooth sphere A, with mass m, is at rest on a smooth horizontal surface and is at a distance d from each wall. An identical smooth sphere B is moving on the horizontal surface with speed u at an angle θ with the line of centres when the spheres collide (see diagram). After the collision, the spheres take the same time to reach a wall. The coefficient of restitution between the spheres is $\frac{1}{2}$.



(a) Find the value of $\tan\theta$. [4]

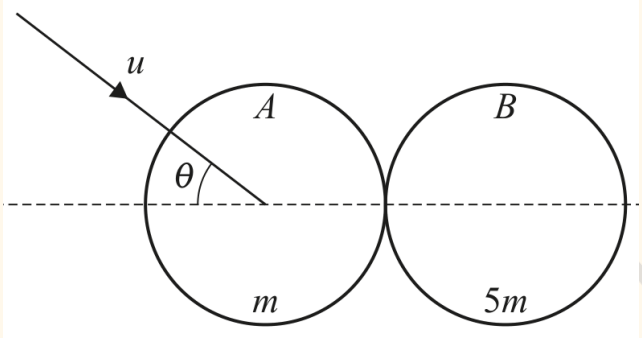
(b) Find the percentage loss in the total kinetic energy of the spheres as a result of this collision. [3]

Q39

9231/31, 9231/32 • May/June 2024 • Question 1

Two smooth uniform spheres A and B of equal radii have masses m and $5m$ respectively. Sphere A is moving on a smooth horizontal surface with speed u when it collides with sphere B which is at rest on the surface. Immediately before the collision, A's direction of motion makes an angle of θ with the line of centres. After the collision, the kinetic energies of A and B are equal. The coefficient of restitution between the spheres is $\frac{1}{2}$.

Find the value of $\tan\theta$. [6]



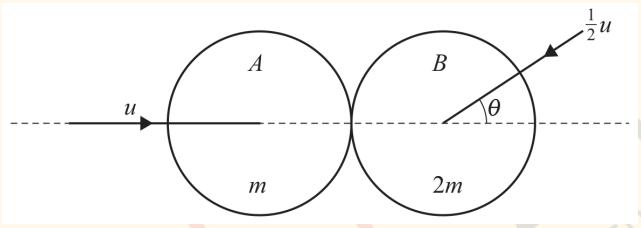
Q40

9231/33 • May/June 2024 • Question 1

Two smooth uniform spheres A and B of equal radii have masses m and $2m$ respectively. The two spheres are moving on a smooth horizontal surface when they collide with speeds u and $\frac{1}{2}u$ respectively. Immediately before the collision, A's direction of motion is along the line of centres, and B's direction of motion makes an angle θ with the line of centres (see diagram).

As a result of the collision, the direction of motion of A is reversed and its speed is reduced to $\frac{1}{4}u$. The direction of motion of B again makes an angle θ with the line of centres, but on the opposite side of the line of centres. The speed of B is unchanged.

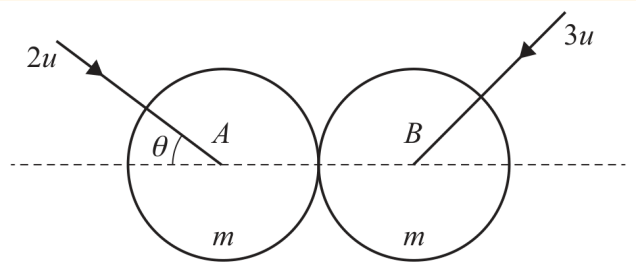
Find the value of the coefficient of restitution between the spheres. [4]



Q41

9231/32 • October/November 2024 • Question 3

The diagram shows two identical smooth uniform spheres A and B of equal radii and each of mass m. The two spheres are moving on a smooth horizontal surface when they collide with speeds $2u$ and $3u$ respectively. Immediately before the collision, A's direction of motion makes an angle θ with the line of centres and B's direction of motion is perpendicular to that of A. After the collision, B moves perpendicular to the line of centres. The coefficient of restitution between the spheres is $\frac{1}{3}$.



(a) Find the value of $\tan\theta$. [3]

(b) Find the total loss of kinetic energy as a result of the collision. [2]

(c) Find, in degrees, the angle through which the direction of motion of A is deflected as a result of the collision. [2]

Q42

9231/31, 9231/32 • May/June 2025 • Question 6

Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere A is moving in a straight horizontal line with speed u, and sphere B is stationary. Sphere A collides directly with B, and they both then move in the same direction with speeds v_A and v_B respectively. After the collision, the kinetic energy of B is $\frac{9}{2}$ times the kinetic energy of A.

(a) Show that $v_B = \frac{6}{5}u$. [3]

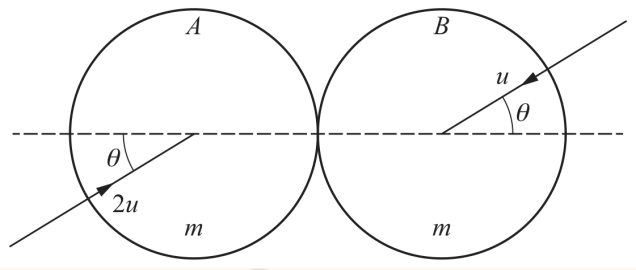
Sphere B then collides with a fixed vertical barrier. Immediately before the collision, the direction of motion of B makes an angle α with the barrier. Immediately after the collision, the direction of motion of B makes an angle β with the barrier. The coefficient of restitution between B and the barrier is $\frac{4}{5}$. As a result of the collision, the velocity of B is reduced to $\frac{12}{25}\sqrt{5}\,u$.

(b) Find the value of $\sin(\alpha + \beta)$. [6]

Q43

9231/33 • May/June 2025 • Question 6

Two identical uniform smooth spheres A and B, each with mass m, are moving on a horizontal surface with speeds $2u$ and u respectively when they collide. Immediately before the collision, the spheres are moving parallel to each other in opposite directions such that their directions of motion each make an angle θ with the line of centres (see diagram). As a result of the collision, B moves in a direction which is perpendicular to its initial direction of motion. The coefficient of restitution between the spheres is e.



(a) Find an expression for $\tan\theta$ in terms of e. [6]

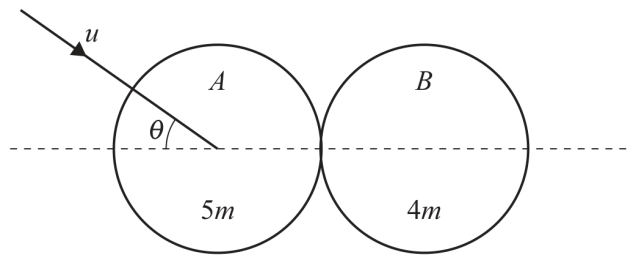
As a result of the collision, A moves in a direction which is perpendicular to the line of centres.

(b) Find the value of θ. [2]

Q44

9231/34 • May/June 2025 • Question 6

Two uniform smooth spheres A and B of equal radii have masses $5m$ and $4m$ respectively. Sphere A is moving with speed u on a horizontal surface when it collides with sphere B which is at rest. Immediately before the collision, A's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is e.



(a) Show that the speed of B after the collision is $\frac{5}{9}u(1+e)\cos\theta$. [3]

After the collision the kinetic energy of A is equal to the kinetic energy of B.

(b) Given that $\tan\theta = \frac{2}{3}$, find the value of e. [6]

Q45

9231/31, 9231/33 • October/November 2025 • Question 1

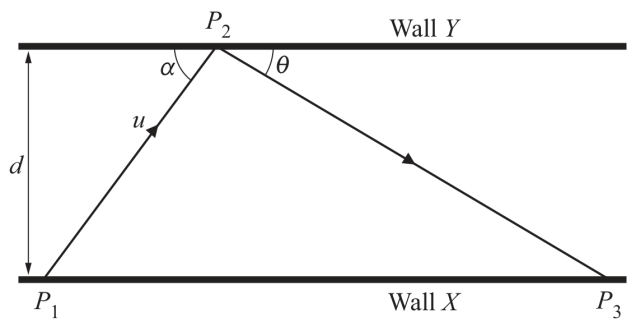
Two uniform smooth spheres A and B of equal radii have masses $4m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides directly with sphere B. After the collision, the momentum of A is three times the momentum of B.

Find the value of the coefficient of restitution e. [4]

Q46

9231/34 • October/November 2025 • Question 7

X and Y are two fixed smooth vertical walls on a smooth horizontal surface. The walls are parallel and at a distance d apart. The points P_1, P_2 and P_3 all lie on the surface.



A particle Q is projected horizontally from the point P_1 on Wall X with speed u, and moves along the surface. The particle Q strikes Wall Y at the point P_2. Immediately before the collision, the direction of motion of Q makes an angle α with Wall Y, where $\sin\alpha = \frac{4}{5}$. Immediately after the collision, the direction of motion of Q makes an angle θ with Wall Y. The particle Q then strikes Wall X at the point P_3 (see diagram).

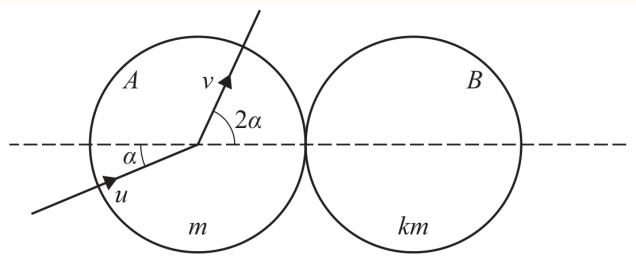
The time that it takes Q to travel the distance P_1P_2 is T. The time that it takes Q to travel the distance P_2P_3 is kT.

Find, in terms of k, the coefficient of restitution between Q and Wall Y. [6]

Q47

9231/32 • October/November 2025 • Question 5

Two uniform smooth spheres, A and B, of equal radii are on a horizontal surface. They have masses of m and km respectively. Sphere A is moving with speed u at an angle α with the line of centres when it collides with sphere B which is stationary. Immediately after the collision, sphere A moves with speed v at an angle 2α with the line of centres (see diagram).



It is given that $\tan\alpha = \frac{3}{4}$.

(a) Find v in terms of u. [2]

(b) Find the coefficient of restitution between the spheres in terms of k. [4]

(c) Find the range of possible values of k. [3]

Answers available in the companion mark scheme booklet.