Further Math with Ali Hashir

Circular Motion

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/21, 9231/22 • May/June 2015 • Question 1

↑ Back to contents

Mark scheme.

Find T by equating dv/dt at $t = T$ to 6: $4T - 4 = 6$.	**M1**
$T = 2.5$.	**A1**
Find radial component v^2/r of acceleration at $t = T$: $v^2/r = (2T^2 - 4T + 3)^2/0.25$ (M0 if T not given a value).	**M1**
$= (11/2)^2 \times 4 = 121$ m s^{-2}. SR: Max M1 (1/4) if linear and angular confused.	**A1**
	Total: 4

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Q2

9231/21, 9231/22 • May/June 2015 • Question 3

↑ Back to contents

Mark scheme.

Find v^2 from conservation of energy: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(1-\cos\theta)$.	**M1**
as above.	**A1**
Find R by using $F = ma$ radially: $R = mg\cos\theta - mv^2/a$.	**B1**
Eliminate v^2 to find R (AG).	**M1**
$R = mg(3\cos\theta - 2) - mu^2/a$.	**A1**
Find u^2 or v^2 in terms of $\cos\theta$ when $R = 0$: $u^2 = ag(3\cos\theta - 2)$ or $v^2 = ag\cos\theta$.	**B1**
EITHER: replace $\cos\theta$ in energy eqn with $v = 2u$: $4u^2 = u^2 + 2ag - \frac{2}{3}(u^2 + 2ag)$; OR: find $\cos\theta$ and substitute in energy eqn: $\cos\theta = 8/11$, $4u^2 = u^2 + 2ag(1 - 8/11)$.	**M1**
as above.	**A1**
Hence find u: $u = \sqrt{2ag/11}$ or $0.426\sqrt{ag}$.	**A1**
	Total: 9

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Q3

9231/21, 9231/22, 9231/23 • October/November 2015 • Question 4

↑ Back to contents

Mark scheme.

$T = mu^2/a - mg$ at the top (from $F = ma$ vertically), or via energy: $T' = mu^2/a + mg(2 - 3\cos\theta)$.	**B1**
Requiring $T \geq 0$ at top gives $u_{\min} = \sqrt{ag}$ (A.G.).	**B1**
Find v at bottom from conservation of energy: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mg \times 2a$, so $v = \sqrt{5ag}$.	**M1A1**
Find new speed V from conservation of momentum: $m'V = mv$ with $m' = m + \frac{1}{4}m$, giving $V = \frac{4}{5}\sqrt{5ag}$.	**M1A1**
Find w^2 at angle θ from conservation of energy: $\frac{1}{2}m'w^2 = \frac{1}{2}m'V^2 - m'ga(1 + \cos\theta)$.	**M1A1**
$T' = m'w^2/a - m'g\cos\theta$.	**B1**
Eliminate w^2: $T' = m'V^2/a - m'g(2 + 3\cos\theta)$.	**A1**
Substitute for m', V: $T' = (5mg/4)(6/5 - 3\cos\theta)$, i.e. $T' = 3mg/2 - (15/4)mg\cos\theta$.	**A1**
String slack when $T' = 0$: $\cos\theta = \frac{1}{3} \times \frac{6}{5} = 2/5$ or 0.4.	**M1A1**
	Total: 13

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Q4

9231/21, 9231/22 • May/June 2016 • Question 4

↑ Back to contents

Mark scheme.

Find v^2 at A from conservation of energy: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1 + \cos\theta)$	**M1 A1**
Use $F = ma$ radially at A with $R = 0$: $mv^2/a = mg\cos\theta$	**B1**
Use $v^2 = 3ag/5$ and eliminate $\cos\theta$ to find u: $\cos\theta = 3/5$, $u^2 = v^2 + 2ag(1 + \cos\theta) = 19ag/5$, $u = \sqrt{19ag/5}$ or $1.95\sqrt{ag}$	**M1 A1**
Find vertical component of speed at A: $V = v\sin\theta$ $[= 4v/5]$	**M1**
Find greatest height h_A reached above A: $h_A = V^2/2g = 24a/125$ or $0.192a$	**M1 A1**
Find greatest height h_O reached above O: $h_O = h_A + a\cos\theta = 99a/125$ or $0.792a$	**M1 A1**
	Total: 10

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Q5

9231/23 • May/June 2016 • Question 1

↑ Back to contents

Mark scheme.

Relate u^2, v^2 at lowest and highest points by energy: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - 2mga$ $[v^2 = u^2 - 4ag]$	**B1**
Find tension T_1 at lowest point by $F = ma$: $T_1 = mu^2/a + mg$	**B1**
Find tension T_2 at highest point by $F = ma$: $T_2 = mv^2/a - mg$	**B1**
EITHER: Relate u^2, v^2 using $T_1 = 2T_2$: $2v^2 = u^2 + 3ag$	**M1**
Eliminate v^2 to find u: $u = \sqrt{11ag}$	**M1 A1**
	Total: 6

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Q6

9231/23 • May/June 2016 • Question 3

↑ Back to contents

Mark scheme.

Find c by equating transverse acceleration to 2 at $t = 3$: $2 \times 3 + c = 2$, $c = -4$	**M1 A1**
EITHER: Find radial force at $t = 3$: $F_R = 0.2(3^2 + 3c + d)^2/0.5$ $[= 0.4(d-3)^2]$	**M1 A1**
Find transverse force at $t = 3$ by $F = ma$	**B1**
Equate $F_T^2 + F_R^2$ to $\{(2\sqrt{17})/5\}^2$: $0.4^2 + 0.4^2(3^2 + 3c + d)^4 = 68/25$ $[F_R^2 = 2.56, F_R = 1.6]$	**M1 A1**
Find d by solving with $c = -4$: $(3^2 + 3c + d)^4 = 17 - 1 = 16 \Rightarrow (d-3)^2 = 4$, $d = 1$ or 5	**A1**
	Total: 8

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Q7

9231/21, 9231/22, 9231/23 • October/November 2016 • Question 4

↑ Back to contents

Mark scheme.

(i)	
Find v_1^2 at lowest point from conservation of energy: $\frac{1}{2}mv_1^2 = \frac{1}{2}mu^2 + 2mga$	**M1 A1**
Verify new v_2 from conservation of momentum: $Mv_2 = mv_1$ with $M = (\lambda{+}1)m$, $v_2 = v_1/(\lambda+1) = \{5/(\lambda+1)\}\sqrt{(\frac{1}{3}ag)}$ A.G.	**M1 A1**
(ii)	
Use $F = ma$ radially at slack point with $T = 0$: $Mv_3^2/a = Mg\cos\theta$ (θ is angle of string with upward vertical)	**B1**
Find v_3^2 at slack point from conservation of energy: $\frac{1}{2}Mv_3^2 = \frac{1}{2}Mv_2^2 - Mg(4a/3)$ (SR: lose max 1 mark if mass is m in either equation)	**M1 A1**
Eliminate v_3^2 [$= ag/3$] using $\cos\theta = \frac{1}{3}$: $ag/3 = v_2^2 - 8ag/3$	**M1**
Substitute for v_2 to find λ: $3ag = \{25/(\lambda+1)^2\}ag/3$, $(\lambda+1)^2 = 25/9$, $\lambda = \frac{2}{3}$ [rejecting $-8/3$]	**M1 A1**
(iii)	
Use $F = ma$ radially just before collision: $T_1 = mv_1^2/a + mg$ [$= (25/3+1)mg = 28mg/3$]	**B1**
Use $F = ma$ radially just after collision: $T_2 = Mv_2^2/a + Mg$ [$= (3+1)(5m/3)g = 20mg/3$]	**B1**
Find change in tension (either sign, AEF): $(25mg/3)\{\lambda/(\lambda{+}1)\} - \lambda mg = 8mg/3$ or $2.67mg$	**A1 A1**
	Total: 14

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Q8

9231/21, 9231/22 • May/June 2017 • Question 5

↑ Back to contents

Mark scheme.

Verify v_1 for string horizontal by conservation of energy: $v_1^2 = ag + 2ag\cos\alpha$, $v_1 = \sqrt{(ag(1+2\cos\alpha))}$ (AG). **(ii)**	**M1,A1**
$T_A = mg(1-\cos\alpha)$ from radial equation of motion at A.	**M1,A1**
Find v_2^2 at C by conservation of energy.	**M1,A1**
$v_2^2 = ag(\frac{1}{3} + 2\cos\alpha)$.	**A1**
$T_C = 3mg\cos\alpha$.	**M1,A1**
Find $\cos\alpha$ from $T_A = T_C$.	**M1,A1**
$\cos\alpha = \frac{1}{4}$.	**A1**
	Total: 12

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Q9

9231/23 • May/June 2017 • Question 5

↑ Back to contents

Mark scheme.

Verify v by conservation of energy (A0 if no m): $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(\cos\alpha + \sin\alpha)$, $v^2 = u^2 + 2ag(4/5 + 3/5) = u^2 + 14ag/5$ (AG). **(ii)**	**M1,A1**
Find tension T_P at P using $F = ma$: $T_P = mu^2/a - mg\cos\alpha$.	**B1**
Find tension T_Q at Q using $F = ma$: $T_Q = mv^2/a + mg\sin\alpha$.	**B1**
Relate u^2, v^2 using $T_Q = 2T_P$ (A0 if required eqn omitted): $v^2 = 2u^2 - ag(2\cos\alpha + \sin\alpha) = 2u^2 - 11ag/5$ (AEF).	**M1,A1**
Eliminate v^2 to find u: $u = \sqrt{(5ag)}$ or $2.24\sqrt{(ag)}$ $[v^2 = 39ag/5]$. **(iii)**	**A1**
Find tension T_{min} at top from $F = ma$ radially: $T_{min} + mg = mV^2/a$.	**B1**
Find V^2 at top by conservation of energy: $\frac{1}{2}mV^2 = \frac{1}{2}mu^2 - mga(1 - \cos\alpha)$ or $\frac{1}{2}mv^2 - mga(1 + \sin\alpha)$.	**M1**
Combine: $[V^2 = (5 - 2/5)ag = (39/5 - 16/5)ag = 23ag/5]$; $T_{min} = 23mg/5 - mg = 18mg/5$ or $3.6mg$.	**A1**
	Total: 10

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Q10

9231/21, 9231/22, 9231/23 • October/November 2017 • Question 1

↑ Back to contents

Mark scheme.

Find p by equating transverse acceleration to 0 at $t = 2$: $-p + 2t = 0$, so $p = 4$.	**M1, A1**
Find radial acceleration a_R at $t = 2$ in terms of p from v^2/r: $a_R = (8 - pt + t^2)^2/0.8$.	**M1**
Evaluate with $p = 4$: $a_R = 4^2/0.8 = 20$.	**A1**
	Total: 4

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Q11

9231/23 • May/June 2018 • Question 1

↑ Back to contents

Mark scheme.

Find radial acceleration a_R at $t = 2$ from v^2/r: $a_R = (2^2 - 2 + 2)^2/0.8 = 4^2/0.8 = 20\ \mathrm{m\,s^{-2}}$	**M1 A1**
Find transverse acceleration a_T at $t = 2$ by differentiation: $a_T = 2t - 1 = 3\ \mathrm{m\,s^{-2}}$	**B1**
	Total: 3

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Q12

9231/23 • May/June 2018 • Question 5

↑ Back to contents

Mark scheme.

Find v_1^2 at the lowest point from conservation of energy: $\frac{1}{2}mv_1^2 = \frac{1}{2}mu^2 + mga$	**M1**
$v_1^2 = (12+2)ag = 14ag$	**A1**
Find new v_2 from conservation of momentum: $Mv_2 = mv_1$ with $M = (1+k)m$	**M1**
$v_2 = v_1/(1+k) = \sqrt{14ag}/(1+k)$	**A1**
Find tension T_1 just before collision using $F = ma$ radially: $T_1 = mv_1^2/a + mg = 15mg$	**M1 A1**
Find tension T_2 just after collision using $F = ma$ radially (needs M, not m, throughout): $T_2 = Mv_2^2/a + Mg$	**M1**
$= (1+k)\left\{\frac{14}{(1+k)^2} + 1\right\} mg$ or $\left\{\frac{14}{1+k} + (1+k)\right\} mg$	**A1**
Equate T_2 and $\frac{1}{2}T_1$: $14 + (1+k)^2 = 15(1+k)/2$	**M1**
Solve resulting quadratic for k (dependent on all previous M marks, requires quadratic equation): $2k^2 - 11k + 15 = 0$, $k = 2.5$ or $k = 3$	**M1 A1**
	Total: 11

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Q13

9231/22 • October/November 2018 • Question 3

↑ Back to contents

Mark scheme.

(i)	
$\frac{1}{2}mv_B^2 = \frac{1}{2}mu^2 - mga(\sin\alpha - \cos\alpha)$ $[v_B^2 = u^2 - (14/13)ag]$ (find speed v_B at B by conservation of energy, A0 if no m; $\sin\alpha = 12/13$, $\cos\alpha = 5/13$)	**M1A1**
$[T_B =]\, mv_B^2/a - mg\sin\alpha = 0$ $[v_B^2 = (12/13)ag]$ (equate tension T_B at B to zero by using $F = ma$ radially)	**B1**
$u^2 = (3\sin\alpha - 2\cos\alpha)ag$ (combine to verify u^2)	**M1**
$= (36/13 - 10/13)ag = 2ag$ AG	**A1**
EITHER:	**(ii)**
$\frac{1}{2}mv_C^2 = \frac{1}{2}mu^2 + mga(1+\cos\alpha)$ or $\frac{1}{2}mv_B^2 + mga(1+\sin\alpha)$ $[v_C^2 = 62ag/13]$ (find v_C^2 at lowest point C by conservation of energy)	**M1**
$T_{max} = mv_C^2/a + mg$ (find tension T_{max} at lowest point from $F = ma$ radially)	**B1**
$= 62mg/13 + mg$ (combine to find T_{max})	**M1**
$= 75mg/13$ or $5.77mg$ or $57.7m$	**A1**
	Total: 9

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Q14

9231/21, 9231/22 • May/June 2019 • Question 1

↑ Back to contents

Mark scheme.

Verify radial acceleration a_R at $t = \pi/6$ from $r\omega^2$: $a_R = 2(\mathrm{d}\theta/\mathrm{d}t)^2 = 2(2\sin 2t)^2 = 2(2\sin(\pi/3))^2$	**M1**
$= 2(\sqrt{3})^2 = 6\,\mathrm{m\,s^{-2}}$ (AG) (ii)	**A1**
Find transverse acceleration a_T at $t = \pi/6$ by differentiation: $a_T = 2\,\mathrm{d}^2\theta/\mathrm{d}t^2 = 2(4\cos 2t) = 2(4\cos(\pi/3))$	**M1**
$= 4\,\mathrm{m\,s^{-2}}$	**A1**
	Total: 4

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Q15

9231/23 • May/June 2019 • Question 2

↑ Back to contents

Mark scheme.

Find tension T_A at A from $F = ma$ radially: $T_A = mv_A^2/a - mg\cos\alpha = m(9ag/4)/a - mg\cos\alpha$	**B1**
Find tension T_B at B from $F = ma$ radially: $T_B = mv_B^2/a + mg\cos\alpha$	**B1**
Apply conservation of energy at B: $\frac{1}{2}mv_B^2 = \frac{1}{2}mv_A^2 + 2mga\cos\alpha$, so $v_B^2 = ag(9/4 + 4\cos\alpha)$	**M1 A1**
Combine using $T_B = 4T_A$ and $v_A^2 = 9ag/4$: $v_B^2 = 4v_A^2 - 5ag\cos\alpha = v_A^2 + 4ga\cos\alpha$	**M1**
$9ag\cos\alpha = 3v_A^2 = 27ag/4$, so $\cos\alpha = 3/4$ (AG) (ii)	**A1**
Find T_B: $T_B = 4T_A = (9 - 4\cos\alpha)mg = 6mg$ (or $v_B^2 = 21ag/4$, $T_B = (21/4 + \cos\alpha)mg = 6mg$)	**M1 A1**
	Total: 8

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Q16

9231/21, 9231/22, 9231/23 • October/November 2019 • Question 1

↑ Back to contents

Mark scheme.

Equate radial acceleration to 8 at $t = T$ from v^2/r: $(T-1)^4/2 = 8$.	**M1 A1**
Hence find positive value of T (or of $T-1$): $T = 3$ (or $T - 1 = 2$).	**A1**
Find magnitude of transverse acceleration at $t = T$: $a_T = 2(T-1) = 4 \text{ m s}^{-2}$.	**M1 A1**
	Total: 5

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Q17

9231/21, 9231/22, 9231/23 • October/November 2019 • Question 4

↑ Back to contents

Mark scheme.

(i)	
Use conservation of energy to slack point P_1: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga\cos\theta$.	**M1**
Equate tension at P_1 to 0 using $F = ma$: $mv^2/a - mg\cos\theta = 0$ (A1 if both equations correct, with m included). AG	**M1 A1**
Combine to verify $\cos\theta$ using $u = \sqrt{2ag}$: $v^2 = 2ag - 2ag\cos\theta = ag\cos\theta$.	**M1**
$\cos\theta = 2/3$.	**A1**
(ii)	
Find vertical speed v_V at P_1: $v_V = v\sin\theta = \sqrt{2ag/3}\,(\sqrt{5}/3)$ or $v_V^2 = (10/27)ag$.	**M1**
Find height risen above P_1 by considering vertical motion: $h = v_V^2/2g = (5/27)a$ or $0.185a$.	**M1 A1**
Find total height risen above level of O: $h + a\cos\theta = (23/27)a$ or $0.852a$.	**A1**
	Total: 9

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Q18

9231/31, 9231/32 • May/June 2020 • Question 2

↑ Back to contents

Mark scheme.

For A: $T = 3mg$. For B: $\uparrow T\cos\theta = mg$.	**M1**
Equate: $3mg\cos\theta = mg \Rightarrow \cos\theta = \dfrac{1}{3}$.	**A1**
$\rightarrow T\sin\theta = mr\omega^2$ with $r = (a-x)\sin\theta$.	**M1**
Equate: $3mg = m(a-x)\omega^2$.	**A1**
$x = \dfrac{a}{4}$.	**A1**
	Total: 5

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Q19

9231/31, 9231/32 • May/June 2020 • Question 7

↑ Back to contents

Mark scheme.

$(N+)mg\cos\theta = \dfrac{mv^2}{a}$.	**B1**
$\dfrac{1}{2}mv^2 - \dfrac{1}{2}m\dfrac{7ag}{2} = -mg(a + a\cos\theta)$.	**M1A1**
Loses contact when $N = 0$, so combine and simplify.	**M1**
$\cos\theta = \dfrac{1}{2}$: $\theta = 60°$ (AG). **(b)**	**A1**
When P is vertically below O, its horizontal displacement is $a\sin 60°$, so time $T = \dfrac{a\sin 60}{v\cos 60} = a\sqrt{3}/v = \sqrt{\dfrac{6a}{g}}$.	**M1**
From (a), $v^2 = \dfrac{1}{2}ag$.	**A1**
Vert: $h = \dfrac{v\sqrt{3}}{2}T - \dfrac{1}{2}gT^2$.	**M1**
$\dfrac{3}{2}a - 3a = -\dfrac{3}{2}a$.	**A1**
This corresponds to the point A. *Alternative method for (b):* $y = x\sqrt{3} - \dfrac{4x^2}{a}$ [**M1A1**]; coordinates of A: $x = \dfrac{1}{2}a\sqrt{3}$, $y = -\dfrac{3}{2}a$ [**B1**]; substitute coordinates into $y = x\sqrt{3} - \dfrac{4x^2}{a}$ and show that these satisfy this equation [**M1A1**].	**A1**
	Total: 10

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Q20

9231/33 • May/June 2020 • Question 1

↑ Back to contents

Mark scheme.

$T = 4mg = ma\omega^2$ so $\omega^2 = \dfrac{4g}{a}$.	**B1**
Time per revolution $= \dfrac{2\pi}{\omega} = \pi\sqrt{\dfrac{a}{g}}$.	**B1**
	Total: 2

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Q21

9231/33 • May/June 2020 • Question 3

↑ Back to contents

Mark scheme.

$T_A - mg\cos 60 = m.\dfrac{4ag}{a}$ (so $T_A = \dfrac{9}{2}mg$).	**B1**
$T_B + mg\cos 60 = m.\dfrac{v^2}{a}$.	**B1**
Energy: $\dfrac{1}{2}m(4ag - v^2) = mg.2a\cos 60$; $v^2 = 2ag$.	**M1A1**
$T_B = \dfrac{3mg}{2}$.	**A1**
Ratio is 3 : 1.	**A1**
	Total: 6

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Q22

9231/31, 9231/33 • October/November 2020 • Question 2

↑ Back to contents

Mark scheme.

At top, tension $= 0$, so $mg = \dfrac{mv^2}{a}$ $(v^2 = ag)$	**B1**
Energy equation: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1 + \cos\theta)$	**M1A1**
Substitute for u and v: $ag = \frac{16}{25}.5ag - 2ag(1 + \cos\theta)$	**M1**
$\cos\theta = \dfrac{1}{10}$	**A1**
	Total: 5

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Q23

9231/31, 9231/33 • October/November 2020 • Question 3

↑ Back to contents

Mark scheme.

Use Hooke's law: $T = 4mg.\dfrac{ka}{a}$	**B1**
N2L horizontally: $T\sin\theta = \left(\dfrac{mrg}{a}\right) = m(k+1)a\sin\theta.\dfrac{g}{a}$ (must see T and k)	**M1**
$T = mg(k+1)$	**A1**
Equate: $k = \dfrac{1}{3}$ (b)	**A1**
$\uparrow$ $T\cos\theta = mg$	**M1**
$\left(T = \frac{4}{3}mg\right)$ $\cos\theta = \dfrac{mg}{\frac{4}{3}mg} = \dfrac{3}{4}$	**A1**
	Total: 6

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Q24

9231/32 • October/November 2020 • Question 1

↑ Back to contents

Mark scheme.

At B, $mg\cos\beta = \dfrac{mv^2}{a}$: $(v^2 = ag\cos\beta)$	**B1**
Energy equation with 4 terms and correct dimensions: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(\cos\alpha - \cos\beta)$ giving $v^2 = \frac{3ga}{2} - 2ag\cos\beta$	**M1A1**
Substitute for u, $\cos\alpha$ and v: $ag\cos\beta = \dfrac{ag}{6} + 2ag\left(\dfrac{2}{3} - \cos\beta\right)$	**M1**
$\cos\beta = \dfrac{1}{2}$	**A1**
	Total: 5

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Q25

9231/32 • October/November 2020 • Question 4

↑ Back to contents

Mark scheme.

$\uparrow\ N\cos\theta = mg$	**B1**
$\leftarrow\ N\sin\theta = mr\sin\theta\,\omega^2$	**B1**
$\cos\theta = \frac{mg}{N}$ so $\cos\theta = \frac{g}{\omega^2 r}$ (AG) (b)	**B1**
$\cos\theta = \frac{r-x}{r} = \frac{g}{\omega^2 r}$ (must involve x)	**B1**
In new situation: $r - 4x = r \times \frac{g}{4\omega^2 r}$ (using $4x$ and 2ω)	**M1**
Combining: $r - x = 4(r - 4x)$	**M1**
$x = \frac{1}{5}r$	**A1**
	Total: 7

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Q26

9231/31, 9231/32 • May/June 2021 • Question 2

↑ Back to contents

Mark scheme.

$R\cos\theta = mg$ (vertical).	**B1**
$R\sin\theta = \dfrac{mv^2}{r}$ (horizontal).	**B1**
$r = a\sin\theta$.	**B1**
$8\cos\theta = 3\big(1 - (\cos\theta)^2\big)$ (quadratic equation in $\cos\theta$).	**M1**
$\cos\theta = \frac{1}{3}$.	**A1**
$x = \frac{2}{3}a$.	**A1**
	Total: 6

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Q27

9231/31, 9231/32 • May/June 2021 • Question 5

↑ Back to contents

Mark scheme.

$T_A - mg\cos\theta = \dfrac{mu^2}{a}$.	**B1**
$T_B + mg\cos\theta = \dfrac{mag}{a}$.	**B1**
Use $T_A = 7T_B$ and combine: $u^2 = ag(7 - 8\cos\theta)$.	**M1**
Energy equation: $\frac{1}{2}mu^2 - \frac{1}{2}mag = mg(a\cos\theta + a\cos\theta)$, so $u^2 = ag(4\cos\theta + 1)$.	**M1 A1**
Equate expressions for u^2.	**M1**
$\cos\theta = \frac{1}{2}$, $\theta = 60°$ (CAO).	**A1**
$u = \sqrt{3ga}$ (CAO).	**A1**
	Total: 8

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Q28

9231/33 • May/June 2021 • Question 3

↑ Back to contents

Mark scheme.

For B: $T = mg$ (may be embedded).	**B1**
For A: $R + T\cos\theta = 3mg$ (all 3 terms, allow sign errors).	**M1**
Use given R to obtain $\cos\theta = \frac{3}{5}$. (b)	**A1**
$T\sin\theta = \dfrac{3mv^2}{r}$ (may be seen in part (a)).	**M1**
$r = AR\sin\theta$.	**B1**
Combine to give $AR = \frac{3a}{4}$, so $BR = \frac{1}{4}a$.	**A1**
	Total: 6

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Q29

9231/33 • May/June 2021 • Question 4

↑ Back to contents

Mark scheme.

Energy equation with 2 KE terms and a two-part GPE term: $\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{2}\right)^2 = mg(a\cos\theta + a\sin\theta)$, allow cos/sin mix.	**M1**
$\frac{3}{4}u^2 = 2ag(\cos\theta + \sin\theta)$.	**A1**
At B, tension in string is zero, so $mg\sin\theta = \dfrac{m(u/2)^2}{a}$, i.e. $u^2 = 4ag\sin\theta$ (N2L).	**B1**
Eliminate u^2.	**DM1**
$\tan\theta = 2$ (OE).	**A1**
At A: $T - mg\cos\theta = \dfrac{mu^2}{a}$ (N2L).	**B1**
Substitute to find $T = \dfrac{9\sqrt{5}}{5}mg$ ($\approx 4.02mg$).	**M1 A1**
	Total: 8

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Q30

9231/31, 9231/33 • October/November 2021 • Question 1

↑ Back to contents

Mark scheme.

$T = \dfrac{3mgx}{a}$ (their tensions equated to obtain a quadratic equation, CAO).	**B1**
$T = \dfrac{4mga}{3(a+x)}$.	**B1**
$9x^2 + 9ax - 4a^2 = 0$ leading to $(3x - a)(3x + 4a) = 0$.	**M1**
$x = \dfrac{1}{3}a$.	**A1**

Total: 4

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Q31

9231/31, 9231/33 • October/November 2021 • Question 6

↑ Back to contents

Mark scheme.

(a)	
At A: $T_A - mg\cos\theta = m \times \frac{5ag}{a}$ (N2L).	**B1**
At B: $T_B + mg\cos\theta = m \times \frac{v^2}{a}$ (N2L).	**B1**
$\frac{1}{2}m \times 5ag - \frac{1}{2}mv^2 = mga \times 2\cos\theta$ (energy equation, correct number of terms).	**M1**
$v^2 = 5ag - 4ga\cos\theta$ (accept multiplied by m and/or divided by a).	**A1**
Use ratio of tensions $= 9 : 5$, simplify to an expression in $\cos\theta$.	**M1**
$\cos\theta = \frac{2}{5}$ (CAO).	**A1**
(b)	
Greatest speed at lowest point: $-\frac{1}{2}m \times 5ag + \frac{1}{2}mV^2 = mga \times (1 - \cos\theta)$ (energy equation including lowest point, correct number of terms).	**M1**
$V = \sqrt{\frac{31ag}{5}}$ (FT their $\cos\theta$ from part (a)).	**A1 FT**
	Total: 8

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Q32

9231/32 • October/November 2021 • Question 3

↑ Back to contents

Mark scheme.

(a)	
$T = 3mg$ and $T\cos\theta = mg$ (must see both of these separately).	**M1**
Combining, $\cos\theta = \frac{1}{3}$ (at least one step of working, AG).	**A1**
(b)	
$\left(\cos\theta = \frac{a-x}{x}, \text{ where } x = AR\right)$: $AR = \frac{3}{4}a$ or $BR = \frac{1}{4}a$ or radius $= \frac{a}{\sqrt{2}}$ $\left(\sin\theta = \frac{\sqrt{8}}{3}\right)$.	**B1**
$T\sin\theta = \frac{mv^2}{r}$.	**M1**
Combining to find an equation in v^2, a and g only.	**DM1**
$v^2 = 2ga$, $v = \sqrt{2ga}$.	**A1**
	Total: 6

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Q33

9231/32 • October/November 2021 • Question 7

↑ Back to contents

Mark scheme.

(a)	
Coordinates of A: $x = a\sin 60$, $y = a - a\cos 60$.	**B1**
$\frac{a}{2} = \frac{a\sqrt{3}}{2}\sqrt{3} - \frac{g\left(\frac{a\sqrt{3}}{2}\right)^2}{2V^2 \cdot \frac{1}{4}}$ (substitute their (x, y) into correct trajectory equation).	**M1**
Rearrange to find V^2.	**M1**
$V^2 = \frac{3}{2}ag$, $V = \sqrt{\frac{3}{2}ag}$.	**A1**
(b)	
$\frac{1}{2}mu^2 - \frac{1}{2}mV^2 = mga(1 + \cos 60)$ (energy equation).	**M1**
$u^2 = \frac{9}{2}ag$ (u is the speed at P).	**A1**
$T - mg = \frac{m}{a}u^2$ (N2L).	**M1**
$T = \frac{11}{2}mg$.	**A1**
	Total: 8

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Q34

9231/31, 9231/32 • May/June 2022 • Question 2

↑ Back to contents

Mark scheme.

Energy equation: $-\frac{1}{2}mv^2 + \frac{1}{2}m \times 3ga = mga(\cos\theta + \cos\alpha)$.	**M1 A1**
N2L: $(T+)mg\cos\theta = \dfrac{m}{a}v^2$.	**B1**
Combine to find $\cos\theta$: $ag\cos\theta = 3ga - 2ga\cos\theta - 2ga \times \dfrac{4}{5}$.	**M1**
$\cos\theta = \dfrac{7}{15}$.	**A1**
	Total: 5

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Q35

9231/31, 9231/32 • May/June 2022 • Question 5

↑ Back to contents

Mark scheme.

For A: N2L horizontal, $T\sin\theta = mr\omega^2$.	**M1**
Correct expression for radius: $r = 3a\sin\theta$.	**B1**
$T = m \times 3a\omega^2$.	**A1**
Similarly, for B: N2L horizontal, $T\cos\theta = \frac{3}{4}m \times r \times k^2\omega^2$.	**M1**
$T = \frac{3}{4}mak^2\omega^2$.	**A1**
Equate expressions for T: $m \times 3a\omega^2 = \frac{3}{4}mak^2\omega^2$.	**M1**
$k^2 = 4$, $k = 2$. Alternative method: use $T\cos\theta = mg$, $T\sin\theta = mr\omega^2$ for A (M1 N2L horizontal and vertical), $r = 3a\sin\theta$ (B1), combine to obtain $\omega^2 = \frac{5}{12}\times\frac{g}{a}$ (A1); similarly for B: $T\cos\theta = \frac{3}{4}m\times r\times k^2\omega^2$ (M1), $T = \frac{3}{4}mak^2\omega^2$ (A1), substitute for T and ω^2 (M1), $k^2 = 4$, $k = 2$ (A1).	**A1**
	Total: 7

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Q36

9231/33 • May/June 2022 • Question 4

↑ Back to contents

Mark scheme.

Energy equation with all necessary terms, GPE terms must be resolved, allow sin/cos mix, allow sign error: $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga(\cos\theta + \cos\alpha)$.	**M1**
$\frac{1}{2}m(2u)^2 - \frac{1}{2}mu^2 = mga(\cos\theta + \cos\alpha)$ ($2u$ may be substituted later; implied by $\frac{3}{2} \times \frac{2}{3}ag = ga(\cos\theta + \cos\alpha)$).	**A1**
At A: $T + mg\cos\theta = \dfrac{m}{a}u^2$ (N2L).	**B1**
Also, $10T - mg\cos\alpha = \dfrac{m}{a}4u^2$ (N2L and use of tension $10T$).	**B1**
Use all three (two N2L and energy) equations to find T in terms of m and g only. Might see $9T - mg(\cos\theta + \cos\alpha) = \dfrac{3m}{a} \times \dfrac{2}{3}ga$; $(\cos\theta + \cos\alpha) = 1$; $(10\cos\theta + \cos\alpha) = 4$.	**M1**
$T = \dfrac{1}{3}mg$. (b)	**A1**
Substitute back, $10 \times \dfrac{1}{3}mg - mg\cos\alpha = \dfrac{4m}{a} \times \dfrac{2}{3}ga$ (any appropriate method to obtain $\cos\alpha$).	**M1**
$\cos\alpha = \dfrac{2}{3}$.	**A1**
	Total: 8

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Q37

9231/31, 9231/33 • October/November 2022 • Question 1

↑ Back to contents

Mark scheme.

Use $F = ma$: $20 = \dfrac{2v^2}{0.6}$ OR $20 = 2 \times 0.6\omega^2$	**M1**
$v^2 = 6$ OR $\omega^2 = \dfrac{50}{3}$	**A1**
FT Number of revolutions per min $= \dfrac{60v}{0.6 \times 2\pi}$ OR $\dfrac{60\omega}{2\pi}$	**A1**
	Total: 3

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Q38

9231/31, 9231/33 • October/November 2022 • Question 5

↑ Back to contents

Mark scheme.

Energy equation: $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga\cos\theta$, with $kag = \frac{1}{3}ag + 2ag\cos\theta$	**B1**
N2L at Q: $T - mg\cos\theta = \dfrac{m}{a}v^2$, so $\frac{11}{6}mg - mg\cos\theta = \frac{m}{a} \cdot kag$, i.e. $\frac{11}{6} - \cos\theta = k$	**B1**
Solve simultaneously.	**M1**
Both values.	**A1**

$$\boxed{k = \frac{4}{3},\ \cos\theta = \frac{1}{2}}$$

(b)

Initial speed (upward) $= \sqrt{kag}\sin\theta$	**B1**
Use $v^2 = u^2 + 2as$: $0 = (\sqrt{kag}\sin\theta)^2 - 2gs$	**M1**
$s = \frac{1}{2}a$	**A1**
FT Height above lowest point $= s + a - a\cos\theta = \frac{1}{2}a + a - \frac{1}{2}a = a$	**A1**
	Total: 8

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Q39

9231/32 • October/November 2022 • Question 1

↑ Back to contents

Mark scheme.

When string goes slack, $mg\cos\beta = \dfrac{m}{a}v^2$, so $v^2 = ag\cos\beta$.	**B1**
Energy equation: $\frac{1}{2}m \cdot 3ag - \frac{1}{2}mv^2 = mg(a\cos\alpha + a\cos\beta)$	**B1**
Combine: $u^2 - ag\cos\beta = 2ag\left(\cos\beta + \frac{2}{3}\right)$	**M1**
$\cos\beta = \dfrac{u^2 - \frac{4}{3}ag}{3ag} = \dfrac{5}{9}$	**A1**
	Total: 4

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Q40

9231/32 • October/November 2022 • Question 6

↑ Back to contents

Mark scheme.

$T = \frac{6}{7}mg$ (may be implied).	**B1**
$T\sin\theta = mr\omega^2 = mh\tan\theta \times \omega^2$	**B1**
Radius of circle $= h\tan\theta$, so $\omega^2 = \dfrac{6g}{7h}\cos\theta$.	**B1**
Second scenario, equivalent result: $\frac{9}{4}\omega^2 = \dfrac{6g}{7h}\cos\alpha$	**M1**
Equate: $\dfrac{6g}{7h}\cos\theta = \frac{4}{9} \times \dfrac{6g}{7h}\cos\alpha$, giving $\cos\theta = \dfrac{4}{9}\cos\alpha$ (AG) **(b)**	**A1**
First scenario: $N + T\cos\theta = mg$; second scenario: $\frac{1}{2}N + T\cos\alpha = mg$.	**B1**
Equate: $mg - \frac{6}{7}mg\cos\theta = 2mg - \frac{12}{7}mg\cos\alpha$	**M1**
$\cos\alpha = \dfrac{3}{4}$	**A1**
$N = \dfrac{5}{7}mg$	**A1**
	Total: 9

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Q41

9231/31, 9231/32 • May/June 2023 • Question 3

↑ Back to contents

Mark scheme.

At B: $mg\sin\theta = \dfrac{m \cdot 4ag}{5a}$. Allow cos instead of sin for M1. Do not award until tension = 0 used. Mass must be seen. No sign error.	**M1**
$\sin\theta = \dfrac{4}{5}$. (b)	**A1**
At A: $T - mg\cos\theta = \dfrac{mu^2}{a}$.	**B1**
Energy equation with 4 terms, dimensionally correct: $\dfrac{1}{2}mu^2 - \dfrac{1}{2}m \times \dfrac{4ag}{5} = mga(\cos\theta + \sin\theta)$. Mass must be present, allow sign errors. Must see $\frac{1}{2}$ in the KE terms.	**M1 A1**
Solve to find T (complete method leading to an expression in mg for T).	**M1**
$T = \dfrac{21}{5}mg$ (CWO).	**A1**
	Total: 7

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Q42

9231/31, 9231/32 • May/June 2023 • Question 5

↑ Back to contents

Mark scheme.

Hooke's law: $T_1 = \frac{\lambda mg}{a}(x-a)$ or $T_2 = \frac{\lambda mg}{a}\left(\frac{3}{4}x - a\right)$.	**B1**
Also $T_1 = \frac{mv^2}{x}$ and equate: $\frac{mv^2}{x} = \frac{\lambda mg}{a}(x-a)$, i.e. $v^2 = \frac{\lambda gx(x-a)}{a}$. Dimensionally correct terms.	**M1**
Similarly: $\frac{\lambda mg\left(\frac{3x}{4}-a\right)}{a} = \frac{2m\left(\frac{1}{2}v\right)^2}{\frac{3}{4}x}$, i.e. $v^2 = \frac{3\lambda gx}{2a}\left(\frac{3}{4}x - a\right)$. Must have $\frac{1}{2}v$ and $\frac{3x}{4}$ on the RHS; their dimensionally correct T_2.	**M1**
Equate expressions for v^2 and solve for x in terms of a.	**M1**
$x = 4a$ (WWW). [SC B3 for answer of $4a$ using λ instead of λmg.] (b)	**A1**
$\lambda = \frac{a}{xg(x-a)}v^2$ or $\lambda = \frac{2a}{3xg\left(\frac{3}{4}x-a\right)}v^2$, and substitute $x = 4a$, $v = \sqrt{12ag}$ (FT their expression for x).	**M1**
$\lambda = 1$ (CAO).	**A1**
	Total: 7

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Q43

9231/33 • May/June 2023 • Question 1

↑ Back to contents

Mark scheme.

Energy equation, 4 terms, dimensionally correct, mass must be present, allow sign errors, allow sin in both terms on RHS: $\frac{1}{2}m \cdot 3ag - \frac{1}{2}mv^2 = mga(\cos\alpha + \cos\theta)$.	**M1**
N2L: $mg\cos\theta = \frac{mv^2}{a}$ (may include tension initially but not awarded until tension $= 0$ used).	**B1**
Dependent on tension $= 0$ and on an energy equation, eliminate v^2: $\frac{3}{2}mag - \frac{1}{2}m \cdot ag\cos\theta = mga\left(\frac{4}{5} + \cos\theta\right)$, i.e. $\frac{3}{2}\cos\theta = \frac{7}{10}$.	**M1**
$\cos\theta = \frac{7}{15}$. [If no m in energy equation and no further errors, award SC B2 for correct final answer.]	**A1**
	Total: 4

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Q44

9231/33 • May/June 2023 • Question 5

↑ Back to contents

Mark scheme.

$\uparrow T\cos\theta = mg$.	**B1**
$\rightarrow T\sin\theta = \dfrac{mv^2}{r} = m \times \dfrac{27ag}{4r}$.	**B1**
$r = 12a\tan\theta$ used.	**M1**
Divide: $\tan\theta = \dfrac{27}{4\times 12\tan\theta}$, so $\tan\theta = \dfrac{3}{4}$ (finds value for $\tan\theta$ OE, reduces to equation in θ or x, no k).	**M1**
$r = 9a$, extension of string $= 3a$. (b)	**A1**
Hooke's law: $T = \dfrac{kmg(L-12a)}{12a}$.	**B1**
Eliminate T: $\dfrac{kmg(L-12a)}{12a} = \dfrac{mgL}{12a}$.	**M1**
$k = \dfrac{L}{L-12a} = 5$.	**A1**
	Total: 8

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Q45

9231/31, 9231/33 • October/November 2023 • Question 6

↑ Back to contents

Mark scheme.

Angular speeds of P and Q are equal, so $\dfrac{v_Q}{x} = \dfrac{v_P}{3a-x}$	M1	**(a)**
$v_Q = \dfrac{2x}{3a-x}\sqrt{ag}$ (AG)	A1	
For P: $T + mg\cos 60° = \dfrac{m \times 4ag}{3a-x}$	B1	**(b)**
For Q: $T - mg\cos 60° = \dfrac{mv_Q^2}{x}$	B1	
Eliminate T: $-mg\cos 60° + \dfrac{m \times 4ag}{3a-x} = mg\cos 60° + \dfrac{mv_Q^2}{x}$	M1	
$\dfrac{m \times 4ag}{3a-x} = 1 + \dfrac{mx \times 4ag}{(3a-x)^2}$; $4a(3a-x) = (3a-x)^2 + 4ax$, giving $x^2 + 2ax - 3a^2 = 0$	M1	
$(x-a)(x+3a) = 0$, $x = a$	A1	
Gain in KE of P: $\frac{1}{2}m(4ag - u^2)$; Loss in KE of Q: $\frac{1}{2}m\left(\left(\frac{u}{2}\right)^2 - v_Q^2\right)$	B1	**(c)**
Loss in GPE of $P = mg(3a-x)(1-\cos 60°)$ $(= mga)$; Gain in GPE of $Q = mgx(1-\cos 60°)$ $(= \frac{1}{2}mga)$	B1FT	
Energy equation: $\frac{1}{2}m(4ag-u^2) - \frac{1}{2}m\left(\left(\frac{u}{2}\right)^2 - v_Q^2\right) = -mgx(1-\cos 60°) + mg(3a-x)(1-\cos 60°)$	M1	
Simplify: $4ag - \frac{5}{4}u^2 + ag = ag$, giving $u^2 = \frac{16}{5}ag$, $u = \frac{4}{5}\sqrt{5ag}$	A1	
		Total: 11

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Q46

9231/32 • October/November 2023 • Question 1

↑ Back to contents

Mark scheme.

$T\cos\theta = mg$	B1
$T\sin\theta = \dfrac{mv^2}{a\sin\theta}$	B1
Eliminate T and substitute for θ	M1
$v = \sqrt{\frac{5}{6}ag}$	A1
	Total: 4

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Q47

9231/32 • October/November 2023 • Question 5

↑ Back to contents

Mark scheme.

At A: $R_A - mg\cos\alpha = \dfrac{mv_A^2}{a}$	B1	**(a)**
At B: $-R_B + mg\cos\theta = \dfrac{mv_B^2}{a}$	B1	
Energy: $\frac{1}{2}mv_A^2 - \frac{1}{2}mv_B^2 = mga(\cos\alpha + \cos\theta)$	M1A1	
Eliminate velocities, use $R_B = \frac{1}{6}R_A$ and substitute angle values: $6R_B - \frac{3}{5}mg + R_B - \frac{4}{5}mg = 2mg \times \frac{7}{5}$	M1	
$R_B = \frac{3}{5}mg$	A1	
From first equation in (a): $6 \times \frac{3}{5}mg - \frac{3}{5}mg = \dfrac{mv_A^2}{a}$	M1	**(b)**
$v_A^2 = 3ag$, so $k = 3$	A1	
	Total: 8	

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Q48

9231/31, 9231/32 • May/June 2024 • Question 5

↑ Back to contents

Mark scheme.

M1: For A: $F_A - T = m \times a\omega^2$ (only allow sign errors).	**(a)**
$F_A = \mu mg = \frac{1}{5}mg$ (accept with g replaced by 10).	**B1**
Combine: $T = \frac{1}{5}mg - \frac{4}{25}mg$ (to reach an equation in T and mg only).	**M1**
$T = \dfrac{1}{25}mg$ (CAO).	**A1**
M1: For B: $F_B + T = km \times 2a\omega^2$ (only allow sign errors).	**(b)**
$F_B = \mu kmg = \frac{1}{5}kmg$ and combine to find k.	**M1**
$k = \dfrac{1}{3}$.	**A1**
	Total: 7

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Q49

9231/33 • May/June 2024 • Question 2

↑ Back to contents

Mark scheme.

B1: Hooke's law: $T = \dfrac{2mgx}{a}$ ($\frac{2mgx}{a}$ seen anywhere).	**(a)**
$T = \dfrac{\frac{mga}{2}}{a - \frac{1}{4}a + x}$ (RHS seen anywhere; may be in terms of radius or extended length).	**B1**
Equate: $\dfrac{2mgx}{a} = \dfrac{\frac{mga}{2}}{\frac{3a}{4} + x}$, giving $4x^2 + 3ax - a^2 = 0$ (equate two expressions for T and obtain a simplified homogeneous quadratic equation).	**M1**
$x = \dfrac{a}{4}$ (single correct answer only).	**A1**
B1: $T = kmg$ seen anywhere in an equation (may be seen in part (a); note that no response in part (b) can earn B1 if kmg seen in part (a)).	**(b)**
$T = \dfrac{2mgx}{a}$, $k = \dfrac{1}{2}$ (CWO; part (a) needs to be correct).	**B1**

Total: 6

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Q50

9231/33 • May/June 2024 • Question 3

↑ Back to contents

Mark scheme.

B1: At lowest point, $T - mg = \dfrac{mu^2}{a}$ (condone r used consistently instead of a throughout).	**(a)**
When string makes angle θ with upward vertical, $S + mg\cos\theta = \dfrac{mv^2}{a}$.	**B1**
Energy: $\dfrac{1}{2}mu^2 - \dfrac{1}{2}mv^2 = mga(1+\cos\theta)$ (must include m; allow $\sin\theta$ instead of $\cos\theta$ for this mark, allow sign errors).	**M1**
Eliminate u^2 and v^2 (need to see at least one line of working).	**M1**
$S = T - 3mg(1+\cos\theta)$ (AG).	**A1**
M1: When string goes slack, $S = 0$ so $T = 3mg(1+\cos\theta)$ (may use $v^2 = ag\cos\theta$ substituted into energy equation).	**(b)**
But $T = mg + \dfrac{mu^2}{a} = mg + 4mg = 5mg$, so $\cos\theta = \dfrac{2}{3}$.	**A1**
	Total: 7

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Q51

9231/31, 9231/32 • May/June 2024 • Question 7

↑ Back to contents

Mark scheme.

*M1: Energy: $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 - mga(1 - \cos\theta)$ (m must be present, dimensionally correct, no missing terms; allow sin instead of cos; allow sign errors).	**(a)**
N2L: $mg\cos\theta = \frac{mv^2}{a}$ (no reaction when P loses contact).	**B1**
Eliminate v^2.	**DM1**
$\cos\theta = \frac{u^2 + 2ag}{3ag}$ (AG).	**A1**
*B1: Vertical component of velocity of P when it leaves the sphere: $v\sin\theta = \sqrt{\frac{55ag}{216}}$ (must not come from u).	**(b)**
$V^2 = (v\sin\theta)^2 + 2g \times a(1 + \cos\theta)$ (use of $v^2 = u^2 + 2as$; allow $\sin\theta$ for $\cos\theta$; allow sign errors).	**DM1**
$V = \sqrt{\frac{847ag}{216}}$ (AEF).	**A1**
M1: $t = \frac{1}{g}\left(\sqrt{\frac{847ag}{216}} - \sqrt{\frac{55ag}{216}}\right)$.	**(c)**
$\frac{1}{6}(\sqrt{847} - \sqrt{55})\sqrt{\frac{a}{6g}} = 1.48\sqrt{\frac{a}{g}}$.	**A1**
	Total: 9

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Q52

9231/31, 9231/33 • October/November 2024 • Question 2

↑ Back to contents

Mark scheme.

At B: $T + mg\sin\theta = \dfrac{mv^2}{a}$.	**B1**
Energy A to B: $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mga(\cos\theta + \sin\theta)$.	**M1A1**
Substitute for u and θ to find T: $T = mg\left(5 - 2\times\frac{4}{5} - 3\times\frac{3}{5}\right)$.	**M1**
$T = \dfrac{8}{5}mg$.	**A1**
	Total: 5

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Q53

9231/31, 9231/33 • October/November 2024 • Question 6

↑ Back to contents

Mark scheme.

(a)	
At P: $T_1 \cos\alpha = T_2 \cos\beta + 0.05g$.	**B1**
At Q: $T_2 \cos\beta = 0.04g$ (OR whole system: $T_1 \cos\alpha = 0.09g$).	**B1**
$T_1 = 0.15g = 1.5\,\text{N}$.	**B1**
(b)	
$T_1 \sin\alpha - T_2 \sin\beta = 0.05 \times 0.8\omega^2$ (allow sin/cos mix).	**M1**
$T_2 \sin\beta = 0.04 \times 1.4\omega^2$.	**M1**
$T_1 \sin\alpha = 0.05 \times 0.8\omega^2 + 0.04 \times 1.4\omega^2$, $\omega^2 = 12.5$, $\omega = \frac{5}{2}\sqrt{2}$.	**A1**
(c)	
$T_2 \cos\beta = 0.04g$ and $T_2 \sin\beta = 0.04 \times 1.4\omega^2$; divide: $\tan\beta = \frac{7}{4}$ (from parts (a) and (b)).	**M1**
$\beta = 60.3°$.	**A1**
	Total: 8

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Q54

9231/32 • October/November 2024 • Question 1

↑ Back to contents

Mark scheme.

Hooke's law: $T = \frac{100}{0.8}x$.	**B1**
N2L: $T = 2 \times (0.8 + x) \times 5^2$.	**B1**
Equate and solve: $50(0.8 + x) = 125x$, $x = 0.533$ $(= \frac{8}{15})$.	**B1**
	Total: 3

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Q55

9231/32 • October/November 2024 • Question 6

↑ Back to contents

Mark scheme.

(a)	
For P to lowest point L: Energy $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(1-\cos\theta)$, giving $v^2 = \frac{28}{9}ag - 2ag\cos\theta$.	**M1***
From L to string goes slack: Energy $\frac{1}{2}mv^2 = \frac{1}{2}mw^2 + \frac{4a}{9}mg(1+\cos\theta)$, giving $w^2 = \frac{20}{9}ag - \frac{26}{9}ag\cos\theta$.	**M1***
Both equations correct, allow unsimplified.	**A1**
When string goes slack: $mg\cos\theta = \frac{mw^2}{\frac{4a}{9}}$.	**B1**
Equate expressions for w^2 to find a value for $\cos\theta$.	**DM1**
$\cos\theta = \dfrac{2}{3}$.	**A1**
(b)	
Tension before: $T_1 - mg = \dfrac{mv^2}{a}$; tension after: $T_2 - mg = \dfrac{mv^2}{\frac{4}{9}a}$ (EITHER equation, dimensionally correct).	**M1**
$v^2 = \dfrac{16}{9}ag$, $T_1 = \dfrac{25}{9}mg$, $T_2 = 5mg$ (EITHER tension correct).	**A1**
Find the other tension (from a valid equation) and find ratio of tensions.	**M1**
Ratio is $5:9$.	**A1**
	Total: 10

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Q56

9231/31, 9231/32 • May/June 2025 • Question 3

↑ Back to contents

Mark scheme.

(a)	
Use $T = \dfrac{\lambda x}{l}$ to find x: $8 = \dfrac{32x}{2}$ leading to $x = 0.5$.	**M1**
Hence $\lvert PA \rvert = 2.5$ and $\sin\alpha = \dfrac{1.5}{2.5} = 0.6$. (AG, shown convincingly.)	**A1**
(b)	
Resolve vertically: $R = 1.6g - 8\cos\alpha$; resolve horizontally: $mr\omega^2 = 8\sin\alpha + \mu R$. (Correct number of terms and signs, dimensionally correct.)	***M1**
Both equations correct.	**A1**
Eliminate R: $8\left(\frac{3}{5}\right) + 0.5\left(1.6g - 8\left(\frac{4}{5}\right)\right) = 1.6 \times 1.5\omega^2$.	**DM1**
$\omega^2 = 4$ leading to $\omega = 2$.	**A1**
Number of revolutions per minute $= \dfrac{60\omega}{2\pi} = 19.1$ (accept 19).	**A1**
	Total: 7

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Q57

9231/31, 9231/32 • May/June 2025 • Question 7

↑ Back to contents

Mark scheme.

(a)	
N2L towards centre at A and B: $R_A = \dfrac{mu^2}{a}$; $R_B = \dfrac{mv^2}{a} - mg\cos\theta$.	**B1**
Use $R_B = \frac{1}{2}R_A$: $\dfrac{mu^2}{2a} = \dfrac{mv^2}{a} - mg\cos\theta$ leading to $v^2 = \frac{1}{2}u^2 + agk$.	***M1**
Energy equation: $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + magk$ leading to $v^2 = u^2 - 2agk$.	***M1**
Combine N2L and energy equations: $\frac{6}{5}ag - 2agk = \frac{3}{5}ag + agk$.	**DM1**
$k = \frac{1}{5}$.	**A1**
(b)	
$v^2 = 0$ leading to $u^2 = \frac{2}{5}ag$, $u = \sqrt{\frac{2}{5}ag}$ (FT their $\sqrt{2agk}$).	**B1 FT**
(c)	
Complete method to find v^2 using vertical/horizontal motion B to C (must use $2a\sin\theta$, not $2a$); FT their k: $v^2 = \dfrac{ag}{\cos\theta}$ (may see $v^2 = \frac{ag}{k}$ or $v^2 = 5ag$).	**M1**
$v^2 = \dfrac{ag}{\cos\theta}$.	**A1**
Use their energy equation from (a) ($v^2 = u^2 - 2agk$), substitute for v^2: $u^2 = \dfrac{ag}{\cos\theta} + 2agk$.	**M1**
$u^2 = \frac{27}{5}ag$, $u = \sqrt{\frac{27}{5}ag}$ (CAO).	**A1**
	Total: 10

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Q58

9231/33 • May/June 2025 • Question 1

↑ Back to contents

Mark scheme.

Both equations SOI: $T\cos\theta = mg$; $T\sin\theta = m\omega^2 r$.	**B1**
Combine and use $r = a\sin\theta$: $\frac{5}{3}mg\sin\theta = m\omega^2 a\sin\theta$ (allow sin/cos mix; must have trig ratio both sides).	**M1**
$\omega = \sqrt{\frac{5g}{3a}}$ (OE, CAO).	**A1**
	Total: 3

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Q59

9231/33 • May/June 2025 • Question 5

↑ Back to contents

Mark scheme.

(a)	
Energy equation, all 3 terms (allow sin/cos and sign errors): $\frac{1}{2}mv^2 + mgr\cos\alpha = \frac{1}{2}m\left(\frac{3}{2}gr\right)$, i.e. $v^2 = \frac{3}{2}gr - 2gr\cos\alpha$.	**M1**
N2L at B: $mg\cos\alpha = \dfrac{mv^2}{r}$.	**B1**
Combine to eliminate v: $mg\cos\alpha = m\left(\frac{3}{2}g - 2g\cos\alpha\right)$.	**M1**
$\cos\alpha = \frac{1}{2}$, $\alpha = 60°$.	**A1**
(b)	
$v^2 = \frac{1}{2}gr$ (FT their 60°).	**B1 FT**
$0 = (\sin 60°\sqrt{v^2})^2 - 2gh$ (allow sign error).	**M1**
$h = \frac{3}{16}r$ (FT their 60° and v^2).	**A1 FT**
Total height $= \frac{3}{16}r + r\cos 60 = \frac{11}{16}r$.	**A1**
	Total: 8

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Q60

9231/34 • May/June 2025 • Question 4

↑ Back to contents

Mark scheme.

For B: $T = mg$ (must be seen explicitly).	**B1**
For A vertically: $R\sin\alpha = T\cos\alpha + mg$ (allow sin/cos mix, sign error; allow mg instead of T).	***M1**
For A horizontally: $R\cos\alpha + T\sin\alpha = mr\omega^2$ (allow sin/cos mix, sign error).	***M1**
$r = h\tan\alpha$ (seen anywhere).	**B1**
Combine equations to find ω^2 in terms of g, h only (may see $R = 3mg$ or $R = \dfrac{mg(1+\cos\alpha)}{\sin\alpha}$).	**DM1**
$\omega = 2\sqrt{\dfrac{g}{h}}$. (If $T = mg$ not seen explicitly, max B0 M1 M1 B1 DM1 A0.)	**A1**
	Total: 6

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Q61

9231/34 • May/June 2025 • Question 7

↑ Back to contents

Mark scheme.

Energy, highest to lowest: $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = 2amg$, giving $v^2 = u^2 - 4ag$ (all terms present, dimensionally correct, allow sign error).	**M1**
Energy, start (at angle θ) to lowest: $\frac{1}{2}mu^2 - \frac{1}{2}m(4ag) = mg(a - a\cos\theta)$, giving $u^2 = ag(6 - 2\cos\theta)$.	**M1**
Energy, start to highest: $\frac{1}{2}m(4ag) - \frac{1}{2}mv^2 = mg(a + a\cos\theta)$, giving $v^2 = 2ag(1 - \cos\theta)$ (one fully correct energy equation).	**A1**
N2L at lowest point: $T_L - mg = \dfrac{mu_L^2}{a}$ (may see $T_L = mg(7 - 2\cos\theta)$).	**B1**
N2L at highest point: $T_H + mg = \dfrac{mv_H^2}{a}$ (may see $T_H = mg(1 - 2\cos\theta)$, must use a different speed).	**B1**
$T_L = 11T_H$ (SOI, correct way around).	**B1**
Combine to find equation involving $\cos\theta$ only.	**M1**
$\cos\theta = \frac{1}{5}$.	**A1**
	Total: 8

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Q62

9231/31, 9231/33 • October/November 2025 • Question 2

↑ Back to contents

Mark scheme.

N2L horizontally and vertically for the first case: $N_1 \cos\theta_1 = mg$, $N_1 \sin\theta_1 = m(r\sin\theta_1)\omega_1^2 \Rightarrow \frac{5}{4}g = r\omega_1^2$.	**B1**
N2L horizontally and vertically for the second case: $N_2 \cos\theta_2 = mg$, $N_2 \sin\theta_2 = m(r\sin\theta_2)\omega_2^2 \Rightarrow \frac{5}{3}g = r\omega_2^2$.	**B1**
Combine to find ratio: $\dfrac{\frac{5}{4}g}{\frac{5}{3}g} = \dfrac{r\omega_1^2}{r\omega_2^2}$.	**M1**
$\dfrac{\omega_1}{\omega_2} = \frac{1}{2}\sqrt{3}$.	**A1**
	Total: 4

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Q63

9231/31, 9231/33 • October/November 2025 • Question 6

↑ Back to contents

Mark scheme.

Dimensionally correct energy equation with a GPE term and two KE terms: $\frac{1}{2}m(17ag) + \frac{3}{2}mag = \frac{1}{2}mv^2$.	**M1**
$v = \sqrt{20ag}$. **(b)**	**A1**
Dimensionally correct equation ($T = 0$ seen or implied): $\frac{mV^2}{a} = T + mg \Rightarrow V = \sqrt{ag}$.	**B1**
Dimensionally correct energy equation with a GPE term and two KE terms: $\frac{1}{2}mw_P^2 = \frac{1}{2}mV^2 + 2mga$.	**M1**
$w_P = \sqrt{5ag}$. **(c)**	**A1**
Momentum conserved (must see masses): $mv = kmw_Q - mw_P$.	**B1**
NEL with consistent signs: $w_P + w_Q = ev\ [= v]$, OR use conservation of kinetic energy.	**B1**
$k = 3$ (must have $e = 1$ seen or implied).	**B1**
	Total: 8

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Q64

9231/34 • October/November 2025 • Question 1

↑ Back to contents

Mark scheme.

Both equations (allow sin/cos mix): For A: $T = mg$; for B vertically: $T\cos\theta = 0.5mg$.	**M1**
Equate and solve for θ, SOI: $\theta = 60°$.	**A1**
For B horizontally (allow sin/cos mix, allow their stated BR): $T\sin\theta = 0.5m(BR\sin\theta)\omega^2$.	**M1**
SOI: $BR\cos\theta = AR$ $[BR = 8a]$.	**B1**
AEF in terms of g and a: $\omega = \frac{1}{2}\sqrt{\frac{g}{a}}$.	**A1**
	Total: 5

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Q65

9231/34 • October/November 2025 • Question 4

↑ Back to contents

Mark scheme.

Accept equality: at top $R \geq 0$, so $\dfrac{mw^2}{a} \geq mg$.	**B1**
Energy inequality with correct number of terms and correct dimensions, accept equality: $\frac{1}{2}mv^2 - \frac{1}{2}mw^2 \geq mga(1 - \cos\theta)$.	**M1**
Fully correct energy inequality, accept equality.	**A1**
Substitute for $\cos\theta$ and w^2: $\frac{1}{2}mv^2 \geq \frac{13}{9}mga$.	**M1**
AG, shown convincingly: $v \geq \frac{1}{3}\sqrt{26ag}$. **(b)**	**A1**
Energy equation with correct number of terms, dimensions and signs (maximum reaction is at bottom): $\frac{1}{2}mv^2 + mga(1 + \cos\theta) = \frac{1}{2}mV^2$ $[V^2 = 5ag]$.	**M1**
$R_{\max} - mg = \dfrac{mV^2}{a}$.	**B1**
$R_{\max} = 6mg$.	**A1**
	Total: 8

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Q66

9231/32 • October/November 2025 • Question 1

↑ Back to contents

Mark scheme.

Correct horizontal equation (allow sin/cos mix): $T_A \cos 45° + T_B \sin 45° = m\omega^2 l \sin 45°$.	**B1**
Correct vertical equation (allow sin/cos mix): $T_A \sin 45° - T_B \cos 45° = mg$.	**B1**
Combine, isolating term in T_A: $2T_A = m\omega^2 l + \sqrt{2}mg$.	**M1**
AEF: $T_A = \frac{1}{2}m(\omega^2 l + \sqrt{2}g)$.	**A1**
	Total: 4

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Q67

9231/32 • October/November 2025 • Question 6

↑ Back to contents

Mark scheme.

One energy equation, all terms present (A to ground): $\frac{1}{2}mu^2 + 2mga = \frac{1}{2}m(3u)^2 \Rightarrow u^2 = \frac{1}{2}ga$.	***M1**
Second energy equation, all terms present (A to B and B to ground).	***M1**
At least one correct energy equation involving $\cos\theta$: $v^2 = u^2 + 2ag(1 - \cos\theta)$; $v^2 = 9u^2 - 2ag(1 + \cos\theta)$.	**A1**
N2L at B: $mg\cos\theta = \dfrac{mv^2}{a} \Rightarrow v^2 = ag\cos\theta$.	**B1**
Eliminate u and v: $ga\cos\theta = \frac{1}{2}ga + 2ag(1 - \cos\theta)$.	**DM1**
$\cos\theta = \frac{5}{6}$.	**A1**
A complete method to find β: horizontally at ground, $v\cos\theta = 3u\cos\beta$.	**M1**
$\beta = 69.0°$.	**A1**
	Total: 8

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Written by Ali Hashir.

Questions available in the companion questions booklet.