Further Math with Ali Hashir

Momentum

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/21, 9231/22 • May/June 2015 • Question 5

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Mark scheme.

For A & B use conservation of momentum, e.g. $3mv_A + 2mv_B = 3mu$.	**M1**
Use Newton's law of restitution (consistent signs): $v_B - v_A = eu$.	**M1**
Combine to find v_B: $v_B = 3(1+e)u/5$.	**A1**
For B & C use conservation of momentum, e.g. $2mv'_B + mv_C = 2mv_B$.	**M1**
Use Newton's law of restitution (consistent signs): $v_C - v'_B = e'v_B$.	**M1**
Combine to find v_C and v'_B: $v_C = 2(1+e')v_B/3 = 2(1+e)(1+e')u/5$ (AG).	**A1**
$v'_B = (2-e')v_B/3 = (1+e)(2-e')u/5$.	**A1**
Find ratios or values of v_A, v'_B, v_C from momentum: $3v_A = 2v'_B = v_C$ $[= u]$.	**B1**
Find e from first collision eqns, e.g. $v_A = (3-2e)u/5 = u/3$, $e = 2/3$.	**M1 A1**
Find e' from second collision eqns, e.g. $2v'_B = v_C$ so $2(2-e') = 2(1+e')$, $e' = 1/2$.	**M1 A1**
	Total: 12

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Q2

9231/23 • May/June 2015 • Question 1

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Mark scheme.

Find speed u_A of A after initial impulse: $2u_A = 8$, $u_A = 4$.	**B1**
For A & B use conservation of momentum, e.g. $2v_A + 3v_B = 2u_A$ $[= 8]$.	**M1**
Use Newton's law of restitution (consistent signs): $v_B - v_A = eu_A$ $[= 4e]$.	**M1**
Combine to find v_A and v_B: $v_A = (2 - 3e)u_A/5$, $v_B = 2(1 + e)u_A/5$.	**A1**
(AEF) $v_A = 4(2 - 3e)/5$, $v_B = 8(1 + e)/5$.	**A1**
Use $v_A < 0$: $2 - 3e < 0$, $e > 2/3$ (AG).	**B1**
	Total: 6

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Q3

9231/23 • May/June 2015 • Question 2

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Mark scheme.

EITHER: find/state components of speed after 1st collision: $u\cos\alpha$ parallel to A and $eu\sin\alpha$ perpendicular to A.	**M1 A1**
Find/state components of speed after 2nd collision: $eu\sin\alpha$ parallel to B and $eu\cos\alpha$ perpendicular to B.	**A1**
Find final angle with barrier B: $\tan^{-1}(eu\cos\alpha/eu\sin\alpha) = \frac{1}{2}\pi - \alpha$ (AG).	**A1**
Find total loss in KE, e.g. $\frac{1}{2}m\{u^2 - (eu\sin\alpha)^2 - (eu\cos\alpha)^2\}$.	**M1 A1**
Find total loss in KE (A0 if wrong sign): $= \frac{1}{2}m(1-e^2)u^2$.	**A1**
	Total: 7

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Q4

9231/21, 9231/22, 9231/23 • October/November 2015 • Question 2

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Mark scheme.

Conservation of momentum for A & B: $2mv_A + mv_B = 2mu$.	**M1**
Newton's law of restitution: $v_B - v_A = eu$.	**M1**
Combine: $v_A = (2-e)u/3$, $v_B = 2(1+e)u/3$.	**A1A1**
Find e from $v_A = v'_B$ with $v'_B = 0.4v_B$: $(2-e) = 0.8(1+e)$, giving $e = 2/3$.	**M1A1**
Equate times in terms of required distance x: $(d-x)/v_A = d/v_B + x/v'_B$ (AEF), noting $v_A = v'_B = 4u/9$, $v_B = 10u/9$.	**M1A1**
Use $v_A = v'_B = 0.4v_B$ to solve for x: $d - x = 0.4d + x$, so $x = 0.3d$.	**M1A1**
	Total: 10

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Q5

9231/21, 9231/22 • May/June 2016 • Question 1

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Mark scheme.

(i)	
Find magnitude of impulse by change in momentum: $(\pm)0.01 \times (320 - 20)$	**M1**
$= (\pm)3 \text{ N s}^{-1}$	**A1**
(ii)	
EITHER: Find thickness d by change in energy: $1000d = \frac{1}{2} \times 0.01 \times (320^2 - 20^2)$	**M1**
$d = 0.51$ m	**A1**
(iii)	
EITHER: Find time t from constant deceleration formula: $d = \frac{1}{2}(320 + 20)t$, $t = 0.003$ s	**B1**
	Total: 5

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Q6

9231/21, 9231/22 • May/June 2016 • Question 2

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Mark scheme.

Use momentum and Newton's law, e.g. $mv_A + mv_B = mu$, $v_B - v_A = eu$	**M1**
Combine to find v_A and v_B: $v_A = \pm(1-e)u/2$, $v_B = \pm(1+e)u/2$	**A1,A1**
Relate speed w_B of B after collision with wall to v_B: $w_B = \pm\frac{1}{2}v_B$ [$= \pm(1+e)u/4$]	**B1**
Relate KEs before and after: $\frac{1}{5}\cdot\frac{1}{2}mu^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mw_B^2$	**M1 A1**
Find e: $\frac{1}{5} = (1-e)^2/4 + (1+e)^2/16 \Rightarrow 25e^2 - 30e + 9 = (5e-3)^2 = 0$	**M1 A1**
	Total: 8

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Q7

9231/23 • May/June 2016 • Question 5

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Mark scheme.

For A & B use conservation of momentum: $5mv_A + mv_B = 5mu$	**M1**
Use Newton's law of restitution: $v_B - v_A = \frac{4}{5}u$	**M1**
Combine to verify $v_A = \frac{7}{10}u$ (A.G.) and find $v_B = \frac{3}{2}u$	**A1**
(i)	
For B & C use conservation of momentum: $mv'_B + kmv_C = mv_B$	**M1**
Use Newton's law of restitution: $v_C - v'_B = \frac{4}{5}v_B$	**M1**
Combine to find k using $v_C = v_A$: $(k+1)v_C = (1+\frac{4}{5})v_B \Rightarrow k+1 = 27/7,\ k = 20/7$ (A.G.)	**M1 A1**
(ii)	
Find $v'_B = -u/2$	**B1**
Find final KE of 3 particles: $\frac{1}{2} \cdot 5mv_A^2 + \frac{1}{2}mv_B'^2 + \frac{1}{2}kmv_C^2$	**M1**
$= \frac{1}{2}mu^2 \left(\frac{49}{20} + \frac{1}{4} + \frac{7}{5}\right) = (41/20)mu^2$ or $2.05mu^2$	**A1**
Find loss in KE: $\frac{1}{2} \cdot 5mu^2 - E_{\text{final}} = (9/20)mu^2$	**M1**
Find percentage loss: $100 \times (9/20)mu^2/(\frac{1}{2} \cdot 5mu^2) = 18\%$	**A1**
	Total: 12

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Q8

9231/21, 9231/22, 9231/23 • October/November 2016 • Question 2

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Mark scheme.

EITHER: Find components of speed after collision at E: $v\cos 45°$ parallel to wall and $\frac{3}{4}v\sin 45°$ perpendicular to wall	**(i)**
	M1 A1
Relate v to u: $\sqrt{(v/\sqrt{2})^2 + (\frac{3}{4}v/\sqrt{2})^2} = \frac{1}{4}u$	**M1**
$(5/4\sqrt{2})v = \frac{1}{4}u$	**A1**
$v = (\sqrt{2}/5)u$ A.G.	**A1**
OR: Relate angle β after collision to u, v: $\frac{1}{4}u\cos\beta = v\cos 45°$ and $\frac{1}{4}u\sin\beta = \frac{3}{4}v\sin 45°$ (M1 A1)	
Find $\tan\beta$, or β: $\tan\beta = \frac{3}{4}$ or $\beta = 36.9°$ (A1)	
Eliminate β: $\frac{1}{4}u \times (4/5) = v/\sqrt{2}$ (M1)	
$v = (\sqrt{2}/5)u$ A.G. (A1)	
(ii)	
Relate components of speed parallel to wall after collision at D: $v\cos 75° = u\cos\alpha$	**M1**
$\cos\alpha = (\sqrt{2}/5)\cos 75°$ [= 0.0732]	**A1**
$\alpha = 85.8°$ or 1.50 rad	**A1**
Relate components of speed perpendicular to wall after collision at D: $v\sin 75° = eu\sin\alpha$	**M1**
$e = (\sqrt{2}/5)\sin 75°/\sin\alpha$ or $= \tan 75°/\tan\alpha = 0.274$	**A1**
	Total: 10

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Q9

9231/21, 9231/22 • May/June 2017 • Question 1

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Mark scheme.

Find equation for exit speed v from e.g. change in momentum $= Ft$: $0.08 \times (300 - v) = 1000 \times 0.02$ (AEF) (if $300 + v$ or equivalent used, allow M1 only).	**M1**
Correct equation.	**A1**
$v = 300 - 250 = 50$.	**A1**
	Total: 3

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Q10

9231/21, 9231/22 • May/June 2017 • Question 3

↑ Back to contents

Mark scheme.

Use momentum and Newton's law: $3mv_A + mv_B = 3mu$, $v_B - v_A = eu$ (AEF) (M0 if inconsistent LHS signs; allow $3v_A + v_B = 3u$).	**M1**
Combine to find velocities of A and B after collision (signs must be consistent with chosen direction): $v_A = \frac{1}{4}(3-e)u$, $v_B = \frac{3}{4}(1+e)u$. **(ii)**	**A1,A1**
Relate velocity v'_B of B after collision with wall to v_B: $v'_B = -\frac{3}{4}v_B$ $[= -\frac{9}{16}(1+e)u]$ (AEF).	**B1**
Use momentum (allow m omitted and $V_A = 0$): $[3mV_A+]\, mV_B = 3mv_A + mv'_B$ $[V_B = 3(9-7e)u/16]$.	**M1**
Use Newton's law: $V_B[-V_A] = -e(v'_B - v_A)$ $[V_B = e(21+5e)u/16]$.	**M1**
EITHER: eliminate V_B with $V_A = 0$ and substitute for v_A and v'_B: $[4V_A =](3-e)v_A + (1+e)v'_B = 0$, giving $\frac{1}{4}(3-e)^2 - \frac{9}{16}(1+e)^2 = 0$ (AEF); OR: $3(9-7e) = e(21+5e)$.	**M1,A1**
Form and solve quadratic for e, rejecting root -9: $5e^2 + 42e - 27 = 0$, $e = 3/5$ or 0.6.	**M1,A1**
	Total: 10

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Q11

9231/23 • May/June 2017 • Question 3

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Mark scheme.

Use conservation of momentum (allow $v_A + v_B = u$): $mv_A + mv_B = mu$ (AEF).	***M1**
Use Newton's restitution law (consistent LHS signs): $v_B - v_A = \frac{2}{3}u$.	***M1**
Combine to find v_B: $v_B = 5u/6$.	**A1**
Verify speed w_B of B after collision with wall (ignore sign): $w_B = \frac{1}{3}v_B = 5u/18$ (AG). **(ii)**	**B1**
Find v_A (dependent on above *M1 *M1): $v_A = u/6$.	**DA1**
EITHER: equate times in terms of required distance x: $(d-x)/v_A = d/v_B + x/w_B$ (AEF).	**M1,A1**
Substitute for speeds to formulate an equation in x: $6(d-x) = 1.2d + 3.6x$.	**M1,A1**
OR: find distance x_A moved by A when B reaches wall: $x_A = (d/v_B)v_A = (6d/5u)u/6 = 0.2d$.	**M1**
Find remaining time t_2: $t_2 = (0.8d)/(v_A + w_B) = 9d/5u$.	**M1,A1**
Find remaining distance moved by A or B: $y_A = v_At_2 = 0.3d$ or $y_B = w_Bt_2 = 0.5d$.	**A1**
Find x: $x = \frac{1}{2}d$.	**A1**
	Total: 10

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Q12

9231/21, 9231/22, 9231/23 • October/November 2017 • Question 3

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Mark scheme.

(i)	
Use conservation of momentum for A & B (allow omission of m): $v_A + kv_B = 2u + ku$.	**M1**
Use Newton's restitution law with consistent signs: $v_B - v_A = \frac{1}{2}(2u - u) = \frac{1}{2}u$.	**M1**
Combine to find $v_B = \dfrac{u(2k+5)}{2(k+1)}$.	**A1**
Find inequality for k from speeds of B and C after the 1st collision: $v_B > \frac{4}{3}u$ if $k < \frac{7}{2}$.	**M1, A1**
(ii)	
Use conservation of momentum for B & C: $kw_B + v_C = kv_B + \frac{4}{3}u$.	**M1**
Use Newton's restitution law and combine to find w_B: $(k+1)w_B = (k-\frac{1}{2})v_B + 2u$.	**M1**
With $k = 2$: $3w_B = \frac{3}{2}v_B + 2u$, and $v_B = \frac{3}{2}u$, so $w_B = \frac{17}{12}u$.	***A1**
Verify no further collisions between A and B: $v_A = u$, $v_A < w_B$.	**DB1**
EITHER find v_C and verify no further collisions between B and C: $v_C = \frac{3}{2}u > w_B$; OR state explicitly that B and C cannot meet again since they move apart after colliding.	**DB1**
	Total: 10

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Q13

9231/21, 9231/22 • May/June 2018 • Question 1

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Mark scheme.

Use change in momentum $= Ft$: $m(250-70) = 450 \times 0.04$	**M1**
$m = \dfrac{18}{180}$	**A1**
$m = 0.1$	**A1**
	Total: 3

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Q14

9231/21, 9231/22 • May/June 2018 • Question 3

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Mark scheme.

Use conservation of momentum: $mv_A = mu - mku$	**M1**
Use Newton's restitution law: $v_A = -e(u + ku)$	**M1**
Combine to verify $e(1 + k) = 1 - k$, so $e = \dfrac{k-1}{k+1}$ (AG) (ii)	**A1**
Attempt to relate initial and final KE: $KE_I - KE_F = 0.6KE_I$ or $0.4KE_I = KE_F$	**M1**
Substitute for KEs: $0.4 \times \frac{1}{2}mu^2(1 + k^2) = \frac{1}{2}mv_A^2 = \frac{1}{2}mu^2(1 - k)^2$	**M1**
Simplify to $0.6k^2 - 2k + 0.6 = 0$ or $3k^2 - 10k + 3 = 0$	**A1**
Solve quadratic: $(3k - 1)(k - 3) = 0$, so $k = 3$ or $k = \frac{1}{3}$	**A1**
Reject $k = \frac{1}{3}$ (since $0 < e < 1$ or $v_A < 0$), so $k = 3$	**A1**
$e = \dfrac{2}{4} = \dfrac{1}{2}$ (FT on k)	**A1**
	Total: 9

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Q15

9231/23 • May/June 2018 • Question 2

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Mark scheme.

Use momentum (allow m omitted): $4mv_A + mv_B = 4mu$	**M1**
Use Newton's law: $v_B - v_A = eu$	**M1**
Combine to verify/find $v_A = \frac{1}{5}(4-e)u$ (AG)	**A1**
$v_B = \frac{4}{5}(1+e)u$ (ii)	**A1**
Relate velocity v_B' of B after collision with wall to v_B: $v_B' = -\frac{3}{4}ev_B = -\frac{3}{5}e(1+e)u$	**M1**
Equate speeds of A and B: $\frac{1}{5}(4-e)u = \frac{3}{5}e(1+e)u$	**M1**
Solve resulting quadratic for e: $3e^2+4e-4=0$, $e=\frac{2}{3}$ (rejecting root -2) (iii)	**A1**
Use momentum (allow m omitted) with $v_A = -v_B' = \frac{2}{3}u$: $4mw_A + mw_B = 3mv_A = 2mu$; use Newton's law $w_B - w_A = e \times 2v_A = \frac{4}{3}v_A$; combine to find velocity of B after final collision: $w_B = \frac{5}{3}v_A = \frac{10}{9}u > 0$, so B collides again with the barrier. (Alternatively: momentum before collision is $3mv_A = 2mu > 0$, so after collision momentum/speed of $B > 0$, hence B collides again.)	**M1 A1**
	Total: 9

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Q16

9231/21, 9231/23 • October/November 2018 • Question 2

↑ Back to contents

Mark scheme.

(i)	
$5mv_A + 2mv_B = 5mu - 4mu = mu$ (use momentum, allow m omitted)	**M1**
$v_B - v_A = e(u + 2u) = 3eu$ (use Newton's law; M0 if LHS signs inconsistent)	**M1**
$v_A = (u/7)(1 - 6e)$ (combine to find/verify speeds of A and B after collision)	**A1**
$v_B = (u/7)(1 + 15e)$ AG (ignore signs)	**A1**
(ii)	
$(u/7)(1 - 6e) = -\frac{1}{2}u$, $e = \frac{3}{4}$ or 0.75 (combine to find e from $v_A = -\frac{1}{2}u$; M0 if direction of motion not reversed)	**M1A1**
(iii)	
$KE_A = \frac{1}{2} \times 5m\{u^2 - (\frac{1}{2}u)^2\} = \frac{15}{8}mu^2$ and $KE_B = \frac{1}{2} \times 2m\{(2u)^2 - (\frac{7}{4}u)^2\} = \frac{15}{16}mu^2$	**M1A1**
$KE_A/KE_B = (15/8)/(15/16) = 2 : 1$ or $2/1$ or 2	**A1**
	Total: 9

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Q17

9231/22 • October/November 2018 • Question 2

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Mark scheme.

(i)	
$2mv_A + mv_B = 2mu$ (AEF) (use momentum, allow m omitted)	**M1**
$v_B - v_A = \frac{2}{3}u$ (use Newton's law; M0 if LHS signs inconsistent)	**M1**
$v_A = 4u/9$, $v_B = 10u/9$ (combine to find speeds of A and B after collision)	**A1,A1**
(ii)	
$w_B = [-]\frac{1}{2}v_B[= \pm 5u/9]$ (relate speed w_B of B after collision with wall to v_B, ignore sign)	**M1**
EITHER: $(d-x)/v_A = d/v_B + x/w_B$ (AEF) – equate times in terms of distance x from wall to 3rd collision	**M1**
$(d-x)/4 = d/10 + x/5$, $x = d/3$ (substitute for speeds to solve for x)	**M1A1**
$t = (d-x)/v_A = (2d/3)/(4u/9) = 3d/2u$ (hence find required time t)	**A1**
	Total: 9

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Q18

9231/21, 9231/22 • May/June 2019 • Question 3

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Mark scheme.

Use conservation of momentum for A and B: $2mv_A + 4mv_B = 4mu - 4mu$, i.e. $v_A + 2v_B = 0$	**M1**
Use Newton's restitution law: $v_B - v_A = e(2u + u)$, i.e. $v_B - v_A = 3eu$	**M1**
Combine to find v_A and v_B: $v_A = -2eu$, $v_B = eu$ (ii)	**A1**
Use conservation of momentum for B and C: $[4mv'_B] + mv_C = 4mv_B - (4/3)mu$	**M1**
Use Newton's restitution law: $v_C[-v'_B] = e(v_B + 4u/3)$	**M1**
Combine to find quadratic equation for e using $v'_B = 0$: $4v_B - (4/3)u = ev_B + 4eu/3$, i.e. $4e - 4/3 = e^2 + 4e/3$	**M1**
$3e^2 - 8e + 4 = 0$, $e = 2/3$ (rejecting $e = 2$) (iii)	**A1**
For A: loss $= \frac{1}{2}2m(2u)^2 - \frac{1}{2}2m(4u/3)^2 = (20/9)mu^2$; for B: loss $= \frac{1}{2}4mu^2 = \frac{1}{2}mu^2$	**M1**
For C: loss $= 0$	**M1**
Hence total loss in KE $= (38/9)mu^2$	**A1**
	Total: 10

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Q19

9231/23 • May/June 2019 • Question 1

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Mark scheme.

Find impulse from change in momentum (may be implied): Impulse $= 0.2 \times (250 - 40) = 42$	**M1 A1**
Find time T from impulse $= Ft$: $T = 42/1200 = 7/200 = 0.035\,\text{s}$	**M1 A1**
	Total: 4

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Q20

9231/23 • May/June 2019 • Question 3

↑ Back to contents

Mark scheme.

Use conservation of momentum for A and B (correct masses): $3mv_A + mv_B = 3mu$	**M1**
Use Newton's restitution law: $v_B - v_A = eu$	**M1**
Combine to find speed of A: $v_A = \frac{1}{4}(3-e)u$, $[v_B = \frac{3}{4}(1+e)u]$	**A1**
Use conservation of momentum for B and C and Newton's restitution law: $w_B + v_C = v_B$ and $v_C - w_B = ev_B$	**M1**
Combine to find w_B and v_C in terms of v_B, then substitute: $w_B = (3/8)(1-e^2)u$ and $v_C = (3/8)(1+e)^2u$ (ii)	**A1 A1**
Find equation in e by equating their v_A and v_C: $\frac{1}{4}(3-e) = (3/8)(1+e)^2$	**M1**
Simplify and solve resulting quadratic equation for e: $3e^2 + 8e - 3 = 0 = (3e-1)(e+3)$	**M1**
$e = 1/3$ (rejecting $e = -3$)	**A1**
	Total: 9

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Q21

9231/21, 9231/22, 9231/23 • October/November 2019 • Question 3

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Mark scheme.

(i)	
Use conservation of momentum for A and B and Newton's restitution law with consistent signs (AEF): $5mv_A + 5mv_B = 5mu$ [$v_A + v_B = u$] and $v_B - v_A = eu$.	**M1**
Combine to verify speed of A: $v_A = \frac{1}{2}(1-e)u$. AG	**A1**
Find speed of B: $v_B = \frac{1}{2}(1+e)u$.	**A1**
(ii)	
Use conservation of momentum for B and C and Newton's restitution law: $5mv'_B + 3mv_C = 5mv_B$, $v_C - v'_B = ev_B$.	**M1**
Combine to find v'_B (v_C not required as B, C cannot collide again): $v'_B = (1/8)(5-3e)v_B$.	**A1**
Find condition on e using $v_A \leq v'_B$: $\frac{1}{2}(1-e)u \leq (1/8)(5-3e) \times \frac{1}{2}(1+e)u$.	**M1**
Simplify to a quadratic inequality: $3e^2 - 10e + 3 \leq 0$.	**A1**
Solve to give a lower bound on e: $\frac{1}{3} \leq e$.	**A1**
Non-strict inequality: $\frac{1}{3} \leq e \leq 1$.	**A1**
	Total: 9

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Q22

9231/31, 9231/32 • May/June 2020 • Question 6

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Mark scheme.

$y = v\cos\alpha = eu\sin\alpha$.	**B1**
$x = v\sin\alpha = u\cos\alpha$.	**B1**
Divide: $\tan\alpha = \dfrac{1}{e\tan\alpha}$: $\tan^2\alpha = \dfrac{1}{e}$. **(b)**	**B1**
$v^2 = \dfrac{1}{3}u^2$.	**B1**
$\left(\dfrac{u\cos\alpha}{\sin\alpha}\right)^2 = \dfrac{1}{3}u^2$.	**M1**
$(\tan\alpha)^2 = 3$.	**M1**
$\alpha = 60°$.	**A1**
$e = \dfrac{1}{3}$. *Alternative method for (b):* KE after impact $= \dfrac{1}{2}m(x^2+y^2) = \dfrac{1}{2}m\left((u\cos\alpha)^2 + e^2(u\sin\alpha)^2\right)$ [**M1**]; from (a) $\sin\alpha = 1/\sqrt{(1+e)}$ and $\cos\alpha = \sqrt{e}/\sqrt{(1+e)}$ [**B1**]; KE $= \dfrac{1}{2}mu^2\left(\dfrac{e}{1+e} + \dfrac{e^2}{1+e}\right) = \dfrac{1}{2}mu^2e$ [**A1**]; this equals $\dfrac{1}{3} \times \dfrac{1}{2}mu^2$ so $e = \dfrac{1}{3}$ [**M1**]; $\tan\alpha = \sqrt{3}, \alpha = 60°$ [**A1**].	**A1**
	Total: 8

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Q23

9231/33 • May/June 2020 • Question 5

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Mark scheme.

$\leftarrow mw = -mu\cos\alpha + mu\sin\alpha$.	**M1**
$w - 0 = e(u\cos\alpha + u\sin\alpha)$.	**M1**
Rearrange: $\sin\alpha(u - eu) = \cos\alpha(u + eu)$.	**M1**
$\tan\alpha = \dfrac{1+e}{1-e}$ (AG). **(b)**	**A1**
$\tan\alpha = 2 \Rightarrow e = \dfrac{1}{3}$.	**B1**
$w = \dfrac{1}{3}u\left(\dfrac{1}{\sqrt{5}} + \dfrac{2}{\sqrt{5}}\right) = \dfrac{u}{\sqrt{5}}$.	**M1**
Speed $= \sqrt{w^2 + (u\sin\alpha)^2}$.	**M1**
$= \sqrt{\dfrac{u^2}{5} + \dfrac{4u^2}{5}} = u$.	**A1**
	Total: 8

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Q24

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Mark scheme.

Momentum equation (with m): $mu = mw + 2mv$	**B1**
Restitution with consistent signs: $v - w = eu$	**B1**
Both correct: $v = \frac{u}{3}(e+1)$, $w = \frac{u}{3}(1-2e)$ (b)	**B1**
Perpendicular to plane: $y = ev\sin\theta$; parallel to plane: $x = v\cos\theta$ (both)	**B1**
Speed of $B = \sqrt{x^2+y^2} = \sqrt{v^2\left(\left(\frac{4}{5}\right)^2 + \left(\frac{2}{3}\cdot\frac{3}{5}\right)^2\right)}\left(= \frac{2}{\sqrt{5}}v\right)$	**M1**
KE of $B = \frac{1}{2}.2m.\frac{4}{5}.\frac{u^2}{9}(e+1)^2$ in terms of u	**M1**
Relate the two KEs: KE of $A = \frac{1}{2}.m.\frac{u^2}{9}(1-2e)^2$; so $\frac{1}{2}.m.\frac{u^2}{9}(1-2e)^2 = \frac{5}{32}.\frac{1}{2}.2m.\frac{4}{5}.\frac{u^2}{9}(e+1)^2$	**M1A1**
Rearrange and simplify to quadratic: $4(1-2e)^2 = (e+1)^2$ or $15e^2 - 18e + 3 = 0$	**M1**
$1 + e = \pm 2(1-2e)$, so $e = \frac{1}{5}, 1$ (both values)	**A1**
	Total: 10

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Q25

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Mark scheme.

Momentum equation with m (correct masses, allow sin instead of cos): $2mv + mw = 2mu\cos\alpha$	**M1**
Restitution with consistent signs: $w - v = eu\cos\alpha$	**M1**
$v = \frac{1}{3}u\cos\alpha(2-e) = \frac{1}{3}u.\frac{3}{5}\left(2-\frac{1}{3}\right) = \frac{1}{3}u$	**A1**
Square of speed of $A = \left(\frac{1}{3}u\right)^2 + (u\sin\alpha)^2$	**M1**
$= \left(\frac{1}{3}u\right)^2 + \left(\frac{4}{5}u\right)^2$; speed $= \frac{13}{15}u$ $(= 0.867u)$	**A1**
	Total: 5

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Q26

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Mark scheme.

Momentum along line of centres, speeds v_1 and v_2: $mv_1 + mv_2 = mu\cos\alpha - mu\cos\beta$ (condone missing masses).	**M1**
Restitution: $v_2 - v_1 = eu(\cos\beta + \cos\alpha)$.	**M1**
Both correct, masses seen.	**A1**
$v_1 = 0$ so A has no speed along line of centres: moves perpendicular to line of centres (AG). (b)	**A1**
$v_2 = \frac{1}{2}u\cos\alpha = u\cos\beta$; use both components.	**M1**
Add KE of A and B after collision and equate to $\frac{3}{4}mu^2$.	**M1**
Simplify to equation in $\sin\alpha$.	**M1**
$\sin\alpha = \frac{1}{\sqrt{2}}$, $\alpha = 45°$.	**A1**
	Total: 8

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Q27

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Mark scheme.

Momentum along line of centres: $mv_1 + kmv_2 = mu\cos\theta$ (must include m, allow cos/sin mix).	**M1**
Restitution, consistent signs: $v_2 - v_1 = \frac{1}{3}u\cos\theta$.	**M1**
Solve: $v_2 = \dfrac{4u\cos\theta}{3(1+k)}$ (AG, shown convincingly). (b)	**A1**
$v_1 = \dfrac{(3-k)u\cos\theta}{3(1+k)}$ (or equivalent, may be unsimplified).	**B1**
Use velocity of A with both components: $v_1^2 + (u\sin\theta)^2$ seen.	**B1**
$\frac{1}{2}kmv_2^2 + \frac{1}{2}m\left(v_1^2 + (u\sin\theta)^2\right) = \frac{3}{10} \times \frac{1}{2}mu^2$ (KE after = 30% KE before, all terms present).	**M1**
Substitute from part (a) and for θ; eliminate trigonometric terms, in terms of k only.	**M1**
Obtain simplified quadratic equation in k: $(3-k)^2 + 16k = 2(1+k)^2$, $k^2 - 6k - 7 = 0$.	**M1**
$k = 7$.	**A1**
	Total: 9

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Q28

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Mark scheme.

(a)	
$u\cos\alpha = v\cos\beta$.	**M1**
$eu\sin\alpha = v\sin\beta$.	**M1**
Divide: $\tan\beta = e\tan\alpha$ (AG, must see divide, OE).	**A1**
(b)	
$v\sin\beta = w\cos\gamma$ $(= eu\sin\alpha)$.	**M1**
$ev\cos\beta = w\sin\gamma$ $(= eu\cos\alpha)$.	**M1**
Divide: $\tan\gamma = 1/\tan\alpha$: $\gamma = 90^\circ - \alpha$.	***A1**
After second rebound, direction of motion is parallel to initial path.	**DB1**
(c)	
Final KE $= \frac{1}{2}m\left((eu\sin\alpha)^2 + (eu\cos\alpha)^2\right)\left(= \frac{1}{2}me^2u^2\right)$ (energy expression in terms of u).	**M1**
So $\frac{1}{2}me^2u^2 = \frac{1}{9} \times \frac{1}{2}mu^2$ giving $e = \frac{1}{3}$.	**A1**
Part (a) gives $\tan(90 - \alpha) = e\tan\alpha$.	**M1**
So $\tan\alpha = \sqrt{3}$.	**A1**
	Total: 11

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Q29

9231/32 • October/November 2021 • Question 2

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Mark scheme.

At the collision of P and Q: $(m+km)v = kmu$ (momentum conserved, allow missing k on RHS).	**M1**
So $v = \dfrac{k\sqrt{4ga}}{(1+k)}$.	**A1**
EPE $= \dfrac{1}{2} \times \dfrac{5mg}{a} \times \left(\dfrac{a}{5}\right)^2 \left(= \dfrac{mga}{10}\right)$.	**B1**
Loss in KE = Gain in EPE: $\dfrac{1}{2}m(k+1)v^2 = \dfrac{1}{2} \times \dfrac{5mg}{a} \times \left(\dfrac{a}{5}\right)^2$ (energy equation, LHS correct, EPE dimensionally correct).	**M1**
Substitute for v and rearrange to form quadratic equation in k: $20k^2 = 1+k$.	**M1**
$k = \dfrac{1}{4}$.	**A1**
	Total: 6

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Q30

9231/32 • October/November 2021 • Question 5

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Mark scheme.

(a)	
Let speeds of A and B along line of centres after collision be v_1 and v_2: $mv_1 + \frac{3}{2}mv_2 = -\frac{3}{2}mu\cos 60° + mu \left(= \frac{u}{4}\right)$ (momentum with masses correct).	**M1**
Restitution, with consistent signs on LHS: $v_2 - v_1 = -\frac{2}{3}(-u\cos 60° - u)$ $(= u)$.	**M1**
$\left(v_1 = -\frac{1}{2}u\right)$, $v_2 = \frac{1}{2}u$.	**A1**
Perpendicular to line of centres, speed of B is $u\sin 60° = \frac{\sqrt{3}}{2}u$.	**B1**
Direction of B is now 60° above line of centres.	**M1**
Angle of deflection is 60° (FT (120°− their direction of B angle)).	**A1 FT**
(b)	
KE before $= \frac{1}{2}mu^2 + \frac{1}{2}\cdot\frac{3m}{2}u^2 = \frac{5}{4}mu^2$.	**B1**
KE after $= \frac{1}{2}m\left(\frac{u}{2}\right)^2 + \frac{1}{2}\cdot\frac{3m}{2}\left(\left(\frac{u}{2}\right)^2 + \left(\frac{\sqrt{3}u}{2}\right)^2\right)$ $\left(= \frac{7}{8}mu^2\right)$ (FT only their speeds from (a)).	**B1 FT**
Loss in KE $= \frac{3}{8}mu^2$.	**B1**
	Total: 9

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Q31

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Mark scheme.

Momentum along line of centres: let v, w be speeds after collision; $mv+kmw = mu\cos\alpha$.	**M1**
NEL, consistent signs: $w - v = \frac{1}{2}u\cos\alpha$.	**M1**
Add to give $\frac{3u\cos\alpha}{2(1+k)}$ (AG, convincing working).	**A1**
Substitute back or re-solve: $v = \left\lvert\frac{(2-k)u\cos\alpha}{2(1+k)}\right\rvert$ (accept without modulus sign). (b)	**A1**
Speed of A (SOI): $\sqrt{(u\sin\alpha)^2 + \left(\frac{(2-k)u\cos\alpha}{2(1+k)}\right)^2}$.	**B1**
Equate KEs: $\frac{1}{2}km\left(\frac{3u\cos\alpha}{2(1+k)}\right)^2 = \frac{1}{2}m\left((u\sin\alpha)^2 + \left(\frac{(2-k)u\cos\alpha}{2(1+k)}\right)^2\right)$, i.e. $9k(\cos\alpha)^2 = 4(1+k)^2(\sin\alpha)^2 + (2-k)^2(\cos\alpha)^2$.	**M1**
Use $\tan\alpha = \frac{2}{3}$: $16(1+2k+k^2) + 9(4-4k+k^2) = 81k$.	**M1**
Obtain quadratic and attempt to solve: $25k^2 - 85k + 52 = 0$ leading to $(5k-4)(5k-13) = 0$.	**M1**
$k = \frac{4}{5}$ or $\frac{13}{5}$.	**A1**
	Total: 9

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Q32

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Mark scheme.

Energy loss: $\frac{1}{2}mv^2 = \frac{2}{5} \times \frac{1}{2}mu^2$, $v^2 = \frac{2}{5}u^2$.	**B1**
Both: $v\cos\alpha = u\cos\theta$; $v\sin\alpha = eu\sin\theta$.	**B1**
Combine to form equation in e only: $v^2 = (u\cos\theta)^2 + (eu\sin\theta)^2 \Rightarrow \frac{2}{5} = \frac{1}{5} + e^2 \times \frac{4}{5}$.	**M1**
$e = \frac{1}{2}$. (b)	**A1**
$\tan\alpha = e\tan\theta$, so $\tan\alpha = 1$, $\alpha = 45°$.	**B1**
For 2nd collision (both, may be implied by the A1): $w\cos\beta = v\cos(180-60-\alpha)$; $w\sin\beta = ev\sin(180-60-\alpha)$.	**M1**
Divide to find β: $\tan\beta = e\tan(120 - \text{their } \alpha)$.	**M1**
$\beta = 61.8°$.	**A1**
	Total: 8

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Q33

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Mark scheme.

Let speed of A after collision be $v_A \rightarrow$ and speed of B perpendicular to line of centres be $v \downarrow$. Along line of centres (momentum): $mu - km\frac{5}{8}u\cos\alpha = mv_A$	**M1**
NEL: $0 - v_A = e\left(\frac{5}{8}u\cos\alpha + u\right)$	**M1**
Solve: $u - \frac{5}{8}ku\cos\alpha = -\frac{2}{3}\left(\frac{5}{8}u\cos\alpha + u\right)$	**M1**
Substitute for $\cos\alpha$, giving $k = 4$.	**A1**

$\boxed{k = 4}$

(b)

$v_B = \frac{5}{8}u\sin\alpha = \frac{3}{8}u$ (velocity perpendicular to line of centres)	**B1**
FT $v_A = -u$	**B1**
KE before $= \frac{1}{2}mu^2 + \frac{1}{2}km\left(\frac{5}{8}u\right)^2 = \frac{41}{32}mu^2$; KE after $= \frac{1}{2}mv_A^2 + \frac{1}{2}kmv_B^2 = \frac{25}{32}mu^2$ (both).	**M1**
Loss $= mu^2\left(\frac{41}{32} - \frac{25}{32}\right) = \frac{1}{2}mu^2$	**A1**

Total: 8

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Q34

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Mark scheme.

Let v, w be speeds of A and B along line of centres after collision. Momentum: $mv + \frac{1}{2}mw = mu\cos\alpha - \frac{1}{2}m \cdot 2u\cos\beta$	**M1**
NEL: $w - v = e(2u\cos\beta + u\cos\alpha)$	**M1**
Use $\alpha + \beta = 90°$ so $\cos\beta = \sin\alpha$, solve for w.	**M1**
$w = \dfrac{2}{3}u\left(\frac{1}{4}\sin\alpha + \frac{13}{8}\cos\alpha\right)$	**A1**

$$\boxed{w = \frac{2}{3}u\left(\tfrac{1}{4}\sin\alpha + \tfrac{13}{8}\cos\alpha\right)}$$

(b)

Perpendicular to line of centres, speed of B before is $2u\sin\beta = 2u\cos\alpha$.	**B1**
After, velocity of B makes angle α with line of centres: $\tan\alpha = \dfrac{2u\cos\alpha}{w}$	**B1**
Obtain homogeneous equation in $\cos\alpha, \sin\alpha$ (or equation in $\tan\alpha$).	**M1***
Obtain quadratic and solve: $2\tan^2\alpha + 13\tan\alpha - 24 = 0$, $(2\tan\alpha - 3)(\tan\alpha + 8) = 0$	**DM1**
$\tan\alpha = \dfrac{3}{2}$	**A1**
	Total: 9

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Q35

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Mark scheme.

Parallel to wall $v\cos\theta = u\cos\alpha$; perpendicular to wall $v\sin\theta = eu\sin\alpha$ (both).	**M1**
Dividing, $e = \dfrac{1}{2\tan\alpha}$ (AEF).	**A1**
KE reduced by 20%, so $\frac{1}{2}mu^2\left(\cos^2\alpha + e^2\sin^2\alpha\right) = \frac{4}{5}\times\frac{1}{2}mu^2$. Dimensionally correct equation in u or v, but not both. Must have either α or θ, but not both. Must see $\frac{4}{5}$ on the correct side of the equation.	**M1**
Eliminate e: $\cos\alpha = \dfrac{4}{5}$.	**A1**
$e = \dfrac{2}{3}$.	**A1**
	Total: 5

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Q36

9231/33 • May/June 2023 • Question 4

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Mark scheme.

Let speeds of A and B along the line of centres after collision be V_A and V_B: $V_A + V_B = u\cos 30°$ (1). Allow sign errors, allow missing m.	**M1**
$-V_A + V_B = eu\cos 30°$ (2). Signs on LHS must be consistent with (1).	**M1**
Speeds perpendicular to the line of centres after collision are $u\sin 30°$ and $2u$. Moving in the same direction, so $\dfrac{V_A}{u\sin 30°} = \dfrac{V_B}{2u}$ (3), SOI $V_B = 4V_A$.	**B1**
Use $V_B = 4V_A$ in (1): $5V_A = u\cos 30°$; from (2): $3V_A = eu\cos 30°$; combine to find equation in e only (a complete method to find equation in e only).	**M1**
$e = \dfrac{3}{5}$. (b)	**A1**
Correct expression for KE after collision, with both components: KE after $= \dfrac{1}{2}m\left(V_A^2 + \left(\dfrac{u}{2}\right)^2\right) + \dfrac{1}{2}m\left((2u)^2 + V_B^2\right)$.	**B1**
KE for A after $= \dfrac{7}{50}mu^2$ or KE for B after $= \dfrac{56}{25}mu^2$, or KE loss for $A = \dfrac{9}{25}mu^2$ or KE gain for $B = \dfrac{6}{25}mu^2$ (implied by total KE after $= \dfrac{119}{50}mu^2$).	**B1**
Total loss in KE $= \dfrac{3}{25}mu^2$ (term $\frac{1}{2}m(2u)^2$ may be omitted from KE of B before and after).	**B1**

Total: 8

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Q37

9231/31, 9231/33 • October/November 2023 • Question 1

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Mark scheme.

PCLM: $mv = -mu\cos 60° + 2mu\cos\theta$; $v = -\frac{1}{2}u + 2u\cos\theta = \frac{1}{2}u(4\cos\theta - 1)$ (AG) B1		**(a)**
KE of $A = \frac{1}{2}m\left(v^2 + (u\sin 60°)^2\right)$ B1		**(b)**
KE of A = 2×KE of B, so $\frac{1}{2}m\left(v^2 + (u\sin 60°)^2\right) = 2 \times \frac{1}{2} \times 2m(u\sin\theta)^2$	M1	
$\left(\frac{1}{2}u(4\cos\theta - 1)\right)^2 + \frac{3}{4}u^2 = 4u^2(\sin\theta)^2$; $8\cos^2\theta - 2\cos\theta - 3 = 0$	M1	
$\cos\theta = \frac{3}{4}$, $-\frac{1}{2}$ but angle is acute, so $\cos\theta = \frac{3}{4}$	A1	
NEL: $v = e(u\cos 60° + u\cos\theta)$ M1		**(c)**
$v = u$, $u = e\left(\frac{1}{2}u + \frac{3}{4}u\right)$; $e = \frac{4}{5}$	A1	
		Total: 7

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Q38

9231/32 • October/November 2023 • Question 4

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Mark scheme.

After collision, A has velocity v_A towards wall on right and B has component of velocity towards lower wall of $u\sin\theta$. Same distance and time, so $v_A = u\sin\theta$ B1		**(a)**
Along line of centres: PCLM: $mv_A + mv_B = mu\cos\theta$; NEL: $v_A - v_B = eu\cos\theta$	M1	
$2\sin\theta = (1+e)\cos\theta$	M1	
$\tan\theta = \frac{3}{4}$	A1	
Final KE $= \frac{1}{2}m\left(v_A^2 + (u\sin\theta)^2 + v_B^2\right)$	B1	**(b)**
Loss $= \frac{1}{2}mu^2 - \frac{1}{2}m\left(v_A^2 + (u\sin\theta)^2 + v_B^2\right) = \frac{1}{2}mu^2\left(1 - \frac{9}{25} - \frac{9}{25} - \frac{1}{25}\right)$	M1	
Loss $= \frac{3}{25}mu^2$; percentage loss $= 24\%$	A1	
	Total: 7	

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Q39

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Mark scheme.

Along line of centres, PCLM: $5mv_B + mv_A = mu\cos\theta$ (must include correct masses).	**M1**
NEL: $v_B - v_A = \frac{1}{2}u\cos\theta$ (signs consistent with PCLM equation).	**M1**
$v_B = \dfrac{u}{4}\cos\theta$, $v_A = -\dfrac{u}{4}\cos\theta$.	**A1**
Perpendicular to line of centres: speed of A is $u\sin\theta$.	**B1**
Equate final kinetic energies, 3 terms, correct masses: $\dfrac{1}{2}m\left[\left(-\dfrac{u}{4}\cos\theta\right)^2 + (u\sin\theta)^2\right] = \dfrac{1}{2}\cdot 5m\left(\dfrac{u}{4}\cos\theta\right)^2$.	**M1**
$(\cos\theta)^2 = \dfrac{4}{5}$, $\cos\theta = \dfrac{2}{\sqrt{5}}$, $\tan\theta = \dfrac{1}{2}$.	**A1**
	Total: 6

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Q40

9231/33 • May/June 2024 • Question 1

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Mark scheme.

Along line of centres, PCLM: $-2m \cdot \frac{1}{2}u\cos\theta + mu = 2m \cdot \frac{1}{2}u\cos\theta - m \cdot \frac{1}{4}u$ (masses must be included, allow sign errors).	**M1**
$\cos\theta = \frac{5}{8}$.	**A1**
NEL: $\frac{1}{2}u\cos\theta + \frac{1}{4}u = e\left(\frac{1}{2}u\cos\theta + u\right)$ (allow sign errors, e must be on correct side).	**M1**
$e = \frac{3}{7}$ (AEF).	**A1**
	Total: 4

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Q41

9231/32 • October/November 2024 • Question 3

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Mark scheme.

(a)	
PCLM along line of centres: $-mv = m2u\cos\theta - m3u\sin\theta$ (must include m, minus sign on RHS).	**B1**
NEL: $v = eu(3\sin\theta + 2\cos\theta)$ (plus sign on RHS).	**M1**
Eliminate v: $6\sin\theta = 8\cos\theta$, $\tan\theta = \dfrac{4}{3}$.	**A1**
(b)	
Only change in KE is along line of centres: Loss $= \frac{1}{2}m\left((2u\cos\theta)^2 + (3u\sin\theta)^2\right) - \frac{1}{2}mv^2$.	**M1**
$\frac{1}{2}mu^2\left(\dfrac{36}{25} + \dfrac{144}{25} - \left(\dfrac{12}{5} - \dfrac{6}{5}\right)^2\right) = \dfrac{72}{25}mu^2$.	**A1**
(c)	
Components of velocity of A after collision are $\dfrac{6}{5}u$ and $\dfrac{8}{5}u$, so angle between line of centres and A's direction is θ.	**M1**
Angle of deflection $= 180° - 2\tan^{-1}(4/3) = 73.7°$ (FT their answer to part (a)).	**A1 FT**
	Total: 7

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Q42

9231/31, 9231/32 • May/June 2025 • Question 6

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Mark scheme.

(a)

KE equation linking v_A and v_B: $\frac{9}{2} \times \frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2$ giving $3v_A = v_B$.	**B1**
Conservation of momentum: $2mu = 2mv_A + mv_B$.	**B1**
AG: $v_A = \frac{2}{5}u$, $v_B = \frac{6}{5}u$, shown convincingly.	**B1**

(b)

Perpendicular to wall: $v_B \cos\alpha = w\cos\beta$ (OE $\sqrt{5}\cos\alpha = 2\cos\beta$).	**B1**
Parallel to wall: $ev_B \sin\alpha = w\sin\beta$ (OE $2\sin\alpha = \sqrt{5}\sin\beta$).	**B1**
Obtain equation involving only one of α, β by squaring both equations and adding: $\frac{576}{625}\sin^2\alpha = \frac{144}{125}\sin^2\beta$; $\frac{36}{25}\cos^2\alpha = \frac{144}{125}\cos^2\beta$.	***M1**
Use $\sin^2\theta + \cos^2\theta = 1$ to reduce to equation with just sine or cosine of one variable: $\frac{576}{625}\sin^2\alpha + \frac{36}{25}(1 - \sin^2\alpha) = \frac{144}{125}$.	**DM1**
$\sin^2\alpha = \frac{5}{9}$, $\sin\alpha = \frac{1}{3}\sqrt{5}$ (or $\cos\alpha = \frac{2}{3}$ or $\cos\beta = \frac{1}{3}\sqrt{5}$ or $\sin\beta = \frac{2}{3}$).	**A1**
$\sin(\alpha + \beta) = 1$ (exact answer).	**A1**
	Total: 9

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Q43

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Mark scheme.

Let v and w denote the horizontal components of the velocities for A and B respectively after the collision.	**(a)**
Momentum along line of centres (allow sign errors, m must be present): $2mu\cos\theta - mu\cos\theta = mw + mv$.	**M1**
Restitution equation, directions consistent: $w - v = e(3u\cos\theta)$.	**M1**
Combine to find w: $w = \frac{1}{2}(3eu\cos\theta + u\cos\theta)$ $\left[= \frac{1}{2}(3e+1)u\cos\theta\right]$.	**A1**
OE using perpendicular motion of B: $\dfrac{\frac{1}{2}(3e+1)u\cos\theta}{u\sin\theta} = \tan\theta$.	**M1**
Obtain expression for $\tan^2\theta$ in terms of e: $\tan^2\theta = \frac{1}{2}(3e+1)$.	**M1**
$\tan\theta = \sqrt{\frac{1}{2}(3e+1)}$ (CWO, reject negative).	**A1**
(b)	
Using $v = 0$ to find e: $w = 3eu\cos\theta = u\cos\theta$ giving $e = \frac{1}{3}$.	**M1**
$\tan\theta = 1$, $\theta = 45°$.	**A1**
	Total: 8

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Q44

9231/34 • May/June 2025 • Question 6

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Mark scheme.

Let v_1, v_2 be speeds of A, B along line of centres after collision.	**(a)**
Momentum: $5mv_1 + 4mv_2 = 5mu\cos\theta$ (m must be present).	**M1**
Restitution: $v_2 - v_1 = eu\cos\theta$ (directions consistent).	**M1**
$v_2 = \frac{5}{9}u(1+e)\cos\theta$ (AG, shown convincingly).	**A1**
(b)	
$v_1 = \frac{1}{9}u(5-4e)\cos\theta$ (may be found in (a), credited here; need not be simplified).	**B1**
Speed of A perpendicular to line of centres $= u\sin\theta$ (seen anywhere).	***B1**
Equate KEs after collision: $\frac{1}{2}(5m)\left(v_1^2 + (u\sin\theta)^2\right) = \frac{1}{2}(4m)v_2^2$.	**M1**
Substitute expressions for speeds and numerical trig values (dependent on second B1): $\frac{5}{2}\left(\left(\frac{1}{9}u(5-4e)\left(\frac{3}{\sqrt{13}}\right)\right)^2 + \left(u\left(\frac{2}{\sqrt{13}}\right)\right)^2\right) = \frac{4}{2}\left(\frac{5}{9}u(1+e)\left(\frac{3}{\sqrt{13}}\right)\right)^2$.	**DM1**
Obtain 3-term quadratic in e (dependent on second B1): $4e^2 + 80e - 41 = 0$.	**DM1**
$e = \frac{1}{2}$ (reject $e = -\frac{41}{2}$ if seen).	**A1**
	Total: 9

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Q45

9231/31, 9231/33 • October/November 2025 • Question 1

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Mark scheme.

Conservation of momentum, masses correct: $4mv_A + mv_B = 4mu$.	**M1**
NEL, signs must be consistent with momentum equation: $-v_A + v_B = eu$.	**M1**
Use momentum condition after the collision to obtain expressions for v_A and v_B in terms of u: $v_A = \frac{3}{4}u$, $v_B = u$.	**A1**
$e = \frac{1}{4}$.	**A1**
	Total: 4

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Q46

9231/34 • October/November 2025 • Question 7

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Mark scheme.

Perpendicular to the wall: $eu \sin\alpha = v \sin\theta$.	**B1**
$P_1P_2 = uT$, $P_2P_3 = vkT$.	**B1**
$\sin\alpha = \dfrac{d}{P_1P_2}$, $\sin\theta = \dfrac{d}{P_2P_3}$.	**M1**
Eliminate P_1P_2 and P_2P_3: $\sin\alpha = \dfrac{d}{uT}$, $\sin\theta = \dfrac{d}{vkT}$.	**M1**
$\dfrac{eud}{uT} = \dfrac{vd}{vkT}$.	**M1**
$e = \dfrac{1}{k}$.	**A1**
	Total: 6

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Q47

9231/32 • October/November 2025 • Question 5

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Mark scheme.

$u \sin\alpha = v\sin(2\alpha)$ (may be implied by correct answer): $\left[\frac{3}{5}u = \frac{24}{25}v\right]$.	**B1**
$v = \frac{5}{8}u$. **(b)**	**B1**
PCLM equation, must include masses: $mu\cos\alpha = mv\cos(2\alpha) + kmw$.	**B1**
NEL equation: $eu\cos\alpha = w - v\cos(2\alpha)$.	**B1**
Combine and substitute for v and w to obtain equation in e and k only: $\frac{5}{8k} = \frac{4}{5}e + \frac{5}{8} \times \frac{7}{25}$.	**M1**
AEF, e.g. $e = \frac{25-7k}{32k}$ (not dependent on first B mark): $e = \frac{25}{32k} - \frac{7}{32}$. **(c)**	**A1**
Form inequality using their expression for e (allow equality or strict): $0 \le \frac{25}{32k} - \frac{7}{32} \le 1$.	**M1**
Rearrange to form $ak \le b$ or $ak \ge b$: $7k \le 25 \le 39k$.	**M1**
CAO: $\frac{25}{39} \le k \le \frac{25}{7}$.	**A1**
	Total: 9

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Written by Ali Hashir.

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