Further Math with Ali Hashir

Hooke's Law

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/21, 9231/22, 9231/23 • October/November 2015 • Question 3

A and B are two fixed points on a smooth horizontal surface, with $AB = 3a$ m. One end of a light elastic string, of natural length a m and modulus of elasticity mg N, is attached to the point A. The other end of this string is attached to a particle P of mass m kg. One end of a second light elastic string, of natural length ka m and modulus of elasticity $2mg$ N, is attached to B. The other end of this string is attached to P. Given that the system is in equilibrium when P is at M, the mid-point of AB, find the value of k. [3]

The particle P is released from rest at a point between A and B where both strings are taut. Show that P performs simple harmonic motion and state the period of the motion. [5]

In the case where P is released from rest at a distance $0.2a$ m from M, the speed of P is 0.7 m s^{-1} when P is $0.05a$ m from M. Find the value of a. [3]

Q2

9231/23 • May/June 2017 • Question 2

A particle P moves on a straight line in simple harmonic motion. The centre of the motion is O, and the amplitude of the motion is 2.5 m. The points L and M are on the line, on opposite sides of O, with $OL = 1.5$ m. The magnitudes of the accelerations of P at L and at M are in the ratio $3 : 4$.

(i) Find the distance OM. [2]

The time taken by P to travel directly from L to M is 2 s.

(ii) Find the period of the motion. [5]

(iii) Find the speed of P when it passes through L. [2]

Q3

9231/21, 9231/22 • May/June 2018 • Question 2

A particle P moves on a straight line in simple harmonic motion. The centre of the motion is O. The points A and B are on the line, on opposite sides of O, with $OA = 1.6$ m and $OB = 1.2$ m. The ratio of the speed of P at A to its speed at B is $3 : 4$.

(i) Find the amplitude of the motion. [4]

The maximum speed of P during its motion is $\frac{1}{3}\pi\,\mathrm{m\,s^{-1}}$.

(ii) Find the period of the motion. [2]

(iii) Find the time taken for P to travel directly from A to B. [3]

Q4

9231/23 • May/June 2018 • Question 3

A particle P moves on the positive x-axis in simple harmonic motion. The centre of the motion is a distance d m from the origin O, where $0 < d < 6.5$. The points A and B are on the positive x-axis, with $OA = 6.5$ m and $OB = 7.5$ m. The magnitude of the acceleration of P when it is at B is twice the magnitude of the acceleration of P when it is at A.

(i) Find d. [2]

The period of the motion is π s and the maximum acceleration of P during the motion is $10\,\text{m}\,\text{s}^{-2}$.

(ii) Find the speed of P when it is 7 m from O. [5]

(iii) Find the time taken by P to travel directly from A to B. [3]

Q5

9231/31, 9231/32 • May/June 2020 • Question 3

One end of a light elastic spring, of natural length a and modulus of elasticity $5mg$, is attached to a fixed point A. The other end of the spring is attached to a particle P of mass m. The spring hangs with P vertically below A. The particle P is released from rest in the position where the extension of the spring is $\frac{1}{2}a$.

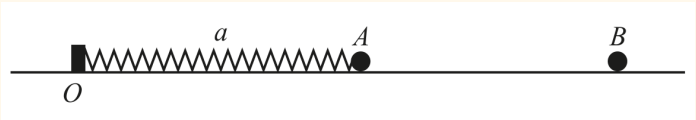
(a) Show that the initial acceleration of P is $\frac{3}{2}g$ upwards. [3]

(b) Find the speed of P when the spring first returns to its natural length. [4]

Q6

9231/33 • May/June 2020 • Question 7

One end of a light spring of natural length a and modulus of elasticity $4mg$ is attached to a fixed point O. The other end of the spring is attached to a particle A of mass km, where k is a constant. Initially the spring lies at rest on a smooth horizontal surface and has length a. A second particle B, of mass m, is moving towards A with speed $\sqrt{\frac{4}{3}ga}$ along the line of the spring from the opposite direction to O (see diagram).



The particles A and B collide and coalesce. At a point C in the subsequent motion, the length of the spring is $\frac{3}{4}a$ and the speed of the combined particle is half of its initial speed.

(a) Find the value of k. [6]

At the point C the horizontal surface becomes rough, with coefficient of friction μ between the combined particle and the surface. The deceleration of the combined particle at C is $\frac{9}{20}g$.

(b) Find the value of μ. [4]

Q7

9231/31, 9231/33 • October/November 2020 • Question 1

A particle P of mass m is placed on a fixed smooth plane which is inclined at an angle θ to the horizontal. A light spring, of natural length a and modulus of elasticity $3mg$, has one end attached to P and the other end attached to a fixed point O at the top of the plane. The spring lies along a line of greatest slope of the plane. The system is released from rest with the spring at its natural length.

Find, in terms of a and θ, an expression for the greatest extension of the spring in the subsequent motion. [3]

Q8

9231/32 • October/November 2020 • Question 6

One end of a light elastic string, of natural length a and modulus of elasticity k, is attached to a particle P of mass m. The other end of the string is attached to a fixed point Q. The particle P is projected vertically upwards from Q. When P is moving upwards and at a distance $\frac{4}{3}a$ directly above Q, it has a speed $\sqrt{2ga}$. At this point, its acceleration is $\frac{7}{3}g$ downwards.

Show that $k = 4mg$ and find in terms of a the greatest height above Q reached by P. [8]

Q9

9231/31, 9231/32 • May/June 2021 • Question 3

One end of a light elastic string, of natural length a and modulus of elasticity kmg, is attached to a fixed point A. The other end of the string is attached to a particle P of mass $4m$. The particle P hangs in equilibrium a distance x vertically below A.

(a) Show that $k = \dfrac{4a}{x-a}$. [1]

An additional particle, of mass $2m$, is now attached to P and the combined particle is released from rest at the original equilibrium position of P. When the combined particle has descended a distance $\frac{1}{3}a$, its speed is $\frac{1}{3}\sqrt{ga}$.

(b) Find x in terms of a. [6]

Q10

9231/33 • May/June 2021 • Question 2

One end of a light elastic string of natural length 0.8 m and modulus of elasticity 36 N is attached to a fixed point O on a smooth plane. The plane is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. A particle P of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane with the particle below the level of O. The particle is projected with speed $\sqrt{2}\,\mathrm{m\,s^{-1}}$ directly down the plane from the position where OP is equal to the natural length of the string.

Find the maximum extension of the string during the subsequent motion. [5]

Q11

9231/31, 9231/33 • October/November 2021 • Question 3

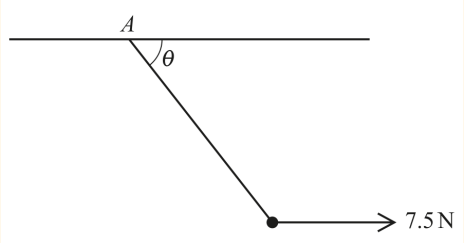
A light elastic string has natural length a and modulus of elasticity $12mg$. One end of the string is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle hangs in equilibrium vertically below O. The particle is pulled vertically down and released from rest with the extension of the string equal to e, where $e > \frac{1}{3}a$. In the subsequent motion the particle has speed $\sqrt{2ga}$ when it has ascended a distance $\frac{1}{3}a$.

Find e in terms of a. [6]

Q12

9231/31, 9231/32 • May/June 2022 • Question 1

A particle of weight 10N is attached to one end of a light elastic string. The other end of the string is attached to a fixed point A on a horizontal ceiling. A horizontal force of 7.5N acts on the particle. In the equilibrium position, the string makes an angle θ with the ceiling (see diagram). The string has natural length 0.8m and modulus of elasticity 50N.



(a) Find the tension in the string. [2]

(b) Find the vertical distance between the particle and the ceiling. [3]

Q13

9231/33 • May/June 2022 • Question 2

A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $\frac{4}{3}mg$. The other end of the string is attached to a fixed point O on a rough horizontal surface. The particle is at rest on the surface with the string at its natural length. The coefficient of friction between P and the surface is $\frac{1}{3}$. The particle is projected along the surface in the direction OP with a speed of $\frac{1}{2}\sqrt{ga}$.

Find the greatest extension of the string during the subsequent motion. [5]

Q14

9231/31, 9231/33 • October/November 2022 • Question 2

A light elastic string has natural length a and modulus of elasticity $4mg$. One end of the string is fixed to a point O on a smooth horizontal surface. A particle P of mass m is attached to the other end of the string. The particle P is projected along the surface in the direction OP. When the length of the string is $\frac{5}{4}a$, the speed of P is v. When the length of the string is $\frac{3}{2}a$, the speed of P is $\frac{1}{2}v$.

(a) Find an expression for v in terms of a and g. [4]

(b) Find, in terms of g, the acceleration of P when the stretched length of the string is $\frac{3}{2}a$. [2]

Q15

9231/32 • October/November 2022 • Question 3

One end of a light elastic string, of natural length a and modulus of elasticity $\frac{16}{3}Mg$, is attached to a fixed point O. A particle P of mass $4M$ is attached to the other end of the string and hangs vertically in equilibrium. Another particle of mass $2M$ is attached to P and the combined particle is then released from rest. The speed of the combined particle when it has descended a distance $\frac{1}{4}a$ is v.

Find an expression for v in terms of g and a. [6]

Q16

9231/31, 9231/32 • May/June 2023 • Question 1

One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O. The other end of the string is attached to a particle P of mass m. The string hangs with P vertically below O. The particle P is pulled vertically downwards so that the extension of the string is $2a$. The particle P is then released from rest.

(a) Find the speed of P when it is at a distance $\frac{3}{4}a$ below O. [3]

(b) Find the initial acceleration of P when it is released from rest. [2]

Q17

9231/33 • May/June 2023 • Question 2

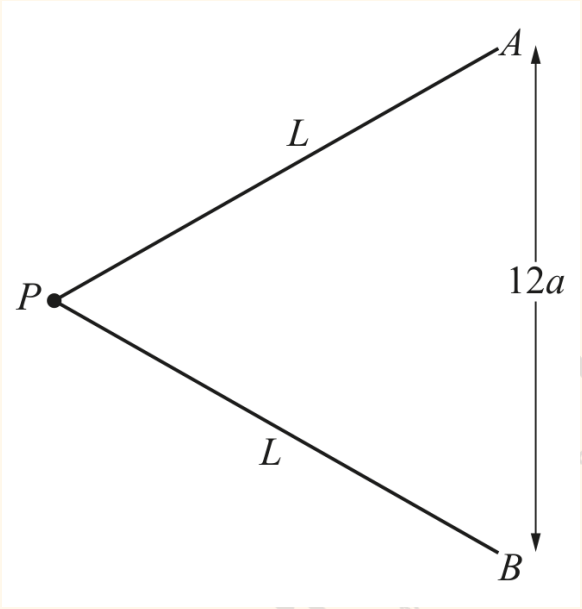
One end of a light elastic string, of natural length a and modulus of elasticity λmg, is attached to a fixed point O. The string lies on a smooth horizontal surface. A particle P of mass m is attached to the other end of the string. The particle P is projected in the direction OP. When the length of the string is $\frac{4}{3}a$, the speed of P is $\sqrt{2ag}$. When the length of the string is $\frac{5}{3}a$, the speed of P is $\frac{1}{2}\sqrt{2ag}$.

Find the value of λ. [4]

Q18

9231/31, 9231/33 • October/November 2023 • Question 4

A light elastic string has natural length $8a$ and modulus of elasticity $5mg$. A particle P of mass m is attached to the midpoint of the string. The ends of the string are attached to points A and B which are a distance $12a$ apart on a smooth horizontal table. The particle P is held on the table so that $AP = BP = L$ (see diagram). The particle P is released from rest. When P is at the midpoint of AB it has speed $\sqrt{80ag}$.



(a) Find L in terms of a. [5]

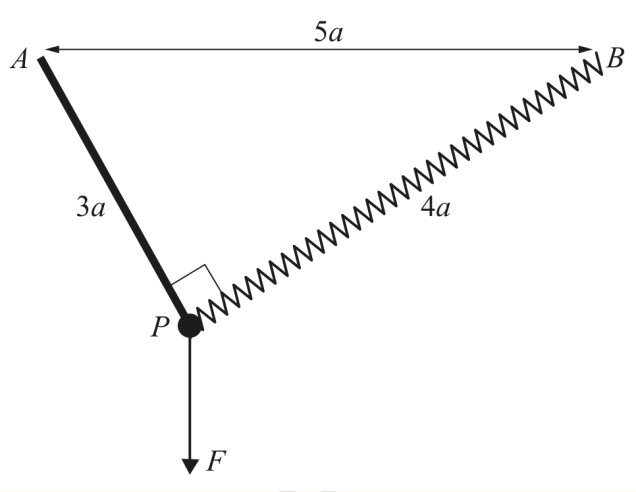
(b) Find the initial acceleration of P in terms of g. [3]

Q19

9231/32 • October/November 2023 • Question 7

A particle P of mass m is attached to one end of a light rod of length $3a$. The other end of the rod is able to pivot smoothly about the fixed point A. The particle is also attached to one end of a light spring of natural length a and modulus of elasticity kmg. The other end of the spring is attached to a fixed point B. The points A and B are in a horizontal line, a distance $5a$ apart, and these two points and the rod are in a vertical plane.

Initially, P is held in equilibrium by a vertical force F with the stretched length of the spring equal to $4a$ (see diagram). The particle is released from rest in this position and has a speed of $\frac{6}{5}\sqrt{2ag}$ when the rod becomes horizontal.



(a) Find the value of k. [5]

(b) Find F in terms of m and g. [2]

(c) Find, in terms of m and g, the tension in the rod immediately before it is released. [2]

Q20

9231/31, 9231/32 • May/June 2024 • Question 2

The points A and B are at the same horizontal level a distance $4a$ apart. The ends of a light elastic string, of natural length $4a$ and modulus of elasticity λ, are attached to A and B. A particle P of mass m is attached to the midpoint of the string. The system is in equilibrium with P at a distance $\frac{3}{2}a$ below M, the midpoint of AB.

(a) Find λ in terms of m and g. [3]

The particle P is pulled down vertically and released from rest at a distance $\frac{8}{3}a$ below M.

(b) Find, in terms of a and g, the speed of P as it passes through M in the subsequent motion. [4]

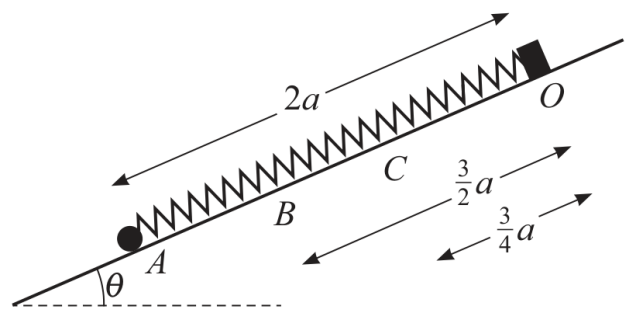
Q21

9231/33 • May/June 2024 • Question 4

A light spring of natural length a and modulus of elasticity kmg is attached to a fixed point O on a smooth plane inclined to the horizontal at an angle θ, where $\sin\theta = \frac{3}{4}$. A particle of mass m is attached to the lower end of the spring and is held at the point A on the plane, where $OA = 2a$ and OA is along a line of greatest slope of the plane (see diagram).

The particle is released from rest and is moving with speed V when it passes through the point B on the plane, where $OB = \frac{3}{2}a$. The speed of the particle is $\frac{1}{2}V$ when it passes through the point C on the plane, where $OC = \frac{3}{4}a$.

Find the value of k. [7]



Q22

9231/31, 9231/33 • October/November 2024 • Question 3

A particle P of mass m kg is attached to one end of a light elastic string of natural length 2 m and modulus of elasticity $2mg$ N. The other end of the string is attached to a fixed point O. The particle P hangs in equilibrium vertically below O. The particle P is pulled down vertically a distance d m below its equilibrium position and released from rest.

(a) Given that the particle just reaches O in the subsequent motion, find the value of d. [6]

(b) Hence find the speed of P when it is 2 m below O. [2]

Q23

9231/32 • October/November 2024 • Question 2

A particle P of mass m is attached to one end of a light elastic spring of natural length a and modulus of elasticity $5mg$. The other end of the spring is attached to a fixed point O. The spring hangs vertically with P below O. The particle P is pulled down vertically and released from rest when the length of the spring is $\frac{3}{2}a$.

Find the distance of P below O when P first comes to instantaneous rest. [4]

Q24

9231/31, 9231/32 • May/June 2025 • Question 5

One end of a light elastic string of natural length 0.5 m and modulus of elasticity 14 N is attached to a fixed point A on a smooth plane. The plane makes an angle α to the horizontal, where $\tan\alpha = \frac{7}{24}$. A particle P of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane. The particle P is initially held on the plane above the level of A, where $AP = 0.8$ m. The particle P is then released from rest.

Find the maximum velocity of P during the subsequent motion. [6]

Q25

9231/33 • May/June 2025 • Question 2

A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity mg. The other end of the string is attached to a fixed point O on a rough plane inclined at an angle of $30°$ to the horizontal. The particle P is held at rest at point O before being released. The frictional force acting on P as it slides down the plane is $\frac{11}{30}mg$.

(a) Find, in terms of a, the distance that P moves down the plane before coming to rest. [5]

(b) It is given that P remains at rest in this new position. Find, in terms of m and g, the magnitude of the frictional force in this position. [3]

Q26

9231/34 • May/June 2025 • Question 1

A light spring of natural length a and modulus of elasticity $20mg$ is placed so that it stands vertically on a horizontal plane. The lower end of the spring is fixed to the plane. A particle of mass m is attached to the upper end of the spring.

The particle is pushed vertically downwards until the length of the spring is $\frac{3}{5}a$. The system is then released from rest.

Find the maximum extension of the spring in the subsequent motion. [5]

Q27

9231/31, 9231/33 • October/November 2025 • Question 4

One end of a light elastic string of natural length a and modulus of elasticity $5mg$ is attached to a fixed point O. Two particles, P and Q, of masses m and $4m$ respectively are attached to the other end of the string and they hang vertically in equilibrium. Particle Q is then detached from the string, hence releasing particle P from rest.

Find, in terms of a, the length of the string when the speed of particle P is first equal to $\sqrt{\frac{7}{5}ag}$. [6]

Q28

9231/34 • October/November 2025 • Question 6

A and B are two fixed points at a distance $22a$ apart, with B vertically below A. A light elastic string of natural length $4a$ and modulus of elasticity $5mg$ has one end attached to A and the other end attached to a particle P of mass km. Another light elastic string of natural length $8a$ and modulus of elasticity $6mg$ has one end attached to B and the other end attached to P. Particle P is vertically above B.

(a) Show that, when the system is in equilibrium, $BP = \dfrac{57a - 2ak}{4}$. [5]

The particle P is pulled vertically upwards so that $BP = 18a$, and is then released from rest. In its subsequent motion, P first comes to instantaneous rest at the point where $BP = 8a$.

(b) Find the value of k. [4]

Q29

9231/32 • October/November 2025 • Question 2

One end of a light elastic string of natural length a and modulus of elasticity $2mg$ is attached to a fixed point A on a rough horizontal surface. The other end of the string is attached to a particle P of mass m. The particle and string rest on the surface. The coefficient of friction between P and the surface is μ. The particle P is initially held in equilibrium at a distance $\frac{4}{3}a$ from A. The particle is then released from rest.

(a) Given that the string never becomes slack, find the minimum value of μ. [3]

It is now given that $\mu = \dfrac{1}{2}$.

(b) Find the extension of the string when the particle comes to rest. [3]

Written by Ali Hashir.

Answers available in the companion mark scheme booklet.