Further Math with Ali Hashir

Equilibrium of a Rigid Body

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/21, 9231/22 • May/June 2015 • Question 4

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Mark scheme.

(i)	
Take moments for rod about B: $W \times a\cos 30° + 3W \times 2a\cos 30° = T \times 2a\cos 30°$.	**M1 A1**
Hence $T = 7W/2$ (can earn M1 A0 A1 if e.g. $\sin 30°$ wrongly used).	**A1**
Find modulus λ using Hooke's Law: $T = \lambda(2a - 3a/5)/(3a/5)$, $\lambda = (3/7)(7W/2) = 3W/2$.	**M1 A1**
(ii)	
Find horizontal component of force F at B: $X = T\cos 30° = (7\sqrt{3}/4)W$ or $3.03W$.	**B1**
Find vertical component: $Y = 4W - T\sin 30° = 9W/4$.	**B1**
Find magnitude of F: $F = \sqrt{X^2 + Y^2} = (\sqrt{57}/2)W$ or $3.77[5]W$.	**B1**
Find direction of F (AEF): upward force at angle to AB of $\tan^{-1}(Y/X) = \tan^{-1}(3\sqrt{3}/7) = 36.6°$ or 0.639 radians (A0 if direction unclear).	**M1 A1**
	Total: 10

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Q2

9231/23 • May/June 2015 • Question 4

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Mark scheme.

Find reaction at C using moments for rod about A: $R_C \times 2a/\sin 30° = W \times 3a\cos 30°$, so $R_C = 3\sqrt{3}W/8$.	**M1 A1**
Find friction F on cube by resolving horizontally: $F = R_C \sin 30°$ $[= 3\sqrt{3}W/16]$.	**B1**
Find reaction R on cube by resolving vertically: $R = W + R_C \cos 30°$ $[= 25W/16]$.	**B1**
Use $F \leqslant \mu R$ (allow $F < \mu R$ for M1) (AG): $3\sqrt{3}/16 \leqslant 25\mu/16$, $\mu \geqslant 3\sqrt{3}/25$ (M0 if $F = \mu R$ with no further explanation).	**M1 A1**
Find horizontal force X at A (ignore sign): $X = F$ $[= 3\sqrt{3}W/16]$.	**B1**
Find vertical force Y at A (ignore sign): $Y = 2W - R$ or $W - R_C \cos 30°$ $[= 7W/16]$.	**B1**
Find magnitude of force at A: $\sqrt{X^2 + Y^2} = (\sqrt{19}/8)W$ or $0.545W$.	**M1 A1**
	Total: 10

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Q3

9231/21, 9231/22, 9231/23 • October/November 2015 • Question 1

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Mark scheme.

Resolve horizontally: $R_B = T\cos\alpha$.	**M1A1**
Resolve vertically: $R_A = W + T\sin\alpha$.	**M1A1**
Take moments about A: $R_B\,3a\sin\theta = W(3a/2)\cos\theta + Ta\sin(\alpha+\theta)$ (or equivalent moment equation about B, C or D).	**M1A1**
$T = W/(2\sin\alpha)$ or $\frac{1}{2}W\csc\alpha$.	**B1**
$R_A = 3W/2$.	**B1**
$R_B = W/(2\tan\alpha)$ or $\frac{1}{2}W\cot\alpha$.	**B1**
	Total: 9

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Q4

9231/23 • May/June 2016 • Question 2

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Mark scheme.

Take moments for rod about e.g. A: $R_P \times AP = W \times a\cos\theta + 2W \times 2a\cos\theta$ [$= 5Wa\cos\theta = 3Wa$] (or equivalent moment equation about P, B, midpoint of AB, or below B)	**M1 A1**
Resolve forces on rod horizontally: $F_A = R_P \sin\theta$ [$= 4R_P/5$]	**B1**
Resolve forces on rod vertically: $3W - R_A = R_P\cos\theta$ [$= 3R_P/5$]	**B1**
Eliminate θ from 3 independent equations for forces [$\sin\theta = 4/5, \cos\theta = 3/5$]	**M1**
Relate F_A and R_A: $F_A = \frac{2}{3}R_A$	**M1**
Find AP from equations in R_P, R_A, F_A: [$R_P = 5W/3, F_A = 4W/3, R_A = 2W$], $AP = 9a/5$ or $1.8a$	**M1 A1**
	Total: 8

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Q5

9231/21, 9231/22, 9231/23 • October/November 2016 • Question 3

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Mark scheme.

(i)

Verify $\sin\theta$ from triangle CDE where E is level with C and vertically below D: $\sin\theta = 6a/10a = 3/5$ A.G.	**B1**

(ii)

Resolve forces on object vertically: $R_A = (k+2)W$	**M1 A1**
Take moments about B (or D, A, C, or centre of rod): e.g. $F_A \cdot 7a + Wa + kWa(1 + 5\cos\theta) + (W - R_A)a(1 + 10\cos\theta) = 0$	**M1 A1**
Find F_A using $R_B = F_A$, $\sin\theta = 3/5$, $\cos\theta = 4/5$: $7F_A = 4kW + 8W$ (as necessary depending on point taken)	**A1**
Find μ: $\mu = F_A/R_A = 4/7 \approx 0.571$	**M1 A1**

(iii)

Equate resultant force at A to $W\sqrt{65}$: $(k+2)^2 + (4/7)^2(k+2)^2 = 65$	**M1 A1**
Solve to verify value of k: $k^2 + 4k - 45 = (k-5)(k+9) = 0$ or $(k+2)^2 = 49$ so $k = 5$ A.G.	**A1**
	Total: 11

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Q6

9231/21, 9231/22 • May/June 2017 • Question 2

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Mark scheme.

Take moments for the rod about some point, using $AP = 3a/4$, $\sin\theta = 4/5$, $\cos\theta = 3/5$, $\tan\theta = 4/3$: e.g. about A: $R_P \times AP - kW \times 3a\cos\theta = W \times (3a/2)\cos\theta$, giving $15R_P - 36kW = 18W$.	**M1,A1**
Find two more independent equations (resolutions or a second moment equation).	**B1,B1**
Relate F_A and R_A: $F_A = \pm R_A/8$.	**B1**
Eliminate θ; find either value of k.	**M1**
Solve: with $F_A \downarrow$: $k = 2$; with $F_A \uparrow$: $k = 4/17$ (or 0.235).	**M1,A1**
	Total: 8

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Q7

9231/23 • May/June 2017 • Question 4

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Mark scheme.

Take moments for rod about A: $T \times 3a\sin\theta = W \times 1.5a\cos 2\theta$.	**M1,A1**
Find tension T: $T = 7W/30$ or $0.233W$. **(ii)**	**A1**
Find length OB of string: $OB = (5a - 3a\cos 2\theta)/\cos\theta = 26a/5$ or $5.2a$.	**M1,A1**
Find modulus λ using Hooke's Law: $T = \lambda(OB - 4a)/4a$.	**M1**
$= \lambda(6a/5)/4a = 3\lambda/10$, so $\lambda = 7W/9$ or $0.778W$. **(iii)**	**A1**
Find horizontal component X of force at A: $X = T\cos\theta\,[= 14W/75$ or $0.187W]$.	**M1**
Find vertical component Y of force at A and angle ϕ: $Y = T\sin\theta + W\,[= 57W/50$ or $1.14W]$; $\phi = \tan^{-1}(Y/X) = \tan^{-1}(171/28)$.	**M1**
$= 80.7°$ or 1.41 radians.	**B1**
	Total: 10

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Q8

9231/21, 9231/22, 9231/23 • October/November 2017 • Question 4

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Mark scheme.

(i)	
Take moments for the rod about A: $T \times 2a = \frac{5W}{2} \times a\cos\theta$.	**M1**
$T = \frac{1}{2} \cdot \frac{5W}{2} \cdot \frac{4}{5} = W$.	**A1**
(ii)	
Use $F_P = \mu R_P$ at P: $T\sin\theta = \mu(W + T\cos\theta)$.	**M1**
$\frac{3}{5}W = \mu \cdot \frac{9}{5}W$, so $\mu = \frac{1}{3}$.	**A1**
(iii)	
Find horizontal component $X = T\sin\theta = \frac{3}{5}W$.	**B1**
Find vertical component $Y = \frac{5W}{2} - T\cos\theta = \frac{17}{10}W$.	**B1**
$R_A^2 = X^2 + Y^2 = \frac{13}{4}W^2$, so $R_A = \frac{1}{2}W\sqrt{13} \approx 1.80W$.	**B1FT**
(iv)	
Find length $PB = (a + 2a\sin\theta)/\cos\theta = \frac{11}{4}a$.	**B1**
$T = \lambda(PB - 2a)/2a$, giving $\lambda = \frac{8}{3}W \approx 2.67W$.	**M1, A1**
	Total: 10

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Q9

9231/21, 9231/22 • May/June 2018 • Question 4

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Mark scheme.

Moments at A: $R_B\, 2a \sin\theta = Wa\cos\theta + 5Wxa\cos\theta$	**M1 A1**
Simplify to give $R_B = \dfrac{W(5x+1)}{2\tan\theta}$ (AG) (ii)	**A1**
Find new $R'_B = \dfrac{W(5x+5+1)}{2\tan\theta}$ by moments at A	**B1**
Find/verify x from $R'_B = 2R_B$: $5x+6 = 2(5x+1)$, $x = \dfrac{4}{5}$ (AG) (iii)	**M1 A1**
Find $R_A = 6W$ and new $F'_A = R'_B$ or $2R_B$ $\left[= \dfrac{10W}{2\tan\theta} = \dfrac{4W}{\tan\theta}\cdot\tan\theta \text{ i.e. } \dfrac{5W}{\tan\theta}\right]$ by resolving	**B1, B1**
Relate R_A and F'_A using $\mu = \dfrac{F}{R}$: $\dfrac{2}{3} = \dfrac{F'_A}{R_A}$	**M1**
$\dfrac{\left(\dfrac{5W}{\tan\theta}\right)}{6W} = \dfrac{2}{3}$ so $\tan\theta = \dfrac{5}{4}$ or 1.25	**M1 A1**
	Total: 11

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Q10

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Mark scheme.

$AC = AD\cos\theta$ and $AD = 2a\cos\theta = 6a/5$, so $AC = 2a\cos^2\theta = 2a(3/5)^2 = 18a/25$ (AG) (D denotes the other end of the rope on the rod's line) (ii)	**M1 A1**
Take moments for the rod about one chosen point (e.g. A): $R_B \cdot 2a\sin\theta - Wa\cos\theta - T \cdot AC = 0$	**M1 A1**
Find a second independent equation, e.g. horizontal resolution: $R_B - F_A = T\sin\theta = 4T/5$	**B1**
Vertical resolution: $R_A - W = T\cos\theta = 3T/5$	**B1**
Find/verify T using $F_A = \frac{1}{4}R_A$, $\sin\theta = 4/5$, $\cos\theta = 3/5$: $T = W/4$ (AG) (iii)	**M1 A1**
$R_A = \dfrac{23W}{20} = 1.15W$ (can assume $T = W/4$); $F_A = \dfrac{23W}{80} = 0.2875W$	**B1**
$R_B = \dfrac{39W}{80} = 0.4875W$	**B1**
	Total: 10

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Q11

9231/21, 9231/23 • October/November 2018 • Question 4

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Mark scheme.

(i)	
$T \times 3a \sin 2\theta = W \times 2a \cos\theta + \frac{1}{2}W \times 4a\cos\theta$ (take moments for rod about A)	**M1A1**
$6aT \sin\theta\cos\theta = 4aW \times a\cos\theta$ (verify tension T)	**M1**
$T = 2W/3\sin\theta = 2W/3(8/17) = 17W/12$ AG (using $\sin\theta = 8/17$, $\cos\theta = 15/17$)	**A1**
EITHER:	**(ii)**
$X = T\cos\theta = (5/4)W$ or $1.25W$ (horizontal component X of force at A)	**B1**
$Y = W + \frac{1}{2}W - T\sin\theta = (5/6)W$ or $0.833W$ (vertical component Y of force at A)	**B1**
$F = \sqrt{(X^2 + Y^2)} = (5\sqrt{13}/12)W$ or $1.50W$ (magnitude of F)	**B1**
Upward force at angle to horizontal of $\tan^{-1}(Y/X) = \tan^{-1}(2/3) = 33.7°$ or 0.588 rad (AEF; A0 if direction unclear)	**M1A1**
(iii)	
$T = \lambda(CD - 2a)/2a$ (find modulus λ using Hooke's Law)	**M1**
$CD = 3a$ so $\lambda = 2T = 17W/6$ or $2.83W$	**A1**
	Total: 11

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Q12

9231/22 • October/November 2018 • Question 4

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Mark scheme.

(i)	
Take moments for rod about one chosen point, e.g. A: $R_C \times x - W\cos 45° \times a = 0$ (or about B, C, G, or D; $F_A \times$ may be replaced by μR_A and $\cos 45°$ by $1/\sqrt{2}$)	**B1**
Horizontally: $F_A - R_C\cos 45° = 0$ (find two more independent equations, e.g. resolution of forces on rod; a second moment equation may be used)	**B1**
Vertically: $R_A + R_C\cos 45° - W = 0$	**B1**
$[R_A = W/(1+\mu), F_A = \mu W/(1+\mu), R_C = \mu W\sqrt{2}/(1+\mu)]$; $x = a(1+\mu)/2\mu$ or $\frac{1}{2}a(1+1/\mu)$ (combine to find x using $F_A = \mu R_A$ and $\cos 45° = 1/\sqrt{2}$)	**M1A1**
(ii)	
$a(1+\mu)/2\mu \leqslant 2a$ so $\mu \geqslant \frac{1}{3}$ AG (verify μ using $x \leqslant 2a$)	**M1A1**
(iii)	
$a(1+\mu)/2\mu = 3a/2$ so $\mu = \frac{1}{2}$ (find μ when $x = 3a/2$ using result in (i))	**M1A1**
$F_A = W/3$, $R_A = 2W/3$ $[R_C = (\sqrt{2})W/3]$; $N_A = \sqrt{(F_A^2 + R_A^2)} = (\sqrt{5}/3)W$ or $0.745W$ (find F_A, R_A and hence magnitude of resultant force N_A at A)	**M1A1**
	Total: 11

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Q13

9231/21, 9231/22 • May/June 2019 • Question 4

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Mark scheme.

Take moments for rod about one chosen point (using $\sin\theta = 2/\sqrt{5}$, $\cos\theta = 1/\sqrt{5}$, $\sin(\pi - 2\theta) = \sin 2\theta = 4/5$)	**M1 A1**
Find two more independent equations, e.g. $R_A = T\sin\theta$	**B1**
$F_A = W - T\cos\theta$ (e.g. resolution of forces on rod; a second moment equation may be used)	**B1**
Relate F_A and R_A (may be implied): $F_A = \mu R_A$	**B1**
Find T by any method (e.g. from moments about A): $T = (2W\sin\theta)/(5/2\sin 2\theta) = 2W/(5\cos\theta)$	**M1**
$T = 2W/\sqrt{5}$ or $(2\sqrt{5}/5)W$ or $0.894W$	**A1**
Find or imply F_A and R_A by any method (e.g. from resolutions) and hence μ: $F_A = (3/5)W$, $R_A = (4/5)W$	**M1 A1**
$\mu = 3/4$ or 0.75	**A1**
	Total: 10

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Q14

9231/23 • May/June 2019 • Question 5

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Mark scheme.

Resolve forces along rod AB (with $\sin\theta = 3/5$, $\cos\theta = 4/5$): $R_B \cos\theta + F_B \sin\theta = W \sin\theta + \frac{1}{4}W \sin\theta$	**M1 A1**
Relate F_B and R_B (may be implied): $F_B = \frac{1}{3}R_B$	**B1**
Combine using $\tan\theta = 3/4$ to verify: $(4/5 + \frac{1}{3} \times 3/5)R_B = (1 + \frac{1}{4})(3/5)W$, so $R_B = \frac{3}{4}W$ (AG) (ii)	**DM1 A1**
Resolve forces perpendicular to rod AB, where C denotes rim: $R_C = F_B \cos\theta - R_B \sin\theta + W \cos\theta + \frac{1}{4}W \cos\theta$	**M1 A1**
Substitute to find R_C: $= (\frac{1}{3} \times \frac{3}{4} \times \frac{4}{5})W - (\frac{3}{4} \times \frac{3}{5})W + (\frac{5}{4} \times \frac{4}{5})W$	**DM1**
$= (1/5 - 9/20 + 1)W = \frac{3}{4}W$ (iii)	**A1**
Find total moments about any point, denoting rod's centre by G (taken on either side of C), where $AC = 2x - 2a\cos\theta = 2x - 8a/5$ and $CG = x - AC = 2a\cos\theta - x = 8a/5 - x$	**M1 A1**
Substitute and equate to zero to find x: $x = a$	**A1**
	Total: 12

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Q15

9231/21, 9231/22, 9231/23 • October/November 2019 • Question 2

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Mark scheme.

(i)	
Take moments about B: $R_E \times 3a = W\cos\theta \times 2a - W\sin\theta \times 2a$.	**M1 A1**
Find normal reaction at E (AEF): $R_E = 4W/(3\sqrt{10})$.	**A1**
Find normal reaction at B by resolving forces vertically: $R_B = W - R_E\cos\theta = 3W/5$.	**M1 A1**
(ii)	
Find friction at B by resolving forces horizontally: $F_B = R_E\sin\theta = 2W/15$.	**M1 A1**
Find μ from $F_B = \mu R_B$: $\mu = (2/15)/(3/5) = 2/9$.	**A1**
	Total: 8

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Q16

9231/31, 9231/32 • May/June 2020 • Question 4

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Mark scheme.

Table of areas and centres of mass: Square area 100, c.o.m. from $BC = 5$, from $DC = 5$; Triangle area $\frac{1}{2}x.\frac{15}{2}$, c.o.m. from $BC = \frac{1}{3}x$, from $DC = \frac{5}{2}$; Shape $ABEFD$ area $100 - \frac{15}{4}x$.	**M1**
Take moments about BC: $\left(100 - \frac{15}{4}x\right)\sigma.\bar{x} = 500\sigma - \frac{15}{4}x\sigma.\frac{1}{3}x$ (M1 for all terms present).	
$\bar{x} = \frac{400 - x^2}{80 - 3x}$ (AG).	**A1**
Take moments about DC: $\left(100 - \frac{15}{4}x\right).\bar{y} = 100 \times 5 - \frac{15}{4}x.\frac{5}{2}$.	**M1**
$\bar{y} = \frac{800 - 15x}{160 - 6x}$. **(b)**	**A1**
Use condition: $\bar{x} \geqslant x$.	**B1**
$2x^2 - 80x + 400 \geqslant 0$.	**M1**
$x = 20 - 10\sqrt{2}$.	**A1**
	Total: 7

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Q17

9231/33 • May/June 2020 • Question 4

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Mark scheme.

Table: Cylinder mass $\pi(kr)^2.3r\rho$, c.o.m. from vertex of cone $= \frac{11}{2}r$; Cone mass $\frac{1}{3}\pi(2r)^2.4r\rho$, c.o.m. from vertex $= 3r$; Combined mass $\left(\pi(kr)^2.3r + \frac{1}{3}\pi(2r)^2.4r\right)\rho$, c.o.m. $= \bar{x}$.	**B1**
Take moments about vertex: $\bar{x} \times \left(\pi(kr)^2.3r + \frac{1}{3}\pi(2r)^2.4r\right) = \frac{11r}{2} \times \pi(kr)^2.3r + 3r \times \frac{1}{3}\pi(2r)^2.4r$ leading to	**M1A1**
$\bar{x} = \frac{(99k^2+96)r}{18k^2+32}$. **(b)**	**A1**
$\tan\alpha = \frac{\bar{x}-4r}{2r}$ $(=1/8)$.	**M1**
$\bar{x} = \frac{17}{4}r$.	**A1**
Equate to answer in (a) and attempt to find k.	**M1**
$45k^2 = 80$: $k = \frac{4}{3}$.	**A1**
	Total: 8

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Q18

9231/31, 9231/33 • October/November 2020 • Question 4

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Mark scheme.

Volumes/centre-of-mass table correct: small cone COM $h + \frac{h}{8} = \frac{9h}{8}$; large cone COM $\frac{3h}{8}$ (unsimplified)	**B1**
Moments equation about AB: $\frac{13}{3}\pi r^2 h.\bar{x} = \frac{27}{6}\pi r^2 h.\frac{3h}{8} - \frac{1}{6}\pi r^2 h.\frac{9h}{8}$ (allow use of relative masses $1, 26, 27$)	**M1A1**
$\bar{x} = \dfrac{9h}{26}$ (b)	**A1**
$\tan\theta = \dfrac{\bar{x}}{3r}\left(= \dfrac{3h}{26r}\right)$	**M1**
Use $h = \frac{13}{4}r$: $\tan\theta = \dfrac{3}{8}$	**A1**
	Total: 6

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Q19

9231/32 • October/November 2020 • Question 3

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Mark scheme.

Volumes and distances from base correct in table (cone: $\frac{1}{3}\pi(3r)^2.4r$, COM $4r + r$; cylinder: $\pi(3r)^2.4r$, COM $2r$)	**B1**
Moments equation about base of cylinder	**M1A1**
$\bar{x} = \dfrac{11}{4}r$ (b)	**A1**
Correct condition for equilibrium: $OG\cos\theta < OA$ (where O is vertex of cone, OA slant height)	**B1**
$\left(4r + \frac{5r}{4}\right) \times \frac{4}{5} < 5r$	**M1**
$21 < 25$, true (correct conclusion with correct working)	**A1**
	Total: 7

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Q20

9231/31, 9231/32 • May/June 2021 • Question 4

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Mark scheme.

Table of volumes and centres of mass from base of cylinder: cone $\frac{1}{3}\pi r^2 kh$ at $h + \frac{kh}{4}$; cylinder $\pi r^2 h$ at $\frac{h}{2}$; combined $\pi r^2 h(\frac{1}{3}k + 1)$ at $\bar{x}$.	**M1**
Moments equation, 2 terms correct.	**A1**
All terms correct.	**A1**
$\bar{x} = \dfrac{h(k^2 + 4k + 6)}{4(3 + k)}$ (AG, shown convincingly). (b)	**A1**
$\tan\theta = \dfrac{r}{\bar{x}}$.	**M1**
Equate to $\frac{4}{3}$ and simplify to quadratic: $2k^2 - k - 15 = 0$.	**M1**
$k = 3$ (CAO, no other solutions).	**A1**
	Total: 7

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Q21

9231/33 • May/June 2021 • Question 1

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Mark scheme.

Areas and centre-of-mass distances from DB: ABD: $24a^2$ at $-a$; BCD: $48a^2$ at $2a$; combined $72a^2$ at $\bar{x}$.	**B1**
Moments equation with masses in correct ratio, e.g. $72a^2\bar{x} = 24a^2 \times (-a) + 48a^2 \times 2a$.	**M1**
$\bar{x} = a$ (CWO).	**A1**
	Total: 3

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Q22

9231/31, 9231/33 • October/November 2021 • Question 4

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Mark scheme.

(a)

Attempt at moments with three terms. Table: Square, area $9a^2$, c.o.m. from $AD = \frac{3}{2}a$; CDF, area $\frac{3}{2}ah$, c.o.m. $= a$; BEC, area $\frac{3}{2}ah$, c.o.m. $= 3a - \frac{1}{3}h$; Resulting $AEFC$, area $9a^2 - 3ah$, c.o.m. $= \bar{x}$.	**M1**
Two terms correct: $(9a^2 - 3ah)\bar{x} = \left(9a^2 \times \frac{3}{2}a\right) - \left(\frac{3}{2}ah \times a\right) - \left(\frac{3}{2}ah \times \left(3a - \frac{1}{3}h\right)\right)$.	**A1**
All correct.	**A1**
$\bar{x} = \dfrac{27a^2 - 12ah + h^2}{6(3a-h)} \left(= \dfrac{9a-h}{6}\right)$ (AEF).	**A1**
$\bar{y} = \bar{x}$ (by symmetry or equal to their $\bar{x}$).	**B1**

(b)

For equilibrium, $\bar{x} \leqslant 3a - h$; $27a^2 - 12ah + h^2 \leqslant 6(3a-h)^2$ (accept strict inequality).	**B1**
$27a^2 - 24ah + 5h^2 \geqslant 0$ (homogeneous 3-term quadratic inequality).	**M1**
$h \leqslant \dfrac{9}{5}a$ (CAO).	**A1**
	Total: 8

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Q23

9231/32 • October/November 2021 • Question 4

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Mark scheme.

(a)	
Attempt at moments, 3 terms. Table: Hemisphere, volume $\frac{2}{3}\pi a^3$, c.o.m. from $AB = \frac{3}{8}a$; Cylinder, volume $\pi ka\left(\frac{a}{2}\right)^2$, c.o.m. $= \frac{ka}{2}$; Remainder, volume $\frac{2}{3}\pi a^3 - \pi ka\left(\frac{a}{2}\right)^2$, c.o.m. $= \bar{x}$.	**M1**
Taking moments about AB: $\left(\frac{2}{3}\pi a^3 - \pi ka\left(\frac{a}{2}\right)^2\right) \times \bar{x} = \left(\frac{2}{3}\pi a^3 \times \frac{3}{8}a\right) - \left(\pi ka\left(\frac{a}{2}\right)^2 \times \frac{ka}{2}\right)$ (any 2 terms correct).	**A1**
All correct.	**A1**
$\bar{x} = \dfrac{3a(2-k^2)}{2(8-3k)}$ (shown convincingly, AG).	**A1**
(b)	
$\tan\theta = \dfrac{\bar{x}}{a}$: $\dfrac{3(2-k^2)}{2(8-3k)} = \dfrac{7}{18}$.	**B1**
$27k^2 - 21k + 2 = 0$ (rearrange to form quadratic).	**M1**
$k = \dfrac{2}{3}$ and $k = \dfrac{1}{9}$ (both answers correct).	**A1**
	Total: 7

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Q24

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Mark scheme.

Moments equation, condone missing ends of cylinder; one expression on the RHS correct. Table: Cylinder area $2\pi ah + 2\pi a^2$, centre of mass from AB is $\frac{1}{2}h$; Shell area $2\pi(2a)^2$, centre of mass from AB is $h + a$.	**M1**
Moments about AB: $\bar{x}\left(2\pi ah + 2\pi a^2 + 2\pi(2a)^2\right) = 2\pi(2a)^2 \times (h+a) + (2\pi ah + 2\pi a^2)\left(\frac{1}{2}h\right)$ (one correct expression on RHS scores A1).	**A1 A1**
$(2h + 10a)\bar{x} = h^2 + ah + 8ah + 8a^2 \Rightarrow \bar{x} = \dfrac{h^2 + 9ah + 8a^2}{2(h + 5a)}$. (b)	**A1**
$\tan\theta \le \dfrac{a}{\bar{x}}$.	**B1**
Form inequality and rearrange to quadratic (condone equation): $\bar{x} = \dfrac{h^2 + 9ah + 8a^2}{2(h + 5a)} \le \dfrac{3}{2}a \Rightarrow h^2 + 6ah - 7a^2 \le 0$.	**M1**
Attempt to solve, condone equation: $(h - a)(h + 7a) \le 0$.	**M1**
$(-7a \le)\ h \le a$.	**A1**
	Total: 8

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Q25

9231/33 • May/June 2022 • Question 1

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Mark scheme.

Attempt at moments equation with all necessary terms. Split into: Triangle OCD area 6, distance from Oy is 2; Rectangle $DEBC$ area 24, distance 6; Triangle BAE area 12, distance 11; Trapezium $OCBA$ area 42, distance $\bar{x}$ (where $D(3,0)$, $E(9,0)$). Other options possible for RHS, e.g. (1) $OAC : 30 \times 6$ and $ABC : 12 \times 9$; (2) $OBC : 12 \times 4$ and $OAB : 30 \times 8$; (3) Subtraction: $60 \times 7.5 - 6 \times 1 - 12 \times 13$.	**M1**
Parts that would give correct total area 42.	**B1**
Moments about Oy: $42\bar{x} = 6 \times 2 + 24 \times 6 + 12 \times 11$ $(= 288)$ (correct equation).	**A1**
$\bar{x} = \dfrac{288}{42} = 6.86$.	**A1**
	Total: 4

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Q26

9231/33 • May/June 2022 • Question 7

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Mark scheme.

Frictional force $= \mu\times$ normal reaction at D and E: $F_{AB} = \mu R$, $F_{BC} = \mu N$.	**B1**
One moments equation about any point involving all relevant forces, resolved if necessary (AEF), e.g. moments about B: $Na - Ra = Wa(\sin\theta - \cos\theta)$; about centre: $F_{AB}a + F_{BC}a = Wa(\cos\theta - \sin\theta)$; about D: $F_{BC}a + Na = Wa(\cos\theta + \sin\theta)$; about E: $Ra - F_{AB}a = Wa(\cos\theta + \sin\theta)$.	**B1**
Two resolutions: all relevant terms, different frictional forces. Parallel to AB: $N - F_{AB} = W\sin\theta + W\sin\theta$; Perpendicular to AB: $F_{BC} + R = W\cos\theta + W\cos\theta$. (Alternative vertical/horizontal resolution equations can also earn this mark.)	**B1**
Combine appropriate equations: $N - R = \frac{1}{2}\left((1-\mu)N - (1+\mu)R\right)$.	**M1**
Collect terms to obtain ratio/fraction in terms of μ only (CWO): $N\left(\frac{1}{2} + \frac{1}{2}\mu\right) = R\left(\frac{1}{2} - \frac{1}{2}\mu\right)$.	**M1**
$R : N = 1+\mu : 1-\mu$. (b)	**A1**
Divide resolution equations (must include μ terms): $\tan\theta = \dfrac{N - \mu R}{\mu N + R}$.	**M1**
Use $R = 2N$ (FT from part (a)): $\tan\theta = \dfrac{\frac{1}{3}N}{\frac{7}{3}N}$.	**M1**
$\tan\theta = \dfrac{1}{7}$.	**A1**
	Total: 9

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Q27

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Mark scheme.

Let F and R be friction and normal reaction at A. Take moments about A, for the rod: $N \times 3a = W \times 2a\cos\theta + kW \times 4a\cos\theta$	**M1**
$3N = (2+4k)W \times \frac{4}{5}$, giving $N = \frac{8}{15}W(1+2k)$ (AG) **(b)**	**A1**
$\uparrow$: $N\cos\theta + R = W + kW$	**B1**
$\rightarrow$: $F = N\sin\theta$ and $F = \frac{6}{7}R$	**B1**
Find $R = \frac{28}{75}W(1+2k)$ or N.	**M1**
Eliminate to find k.	**M1**
$k = \frac{1}{3}$.	**A1**
	Total: 7

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Q28

9231/32 • October/November 2022 • Question 2

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Mark scheme.

Taking moments about AB: $\bar{x}\left(27a^2 - \frac{9}{2}ax\right) = 27a^2 \times 2a - \frac{9}{2}ax \times \frac{1}{3}x$	**M1**
At least 2 terms correct.	**A1**
All correct, giving $\bar{x} = \dfrac{54a^3 - \frac{3}{2}ax^2}{27a^2 - \frac{9}{2}ax}$	**A1**
For equilibrium, $x \le \bar{x}$ (allow strict inequality).	**B1**
Simplify to $54a^2 - 27ax + 3x^2 \ge 0$, i.e. $(x-3a)(x-6a) \ge 0$, and solve.	**M1**
$(0 \le)x \le 3a$ (CAO).	**A1**
	Total: 6

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Q29

9231/31, 9231/32 • May/June 2023 • Question 4

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Mark scheme.

[Mass proportional to volume] Table of volumes and centre-of-mass distances for hemisphere, cylinder and object; moments equation, dimensionally correct, correct number of terms, allow sign errors: $\left(\frac{2}{3}\pi(2a)^3 + \pi a^2 d\right)\bar{x} = \frac{16}{3}\pi a^3 \times 2a + \pi a^2 d \times a$.	**M1 A1**
Simplify to $\bar{x} = \dfrac{32a^2 + 3ad}{16a + 3d}$ (AG, at least one line of intermediate working).	**A1**
Moments equation for $\bar{y}$, dimensionally correct, correct number of terms, allow sign errors: $\left(\frac{2}{3}\pi(2a)^3 + \pi a^2 d\right)\bar{y} = \frac{16}{3}\pi a^3 \times \left(-\frac{3}{8} \times 2a\right) + \pi a^2 d \times \frac{1}{2}d$.	**M1**
$\bar{y} = \dfrac{3(d^2 - 8a^2)}{2(16a + 3d)}$ (AEF). (b)	**A1**
$\sin\theta = \dfrac{2a - \bar{x}}{2a}$.	**B1**
Remove fractions: $2a \times \dfrac{1}{6} = 2a - \dfrac{32a^2 + 3ad}{16a + 3d}$, i.e. $\dfrac{5}{3}(16a + 3d) = (32a + 3d)$.	**M1**
$d = \dfrac{8}{3}a$.	**A1**
	Total: 8

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Q30

9231/33 • May/June 2023 • Question 3

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Mark scheme.

[Mass proportional to area] Table of areas and centre-of-mass distances from AC for ABC $(24a^2, 2a)$, DEC $(\frac{1}{2}x \cdot 5a, \frac{1}{3}x)$ and $ADEB$ $(24a^2 - \frac{5}{2}xa, \bar{x})$; all correct for ABC and DEC.	**B1**
Moments about AC: $\bar{x}\left(24a^2 - \frac{5}{2}xa\right) = 24a^2 \times 2a - \frac{1}{3}x \times \frac{5}{2}ax$. All moment terms present, dimensionally correct, allow sign error.	**M1**
All correct moments about AC.	**A1**
$\bar{x} = \dfrac{288a^2 - 5x^2}{3(48a - 5x)}$ (AEF). (b)	**A1**
On the point of toppling about E: $\bar{x} = x$, so $\dfrac{288a^2 - 5x^2}{3(48a - 5x)} = x$ (FT their expression for $\bar{x}$ from (a)).	**B1 FT**
Rearrange to 3-term quadratic: $10x^2 - 144ax + 288a^2 = 0$ (allow 3-term inequality).	**M1**
$2(5x - 12a)(x - 12a) = 0$, $x = \dfrac{12}{5}a$ (single correct answer, no inequality, CWO).	**A1**
	Total: 7

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Q31

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Mark scheme.

Let N be normal reaction at B and F the frictional force acting downwards: $T\cos\theta = F + W$	B1
$T\sin\theta = N$	B1
Moments about B: $Ta = W\sin\theta \times a + W\cos\theta \times a$	M1A1
$F = \frac{1}{2}N$ used	M1
$(\cos\theta + \sin\theta)\left(\cos\theta - \frac{1}{2}\sin\theta\right) = 1$	M1
$\frac{1}{2}\cos\theta\sin\theta = \frac{3}{2}(\sin\theta)^2$, so $\sin\theta(\cos\theta - 3\sin\theta) = 0$	M1
$\tan\theta = \frac{1}{3}$	A1
	Total: 8

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Q32

9231/32 • October/November 2023 • Question 3

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Mark scheme.

Mass proportional to area. Areas and distances from AC: ABC: area $\frac{1}{2} \times 2a \times a$, distance $\frac{1}{3}a$; BDE: area $\frac{1}{2} \times 2\left(\frac{a}{k}\right)^2$, distance $a - \frac{2}{3}\left(\frac{a}{k}\right)$; $ADEC$: area $-\frac{1}{2} \times 2\left(\frac{a}{k}\right)^2 + a^2$, distance $\bar{x}$	B1	**(a)**
Moments equation: $\bar{x}\left(-\frac{1}{2} \times 2\left(\frac{a}{k}\right)^2 + a^2\right) = \frac{1}{3}a \times a^2 - \left(a - \frac{2}{3}\left(\frac{a}{k}\right)\right) \times \left(\frac{a}{k}\right)^2$	M1A1	
$\bar{x} = \dfrac{a(k^2+k-2)}{3k(k+1)}$	A1	
$\tan\theta = \dfrac{\bar{x}}{a}\left[= \dfrac{5}{18}\right]$, with their $\bar{x}$ from (a)	B1	**(b)**
$18(k^2+k-2) = 5(3k^2+3k)$, so $k^2+k-12=0$	M1	
$(k+4)(k-3)=0$, $k=3$ only	A1	
	Total: 7	

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Q33

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Mark scheme.

M1: Frictional force F and normal reaction R at point of contact of ring with plane. Resolve parallel to plane: $F + T\cos\alpha = W\sin\alpha$ (only allow cos/sin errors for T and W components, sign errors; accept equations for vertical and horizontal, both needed).	**(a)**
Moments about O: $Fa = Ta\sin\alpha$.	**B1**
Combine and substitute for α: expression for T in terms of W.	**M1**
$T = \frac{1}{3}W$ (CAO). [Alternative: moments about point where ring touches plane: $Ta\sin\alpha + Ta\cos\alpha = Wa\sin\alpha$, then rearrange – same M1 A1 M1 A1 pattern.]	**A1**
M1: Resolve perpendicular to plane: $R = T\sin\alpha + W\cos\alpha$ (only allow cos/sin errors).	**(b)**
Use $F = \mu R$ and combine to reach an equation in μ only (from part (a), $F + T\cos\alpha = W\sin\alpha$ or $F = T\sin\alpha$).	**M1**
$\mu = \frac{1}{7}$.	**A1**
	Total: 7

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Q34

9231/33 • May/June 2024 • Question 5

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Mark scheme.

M1 A1: Tabulate areas and centroid distances from OC for OBC (area $\frac{1}{2} \times 24a \times 18a$, centroid $8a$), OAC (area $9ax$, centroid $\frac{1}{3}x$), ABC (area $216a^2 - 9ax$, centroid $\bar{x}$); moments equation about OC with all terms present, allow sign error, dimensionally correct: $(216a^2 - 9ax)\bar{x} = 216a^2 \times 8a - 9ax \times \frac{1}{3}x$; all correct for A1.	**(a)**
$\bar{x} = \dfrac{576a^2 - x^2}{72a - 3x}$ or $\dfrac{x + 24a}{3}$ (accept any equivalent form). [Alternative: treat as particles at $(0, 18a)$, $(x, 0)$, $(24a, 0)$; x-coordinate of centre of mass is $\frac{1}{3}(x + 24a)$ – B1 M1 A1.]	**A1**
M1 A1: $\tan\theta = \dfrac{18a - 6a}{\bar{x}}$ or $\dfrac{18a - \bar{y}}{\bar{x}}$ etc. (either way up, their value for $\bar{x}$ may be substituted in).	**(b)**
Obtain homogeneous (quadratic) equation $x^2 - 30ax + 144a^2 = 0$ (or, with simplified form, $30a = x + 24a$, linear).	**M1**
$x = 6a$ (single correct answer only).	**A1**
	Total: 7

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Q35

9231/31, 9231/33 • October/November 2024 • Question 4

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Mark scheme.

(a)	
Correct volumes and distances of centre of mass from AB for large and small cylinders.	**B1**
Moments equation with 3 terms, dimensionally correct.	**M1**
Correct, unsimplified: $\pi a^2 h\left(1 - \frac{4}{9}k\right)\bar{y} = \pi a^2 h \times \frac{1}{2}h - \pi\left(\frac{2}{3}a\right)^2 kh \times \frac{1}{2}kh$.	**A1**
$\bar{y} = \dfrac{(9-4k^2)h}{2(9-4k)}$.	**A1**
(b)	
$\tan\theta = \dfrac{\bar{y}}{a}$: $\dfrac{(9-4k^2)h}{2(9-4k)a} = \dfrac{3}{2}$ (FT their part (a)).	**B1 FT**
Use $h = \frac{8}{3}a$ and simplify to quadratic in k: $32k^2 - 36k + 9 = 0$.	**M1**
$k = \dfrac{3}{8}, \dfrac{3}{4}$.	**A1**
	Total: 7

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Q36

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Mark scheme.

(a)	
$\uparrow\ T\cos\theta + F = 2W$.	**B1**
$\rightarrow\ T\sin\theta = R$.	**B1**
Moments about A: $T\cos\theta \times 3a\sin\theta + T\sin\theta \times 3a\cos\theta = W \times 3a\sin\theta + W \times ka\sin\theta$ (all relevant terms, dimensionally correct).	**M1**
Use $F = \frac{1}{3}R$ and eliminate T and W to obtain an expression in k and θ: $12\cos\theta = (3+k)\left(\cos\theta + \frac{1}{3}\sin\theta\right)$.	**M1**
$k = 5$.	**A1**
(b)	
Any complete method to find F in terms of W (e.g. substitute into moments equation to obtain $T = \frac{2}{3}\sqrt{13}\,W$, $R = 2W$).	**M1**
$F = \dfrac{2}{3}W$.	**A1**
	Total: 7

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Q37

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Mark scheme.

(a)	
Cone: volume $18\pi r^3$, CoM above base $\frac{3}{2}r$; hemisphere: volume $\frac{16}{3}\pi r^3$, CoM above base $\frac{3}{4}r$. (Allow one error.)	**M1**
Moments equation with 3 terms: $\left(18\pi r^3 - \frac{16}{3}\pi r^3\right)\bar{y} = 18\pi r^3\left(\frac{3}{2}r\right) - \frac{16}{3}\pi r^3\left(\frac{3}{4}r\right)$.	**M1**
Fully correct equation.	**A1**
$\bar{y} = \dfrac{69}{38}r$.	**A1**
(b)	
At point of toppling: $\tan\alpha = \dfrac{3r}{\bar{y}}$ (or reciprocal), $\left[= \frac{38}{23}\right]$.	**M1**
FT their $\bar{y}$.	**A1 FT**
$\alpha = 58.8°$ (CAO).	**A1**
	Total: 7

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Q38

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Mark scheme.

Areas: rectangle $2a^2$ at distance $-\frac{1}{2}a$ from D; triangle $\frac{1}{2}a(2a)$ at distance $\frac{2}{3}a$ from D. (Allow one error in first two columns.)	**B1**
Moments equation with 3 terms, dimensionally correct: $-2a^2\left(\frac{1}{2}a\right) + \frac{1}{2}(2a^2)\left(\frac{2}{3}a\right) = (3a^2)\bar{x}$.	***M1**
Moments equation about toppling point D, 2 terms only, must include k: $m\left(\frac{1}{9}a\right) = km(2a)$.	**DM1**
$k = \frac{1}{18}$.	**A1**
	Total: 4

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Q39

9231/34 • May/June 2025 • Question 5

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Mark scheme.

Split into rectangle $AGED$ and triangle GFE (DA parallel to EG).	**(a)**
Moments about AD, all terms present and dimensionally correct: $18a^2\bar{x} = 6ah\left(\frac{1}{2}h\right) + (18a^2 - 6ah)\left(\frac{1}{3}h + 2a\right)$.	**M1**
Fully correct equation.	**A1**
$\bar{x} = \dfrac{h^2 - 6ah + 36a^2}{18a}$ (AG, shown convincingly).	**A1**
Moments about AB: $18a^2\bar{y} = 6ah(3a) + (18a^2 - 6ah)(2a)$.	**M1**
$\bar{y} = 2a + \frac{1}{3}h$ (must expand and collect like terms).	**A1**
(b)	
$\tan\theta = \dfrac{\bar{x}}{6a - \bar{y}}$.	**B1**
Equate to $\frac{7}{15}$, substitute $\bar{x}$ and their $\bar{y}$, simplify to 3-term quadratic: $\dfrac{h^2 - 6ah + 36a^2}{18a} \Big/ \left(6a - \left(2a + \frac{1}{3}h\right)\right) = \frac{7}{15}$ leading to $5h^2 - 16ah + 12a^2 = 0$, i.e. $(h - 2a)(5h - 6a) = 0$.	**M1 FT**
$h = \frac{6}{5}a$ or $h = 2a$ (CAO, both required).	**A1**
	Total: 8

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Q40

9231/32 • October/November 2025 • Question 3

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Mark scheme.

Vertical distance of COM of sector from OA (SOI, allow unsimplified): $\dfrac{(2a\cos(\frac{1}{4}\pi))\sin(\frac{1}{4}\pi)}{3(\frac{1}{4}\pi)} = \dfrac{4a}{3\pi}$.	**B1**
3-term dimensionally correct moments equation: $ka^2(\frac{1}{2}a) + \frac{1}{4}\pi a^2\left(a - \dfrac{4a}{3\pi}\right) = (ka^2 + \frac{1}{4}\pi a^2)\bar{y}$.	**M1**
Correct moments equation, allow unsimplified: $6ka + 3\pi a - 4a = 3(4k+\pi)\bar{y}$.	**A1 FT**
AEF (not allowing fractions within numerator/denominator): $\bar{y} = \dfrac{6k+3\pi-4}{3(4k+\pi)}a$. **(b)**	**A1**
Dimensionally correct moments equation about OB using their horizontal distance of COM of sector from OB: $ka^2(\frac{1}{2}ka) = \frac{1}{4}\pi a^2\left(\dfrac{4a}{3\pi}\right)$.	**M1**
$k^2 = \frac{2}{3}$, $k = \sqrt{2/3} = 0.816$.	**A1**
	Total: 6

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Q41

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Mark scheme.

$\bar{x} = ka + \frac{1}{3}h$ (may be seen in a table).	**M1**
$\bar{x} = \frac{1}{3}(3ka + h)$, AG, shown convincingly.	**A1**
$\bar{y} = a + \frac{1}{3}(h - a) = \frac{1}{3}(2a + h)$. **(b)**	**B1**
Moments equation with 3 terms: $\bar{x}(2ka^2 + ah) = 2ka^2 \times \frac{1}{2}ka + ah(ka + \frac{1}{3}h)$.	**M1**
Fully correct moments equation (may be seen in part (a)).	**A1**
Use correct condition $\bar{x} \leq ka$ with their $\bar{x}$ for $OABCD$: $\dfrac{a^2k^2 + ahk + \frac{1}{3}h^2}{2ak + h} \leq ka$.	**M1**
CAO: $(0 \leq)\, h \leq \sqrt{3}ka$. **(c)**	**A1**
Point of toppling means $h = \sqrt{3}ka\,[= a]$, so $\bar{x} = \frac{1}{3}a(\sqrt{3} + 1)$.	**B1**
[$h = a$, meaning triangle ABC is isosceles.] $\bar{y} = a$ (by symmetry).	**B1**
	Total: 9

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Q42

9231/34 • October/November 2025 • Question 3

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Mark scheme.

Areas and distances from AD correct (need not be presented in a table): $ABCD$: area $4a^2$, from AD: a; AED: area a^2, from AD: $\frac{1}{3}a$; BCF: area ah, from AD: $2a - \frac{1}{3}h$; shape $BFDE$: area $3a^2 - ah$.	**B1**
Dimensionally correct moments equation about AD with at least three correct terms: $(3a^2 - ah)\bar{x} = 4a^2 \times a - a^2 \times \frac{1}{3}a - ah \times (2a - \frac{1}{3}h)$.	**M1**
AG, shown convincingly: $\bar{x} = \dfrac{h^2 - 6ah + 11a^2}{3(3a - h)}$.	**A1**
Dimensionally correct moments equation about AB with at least three correct terms: $(3a^2 - ah)\bar{y} = 4a^2 \times a - a^2 \times \frac{2}{3}a - ah \times \frac{4}{3}a$.	**M1**
$\bar{y} = \dfrac{2a(5a - 2h)}{3(3a - h)}$. **(b)**	**A1**
Use condition $\bar{x} \geq a$ and simplify to three-term homogeneous quadratic inequality: $h^2 - 3ah + 2a^2 \geq 0$.	**M1**
$[0 <]\ h \leq a$, $h = 2a$ (accept $h = 0$).	**A1**
	Total: 7

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Questions available in the companion questions booklet.

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