$$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

Further Math with Ali Hashir

Matrices

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/11, 9231/12 • May/June 2015 • Question 2

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Mark scheme.

$\begin{vmatrix} 2 & -3 & 4 \\ 3 & -1 & 0 \\ 1 & 2 & k \end{vmatrix} = 0 \Rightarrow 7k = -28 \Rightarrow k = -4$	**M1A1**
Add 1st and 3rd $\Rightarrow 3x - y = 2$ (Same as 2nd) (OE)	**M1**
Set $x = t$ (for example) (OE)	**M1**
$\Rightarrow y = 3t - 2, \quad z = \frac{7}{4}t - \frac{5}{4}$ (OE) – many forms	**A1A1**
	Total: 6

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Q2

9231/11, 9231/12 • May/June 2015 • Question 10

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Mark scheme.

$\begin{pmatrix} 2 & 2 & -3 \\ 2 & 2 & 3 \\ -3 & 3 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -3 \end{pmatrix} \Rightarrow$ eigenvalue is -3.	**M1A1**
$\lambda = 4$: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 2 & -3 \\ -3 & 3 & -1 \end{vmatrix} = \begin{pmatrix} 7 \\ 7 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ (Or by equations)	**M1A1**
$\lambda = 6$: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 2 & -3 \\ -3 & 3 & -3 \end{vmatrix} = \begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix} \sim \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$	**A1**
$\mathbf{P} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 0 & -2 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{pmatrix}$	**B1B1**
$\mathbf{B} = \mathbf{QAQ}^{-1} = \mathbf{QPAP}^{-1}\mathbf{Q}^{-1}; = \mathbf{QPA}(\mathbf{QP})^{-1}$	**M1;A1**
	B1
$\mathbf{QP} = \begin{pmatrix} -2 & 15 & -17 \\ -1 & 5 & -7 \\ 0 & 3 & -3 \end{pmatrix}$ (CAO)	
Eigenvalues are $-3, 4$ and 6 (same as those for $\mathbf{A}$). (CAO)	**B1**
Eigenvectors are: $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 15 \\ 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 17 \\ 7 \\ 3 \end{pmatrix}$.	**B1**
	Total: 12

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Q3

9231/13 • May/June 2015 • Question 11

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Mark scheme.

$\begin{vmatrix} 12 & -4 & 2 \\ -4 & \alpha+9 & 6 \\ 2 & 6 & 7 \end{vmatrix} = 0 \Rightarrow 80\alpha + 80 = 0 \Rightarrow \alpha = -1$	**M1A1**
	A1
(i) $\begin{vmatrix} 3-\lambda & -4 & 2 \\ -4 & -1-\lambda & 6 \\ 2 & 6 & -2-\lambda \end{vmatrix} = 0$	**M1**
$\Rightarrow \lambda^3 - 63\lambda + 162 = 0$	**M1A1**
$\Rightarrow (\lambda+9)(\lambda-3)(\lambda-6) = 0 \Rightarrow \lambda_1 = 6$ and $\lambda_2 = 3$.	**M1A1**
(ii) $\lambda = -9 \Rightarrow \mathbf{e} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ (Award M1A1 for any one	**M1A1**
$\lambda_1 = 6 \Rightarrow \mathbf{e}_1 = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3 \Rightarrow \mathbf{e}_2 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and A1 for the other two).	**A1**
$\mathbf{Mx} = \mathbf{M}(a\mathbf{e}_1 + b\mathbf{e}_2) = a\mathbf{Me}_1 + b\mathbf{Me}_2$	**B1**
$= a\lambda_1\mathbf{e}_1 + b\lambda_2\mathbf{e}_2$	**B1**
$= 6a\mathbf{e}_1 + 3b\mathbf{e}_2$	**B1**
	Total: 14

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Q4

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 6

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Mark scheme.

$\lambda = 1: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & -8 & 10 \\ 7 & -5 & 7 \end{vmatrix} = \begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ oe	**M1A1**
$\lambda = 3: \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 0 & 0 \\ 10 & -10 & 10 \end{vmatrix} = \begin{pmatrix} 0 \\ 20 \\ 20 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ oe	**A1**
$\begin{pmatrix} 1 & 0 & 0 \\ 10 & -7 & 10 \\ 7 & -5 & 8 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ -2 \end{pmatrix} = -2 \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \Rightarrow \lambda = -2$	**M1A1**
$\mathbf{D} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}$ (or other multiples or permutations).	**B1 B1**
Det $\mathbf{P} = -1$ (or 1 depending on permutation).	**B1**
Adj $\mathbf{P} = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 0 & 0 \\ -2 & 1 & 2 \end{pmatrix} \Rightarrow \mathbf{P}^{-1} = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & -1 & 2 \end{pmatrix}$ (or other permutations).	**M1A1**
	Total: 10

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Q5

9231/11, 9231/12 • May/June 2016 • Question 10

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Mark scheme.

Eigenvalues are: $-2, -1, 1$	**B1**
Eigenvectors are; $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}$ (oe)	**M1A1**
	A1
$\mathbf{P} = \begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix}, \mathbf{D} = \begin{pmatrix}-2&0&0\\0&-1&0\\0&0&1\end{pmatrix}$ or equivalent (in correct order)	**B1B1**
$\mathbf{P}^{-1} = \begin{pmatrix}1&-1&1\\0&1&-1\\0&0&1\end{pmatrix}$	**M1A1**
$\left(\mathbf{P}^{-1}\mathbf{A}\mathbf{P}\right)^n = \mathbf{P}^{-1}\mathbf{A}^n\mathbf{P} = \mathbf{D}^n$	
$\Rightarrow \mathbf{A}^n = \begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix}\begin{pmatrix}(-2)^n&0&0\\0&(-1)^n&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&-1&1\\0&1&-1\\0&0&1\end{pmatrix}$ Accept $\mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$ here.	**M1A1**
$\mathbf{A}^n = \begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix}\begin{pmatrix}(-2)^n&-(-2)^n&(-2)^n\\0&(-1)^n&(-1)^{n+1}\\0&0&1\end{pmatrix}$	**M1**
$= \begin{pmatrix}(-2)^n&-(-2)^n+(-1)^n&(-2)^n+(-1)^{n+1}\\0&(-1)^n&(-1)^{n+1}+1\\0&0&1\end{pmatrix}$	**A1**
	Total: 12

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Q6

9231/13 • May/June 2016 • Question 3

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Mark scheme.

$\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0 \Rightarrow k^3 - 3k + 2 = 0$	**M1A1**
$(k-1)^2(k+2) = 0 \Rightarrow k = 1, -2$	**M1A1**
$k = 1 :\Rightarrow x + y + z = 2$ and $x + y + z = -1 \Rightarrow$ inconsistent.	**B1**
$k = -2 :\Rightarrow x = z - 1$	**M1**
Hence $(x, y, z) = ([-1 + t], t, t)$	**A1**
	Total: 7

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Q7

9231/13 • May/June 2016 • Question 11

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Mark scheme.

$\lambda = 1$: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -3 & -3 \\ -8 & 5 & -3 \end{vmatrix} = \begin{pmatrix} 24 \\ 24 \\ -24 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$	(Or by equations.)	**M1A1**
$\lambda = 4$: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -3 & -3 \\ -8 & 2 & -3 \end{vmatrix} = \begin{pmatrix} 15 \\ 15 \\ -30 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$		**A1**
$\begin{pmatrix} 1 & -3 & -3 \\ -8 & 6 & -3 \\ 8 & -2 & 7 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 9 \\ -9 \end{pmatrix} \Rightarrow \lambda = 9$		**M1A1**
$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & -2 & -1 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{pmatrix}$		**B1B1**
Det $\mathbf{P} = 1$		**B1**
Adj $\mathbf{P} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{pmatrix} \Rightarrow \mathbf{P}^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{pmatrix}$.	(Or by row operations)	**M1A1**
$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$		**B1**
$\mathbf{P}^{-1}\mathbf{AP} = \mathbf{C}^2 \Rightarrow \mathbf{A} = \mathbf{PC}^2\mathbf{P}^{-1} = \mathbf{PCP}^{-1}.\mathbf{PCP}^{-1}$ (i.e. $\mathbf{B} = \mathbf{PCP}^{-1}$)		**M1**
$\mathbf{PC} = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 2 & 3 \\ -1 & -4 & -3 \end{pmatrix} \Rightarrow \mathbf{B} = \begin{pmatrix} 1 & -1 & -1 \\ -2 & 2 & -1 \\ 2 & 0 & 3 \end{pmatrix}$	(Intermediate step required.)	**M1A1**
(Note: $\mathbf{B}$ may not be unique.)		
		Total: 14

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Q8

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 3

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Mark scheme.

$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ soi	**B1B1**
$\mathbf{P}^{-1}\mathbf{AP} = \mathbf{D} \Rightarrow \mathbf{A} = \mathbf{PDP}^{-1}$ soi	**M1**
$\mathbf{P}^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$	**M1A1**
$\mathbf{PD} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$ or $\mathbf{DP}^{-1} = \begin{pmatrix} -1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$	**M1**
$\mathbf{A} = \begin{pmatrix} -1 & 2 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$	**A1**
ALT METHOD: $\mathbf{A}v_1 = -v_1, \mathbf{A}v_2 = v_2, \mathbf{A}v_3 = 2v_3$ **M1, A1** Multiply out, solve each equation **M1, A1, A1, A1** $\mathbf{A} = \begin{pmatrix} -1 & 2 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ **B1✓**	

Total: 7

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Q9

9231/11, 9231/12 • May/June 2017 • Question 4

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Mark scheme.

$\begin{vmatrix} 1 & 3 & k \\ 2 & -1 & -5 \\ 1 & 1 & 2 \end{vmatrix} = 0$ (Using the determinant. Alt method: Uses row operations)	**M1 M1**
$\Rightarrow k = 8$	**A1**
$x + 3y + 8z = 4 \quad (1)$	**M1**
$4x - 2y - 10z = -5 \quad (2)$	
From (1) and (2) obtain $y = -3x$ or other correct expression; $x + y + 2z = 1$ (3); Substitute $x = t$ and $y = -3t$ in (3) to obtain z. (Alternative: Find REF for augmented matrix and form equations (**M1**) Correctly (**A1**))	**A1 FT**
$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} + t \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} \frac{1}{-2} \\ \frac{3}{2} \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ (OE)	**A1**
	Total: 6

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Q10

9231/11, 9231/12 • May/June 2017 • Question 5

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Mark scheme.

Eigenvectors are $\mathbf{i}+2\mathbf{j}+2\mathbf{k}$, $\mathbf{i}+\mathbf{k}$ and $\mathbf{j}+\mathbf{k}$ (OE) for $\lambda = 1, -1$ and -2 respectively.	**M1A1**
(Award **M1A1** for any one correct and **A1** for each of the other two.)	**A1A1**
Eigenvalues for **B** are -1, -3 and -4.	**B1**
Eigenvectors are the same as for **A** respectively.	**B1 FT**
	Total: 6

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Q11

9231/13 • May/June 2017 • Question 10

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Mark scheme.

$\begin{pmatrix} 6 & -8 & 7 \\ 7 & -9 & 7 \\ 6 & -6 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} \Rightarrow \lambda_1 = -2$	**M1A1**
$\lambda_2 = -1 \Rightarrow \mathbf{e}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (OE)	**M1A1**
Det $\mathbf{A} = 10 \Rightarrow \lambda_3 = 5 \Rightarrow \mathbf{e}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (OE)	**B1 B1 B1**
$\mathbf{P} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ (Check for consistency.)	**B1FT**
	B1FT
Det $\mathbf{P} = -1$ (From working) Adj $\mathbf{P} = \begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ -1 & 1 & -1 \end{pmatrix} \Rightarrow \mathbf{P}^{-1} = \begin{pmatrix} -1 & 2 & -1 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$ (Alternative Method: Reduced row echelon form M1, A1)	**M1A1**
$\mathbf{A}^n = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} (-2)^n & 0 & 0 \\ 0 & (-1)^n & 0 \\ 0 & 0 & 5^n \end{pmatrix}\begin{pmatrix} -1 & 2 & -1 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$ (Allow $\mathbf{A}^n = \mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$)	**M1**
$= \ldots = \begin{pmatrix} [5^n + (-1)^n - (-2)^n] & [2.(-2)^n + (-1)^{n+1} - 5^n] & [5^n - (-2)^n] \\ [5^n - (-2)^n] & [2.(-2)^n - 5^n] & [5^n - (-2)^n] \\ [5^n - (-1)^n] & [(-1)^n - 5^n] & [5^n] \end{pmatrix}$	**A1**

Total: 13

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Q12

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 11

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Mark scheme.

$\mathbf{Ae} = \lambda\mathbf{e}$ and $\mathbf{Be} = \mu\mathbf{e}$	**M1A1**
$\mathbf{ABe} = \mathbf{A}\mu\mathbf{e} = \mu\mathbf{Ae} = \mu\lambda\mathbf{e} = \lambda\mu\mathbf{e}$ (AG)	**M1**
$(\lambda+1)(\lambda^2 - 5\lambda + 6) = 0$	**A1**
$(\lambda+1)(\lambda-2)(\lambda-3) = 0$	**A1**
$\lambda = -1, 2, 3.$	**M1**
Eigenvectors are $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ respectively. (Uses either vector product or equations to find eigenvectors)	**A1A1A1**
$\begin{pmatrix} 3 & 6 & 1 \\ 1 & -2 & -1 \\ 6 & 6 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix} \Rightarrow \mu_1 = -3$	**M1**
Similarly, other two eigenvalues of $\mathbf{B}$ are -2 and 4.	**A1**
Eigenvalues of $\mathbf{AB}$ are 3, -4 and 12	**A1**
Corresponding eigenvectors are $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.	**A1**
	Total: 13

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Q13

9231/11, 9231/12 • May/June 2018 • Question 11

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Mark scheme.

e.g. $2\mathbf{e}$. Allow any scalar multiple $\mu\mathbf{e}$ where $\mu \neq 0, 1$	**B1**
Eigenvector: $\mathbf{e}$, Eigenvalue: λ^n. Allow any scalar multiple $\mu\mathbf{e}$ where $\mu \neq 0$. Note: $A^n\mathbf{e} = \lambda^n\mathbf{e}$ SCB1	**B1B1**
Eigenvalues of (diagonal matrix) $\mathbf{A}$: $\lambda = 3, 7, 1$ (Or from characteristic equation: $(\lambda-3)(\lambda-7)(\lambda-1) = 0$)	**B1**
$\lambda = 3$: $\mathbf{e}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & 0 \\ 4 & 8 & -2 \end{vmatrix} = \begin{pmatrix} -8 \\ 4 \\ 0 \end{pmatrix} = t\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$. Uses vector product (or equations) to find corresponding eigenvectors	**M1A1**
$\lambda = 7$: $\mathbf{e}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 0 & 0 \\ 4 & 8 & -6 \end{vmatrix} = \begin{pmatrix} 0 \\ -24 \\ -32 \end{pmatrix} = t\begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix}$	**A1**
$\lambda = 1$: $\mathbf{e}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 6 & 0 \\ 4 & 8 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ -8 \end{pmatrix} = t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	**A1**
Thus $\mathbf{P} = \begin{pmatrix} -2 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 3^n & 0 & 0 \\ 0 & 7^n & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Or correctly matched permutations of columns. FT on non-zero and distinct eigenvalues and vectors	**B1FT** **B1FT**
$\sum_{n=1}^{N} (k^nA^n - k^{n+1}A^{n+1}) = kA - k^{N+1}A^{N+1}$. Method of differences	**M1A1**
$k^{N+1}A^{N+1} \to 0$ as $N \to \infty$ for	**M1**
$-\frac{1}{7} < k < \frac{1}{7}$.	**A1**
	Total: 14

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Q14

9231/13 • May/June 2018 • Question 5

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Mark scheme.

$\mathbf{Ae} = \lambda\mathbf{e}$ SOI	**B1**
$\mathbf{A}^3\mathbf{e} = \mathbf{A}^2(\mathbf{Ae}) = \lambda\mathbf{A}(\mathbf{Ae}) = \lambda^2(\mathbf{Ae}) = \lambda^3\mathbf{e}$. Substitutes for $\mathbf{Ae}$	**M1**
So eigenvalue is λ^3. Special case: states eigenvalue is λ^3 B1	**A1**
Eigenvalues of $\mathbf{A}$ are 2 and 3.	**B1**
Eigenvectors of $\mathbf{A}$ are $\begin{pmatrix}1\\1\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$. AEF (allow any non-zero scalar multiple).	**M1**
So $\mathbf{P} = \begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$. Alt method: Find $\mathbf{A}^3 + \mathbf{I} = \begin{pmatrix}9 & 0\\-19 & 28\end{pmatrix}$ B1	**A1**
and $\mathbf{D} = \begin{pmatrix}2^3+1 & 0\\0 & 3^3+1\end{pmatrix} = \begin{pmatrix}9 & 0\\0 & 28\end{pmatrix}$. Columns of $\mathbf{P}$ and $\mathbf{D}$ can be permuted, but must match. Eigenvalues $(9, 28)$ and vectors $\begin{pmatrix}1\\1\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ M1, A1. $\mathbf{P}$ and $\mathbf{D}$ FT on eigenvalues M1, A1FT	**M1A1FT**

Total: 8

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Q15

9231/11, 9231/13 • October/November 2018 • Question 5

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Mark scheme.

$\mathbf{Ae} = \lambda\mathbf{e}$ and $\mathbf{Be} = \mu\mathbf{e} \Rightarrow \mathbf{Ae} + \mathbf{Be} = \lambda\mathbf{e} + \mu\mathbf{e}$ (Adds equations.)	**M1**
$\Rightarrow (\mathbf{A} + \mathbf{B})\mathbf{e} = (\lambda + \mu)\mathbf{e}$	**A1**
$\begin{pmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix} \Rightarrow \lambda = 3$ (Multiplies matrix with vector.)	**M1**
$\Rightarrow \lambda = 3$ (Finds one eigenvalue.)	**A1**
$\lambda = 1, 2$ (Finds other two.)	**A1**
$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 4 & 1 \\ 1 & -1 & 0 \end{pmatrix}$	**B1**
$\mathbf{D} = \begin{pmatrix} 7^3 & 0 & 0 \\ 0 & 6^3 & 0 \\ 0 & 0 & 3^3 \end{pmatrix} = \begin{pmatrix} 343 & 0 & 0 \\ 0 & 216 & 0 \\ 0 & 0 & 27 \end{pmatrix}$ (Or correctly matched permutations of columns.)	**M1A1**
	Total: 8

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Q16

9231/12 • October/November 2018 • Question 2

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Mark scheme.

2 (Stated)	**B1**
Negative eigenvalue $= -2$ (Stated)	**B1**
$\mathbf{A} + 2\mathbf{I} = \begin{pmatrix} 4 & 3 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{pmatrix}, \begin{vmatrix} i & j & k \\ 4 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix}$ (Uses vector product (or equations) to find corresponding eigenvector.)	**M1**
$\begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ (Accept any non-zero scalar multiple of $\begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$.)	**A1**
An eigenvalue of $A + A^6$ is $2 + 2^6 = 66$, 62 or 2	**B1**
Corresponding eigenvector is $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} -6 \\ 1 \\ 3 \end{pmatrix}$ oe	**B1**

Total: 6

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Q17

9231/11, 9231/12 • May/June 2019 • Question 9

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Mark scheme.

$\mathbf{A}^2\mathbf{e} = \mathbf{A}(\mathbf{A}\mathbf{e}) = \lambda\mathbf{A}\mathbf{e} = \lambda^2\mathbf{e}$	**M1A1**
Eigenvalues of $\mathbf{A}$ are $n, 2n$ and $3n$.	**B1**
$\lambda = n$: $\mathbf{e}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 3 \\ 0 & n & 0 \end{vmatrix} = \begin{pmatrix} -3n \\ 0 \\ 0 \end{pmatrix} = t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	**M1A1**
$\lambda = 2n$: $\mathbf{e}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -n & 1 & 3 \\ 0 & 0 & n \end{vmatrix} = \begin{pmatrix} n \\ n^2 \\ 0 \end{pmatrix} = t\begin{pmatrix} 1 \\ n \\ 0 \end{pmatrix}$	**A1**
$\lambda = 3n$: $\mathbf{e}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2n & 1 & 3 \\ 0 & -n & 0 \end{vmatrix} = \begin{pmatrix} 3n \\ 0 \\ 2n^2 \end{pmatrix} = t\begin{pmatrix} 3 \\ 0 \\ 2n \end{pmatrix}$	**A1**
Eigenvalues of $\mathbf{A} + n\mathbf{I}$ are $2n, 3n$ and $4n$	**B1**
Thus $\mathbf{P} = \begin{pmatrix} 1 & 1 & 3 \\ 0 & n & 0 \\ 0 & 0 & 2n \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} (2n)^2 & 0 & 0 \\ 0 & (3n)^2 & 0 \\ 0 & 0 & (4n)^2 \end{pmatrix}$	**M1A1**
	Total: 10

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Q18

9231/13 • May/June 2019 • Question 11

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Mark scheme.

$\mathbf{P} = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & b & -1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	**B1B1**
$\det \mathbf{P} = -b - 1$	**B1**
$\mathbf{P}^{-1} = \frac{1}{b+1} \begin{pmatrix} b & -1 & -b \\ 1 & 1 & b \\ 1 & 1 & -1 \end{pmatrix}^{\mathrm{T}} = \frac{1}{b+1} \begin{pmatrix} b & 1 & 1 \\ -1 & 1 & 1 \\ -b & b & -1 \end{pmatrix}$	**M1A1**
$\mathbf{A} = \mathbf{PDP}^{-1}$	**M1**
$\mathbf{A} = \frac{1}{b+1} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & b & -1 \end{pmatrix} \begin{pmatrix} 2b & 2 & 2 \\ -1 & 1 & 1 \\ -3b & 3b & -3 \end{pmatrix}$	**M1A1**
$\mathbf{A} = \frac{1}{b+1} \begin{pmatrix} 1+2b & 1 & 1 \\ -b & 3b+2 & -1 \\ 2b & -2b & 3+b \end{pmatrix}$	**A1**
$\mathbf{A}^{-1} \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} = 2\mathbf{A}^{-1} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \frac{2}{3} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$	**M1A1**
$\mathbf{A}^n \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 2^n \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 2^2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow n = 2$	**B1**
$\mathbf{A}^n \begin{pmatrix} -1 \\ 0 \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ b^{-1} \end{pmatrix} \Rightarrow b = b^{-1} \Rightarrow b = 1$	**M1A1**

Total: 14

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Q19

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 8

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Mark scheme.

Eigenvalues of (upper diagonal matrix) $\mathbf{A}$ are $2, m$ and 1. (Or from characteristic equation: $(\lambda-2)(\lambda-m)(\lambda-1)=0$)	**B1**
$\lambda=2$: $\mathbf{e}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & m-2 & 1 \\ 0 & 0 & -1 \end{vmatrix} = \begin{pmatrix} 2-m \\ 0 \\ 0 \end{pmatrix} = t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	**M1A1**
Uses vector product (or equations) to find corresponding eigenvectors.	
$\lambda=m$: $\mathbf{e}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2-m & m & 1 \\ 0 & 0 & 7 \end{vmatrix} = \begin{pmatrix} 7m \\ 7(m-2) \\ 0 \end{pmatrix} = t\begin{pmatrix} m \\ m-2 \\ 0 \end{pmatrix}$	**A1**
$\lambda=1$: $\mathbf{e}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & m & 1 \\ 0 & m-1 & 7 \end{vmatrix} = \begin{pmatrix} 6m+1 \\ -7 \\ m-1 \end{pmatrix} = t\begin{pmatrix} 6m+1 \\ -7 \\ m-1 \end{pmatrix}$	**A1**
Thus $\mathbf{P} = \begin{pmatrix} 1 & m & 6m+1 \\ 0 & m-2 & -7 \\ 0 & 0 & m-1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{pmatrix}$	**M1A1FT**
Or correctly matched permutations of columns. No follow through on two or more zero eigenvectors.	
$\mathbf{M}^7\mathbf{P} = \mathbf{P}\mathbf{D}^7\mathbf{P}^{-1}\mathbf{P} = \mathbf{P}\mathbf{D}^7 = \begin{pmatrix} 1 & m & 6m+1 \\ 0 & m-2 & -7 \\ 0 & 0 & m-1 \end{pmatrix}\begin{pmatrix} 2^7 & 0 & 0 \\ 0 & m^7 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	**M1A1FT**
Applies $\mathbf{M}^7 = \mathbf{P}\mathbf{D}^7\mathbf{P}^{-1}$.	
$= \begin{pmatrix} 2^7 & m^8 & 6m+1 \\ 0 & m^8-2m^7 & -7 \\ 0 & 0 & m-1 \end{pmatrix}$	**A1**
Order of columns might be swapped depending on $\mathbf{P}$.	
	Total: 10

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Q20

9231/21, 9231/22 • May/June 2020 • Question 8

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Mark scheme.

(a)

$\begin{vmatrix} 3 & 1 & 1 \\ a & 6 & -1 \\ 0 & a & -2 \end{vmatrix} = 0 \Rightarrow 3(-12+a) - (-2a) + a^2 = 0$	**M1**
$a^2 + 5a - 36 = 0 \Rightarrow a = 4, -9$	**M1 A1**

(b)

characteristic equation $(\lambda - 3)(\lambda - 6)(\lambda + 2) = \lambda^3 - 7\lambda^2 + 36 = 0$	**B1**
$36\mathbf{I} = 7\mathbf{A}^2 - \mathbf{A}^3 \Rightarrow 36(\mathbf{A}^{-1})^2 = 7\mathbf{I} - \mathbf{A}$	**M1**
$(\mathbf{A}^{-1})^2 = \frac{1}{36}\begin{pmatrix} 4 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 9 \end{pmatrix}$	**M1 A1**

(c)

eigenvalues of $\mathbf{A}$ are $3, 6, -2$	**B1**
$\lambda = 3$: eigenvector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	**M1 A1**
$\lambda = 6$: eigenvector $\begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$; $\lambda = -2$: eigenvector $\begin{pmatrix} -9 \\ 5 \\ 40 \end{pmatrix}$	**A1 A1**
$\mathbf{P} = \begin{pmatrix} 1 & 1 & -9 \\ 0 & 3 & 5 \\ 0 & 0 & 40 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 243 & 0 & 0 \\ 0 & 7776 & 0 \\ 0 & 0 & -32 \end{pmatrix}$	**M1 A1**

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Q21

9231/23 • May/June 2020 • Question 3

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Mark scheme.

(a)	
$\begin{vmatrix} 5-\lambda & -1 & 7 \\ 0 & 6-\lambda & 0 \\ 7 & 7 & 5-\lambda \end{vmatrix} = 0$	**B1**
$-\lambda^3 + 16\lambda^2 - 36\lambda - 144 = 0 \Rightarrow (\lambda+2)(\lambda-6)(\lambda-12) = 0$	**M1**
$\lambda = -2,\ 6,\ 12$	**A1 A1**
(b)	
$-\mathbf{A}^3 + 16\mathbf{A}^2 - 36\mathbf{A} - 144\mathbf{I} = 0$	**B1**
$144\mathbf{A}^{-1} = -\mathbf{A}^2 + 16\mathbf{A} - 36\mathbf{I}$	**M1**
$\mathbf{A}^2 = \begin{pmatrix} 74 & 38 & 70 \\ 0 & 36 & 0 \\ 70 & 70 & 74 \end{pmatrix} \Rightarrow \mathbf{A}^{-1} = \frac{1}{24}\begin{pmatrix} -5 & -9 & 7 \\ 0 & 4 & 0 \\ 7 & 7 & -5 \end{pmatrix}$	**M1 A1**
	Total: 8

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Q22

9231/21, 9231/23 • October/November 2020 • Question 3

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Mark scheme.

(a) Shows that determinant is zero: $\begin{vmatrix} 1 & -2 & -4 \\ 1 & -2 & k \\ -1 & 2 & 2 \end{vmatrix} = -4-2k+2(2+k)-4(0) = 0$.	**M1 A1**
(b) Derives one equation with two unknowns or states that third plane is not parallel to the repeated one: $z = -1$, $x - 2y = -3$.	**B1**
Two of the planes are identical (or coincident).	**B1**
There is a line of intersection with the other plane.	**B1**
(c) Derives contradiction: $-1 = 1$.	**B1**
Two parallel planes, not identical.	**B1**
(d) Derives contradiction: $(-4-k)z = 0$, $-2z = 2 \Rightarrow 0 = 2$.	**B1**
The three planes form a triangular prism.	**B1**
	Total: 9

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Q23

9231/21, 9231/23 • October/November 2020 • Question 7

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Mark scheme.

(a) Eigenvalues: $1, -1, 2$.	**B1**
(b) States that $\mathbf{P}$ satisfies its characteristic equation: $\mathbf{P}^3 - 2\mathbf{P}^2 - \mathbf{P} + 2\mathbf{I} = 0$.	**B1**
Multiplies through by $\mathbf{P}^{-1}$: $2\mathbf{P}^{-1} = \mathbf{I} + 2\mathbf{P} - \mathbf{P}^2$.	**M1**
$\mathbf{P}^2 = \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix} \Rightarrow \mathbf{P}^{-1} = \begin{pmatrix} 1 & 4 & -3 \\ 0 & -1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$.	**M1 A1**
(c) Applies $\mathbf{A} = \mathbf{PDP}^{-1}$: $\mathbf{A} = \mathbf{P}\begin{pmatrix} b & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\mathbf{P}^{-1}$.	**M1**
Multiplies two adjacent matrices.	**M1 A1**
$\mathbf{A} = \begin{pmatrix} b & 4b+4 & -3b-1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.	**A1**
	Total: 9

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Q24

9231/22 • October/November 2020 • Question 9

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Mark scheme.

(a) Shows that determinant is non-zero or solves equations and finds at least two correct values: $4a \neq 0$, or $x = \frac{11}{2} + \frac{8}{a}$, $y = -\frac{1}{2}$, $z = -\frac{9}{2}$.	**M1 A1**
The three planes intersect at a single point.	**B1**
(b) Equates determinant to zero: $\begin{vmatrix} a-\lambda & 2a+5 & a+1 \\ 0 & -4-\lambda & 0 \\ 0 & 3 & -1-\lambda \end{vmatrix} = 0.$	**M1**
AG: $(a-\lambda)(-4-\lambda)(-1-\lambda) = 0 \Rightarrow \lambda = a, -1, -4$.	**A1**
(c) Uses vector product (or equations) to find corresponding eigenvectors: $\lambda = a$: $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.	**M1 A1**
$\lambda = -1$: $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.	**A1**
$\lambda = -4$: $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.	**A1**
$\mathbf{P} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$ OE.	**A1**
(d) Finds characteristic equation: $(\lambda - a)(\lambda+1)(\lambda+4) = \lambda^3 + (5-a)\lambda^2 + (4-5a)\lambda - 4a = 0$.	**B1**
Multiplies through by $\mathbf{A}^{-1}$: $4a\mathbf{A}^{-1} = \mathbf{A}^2 + (5-a)\mathbf{A} + (4-5a)\mathbf{I}$.	**M1 A1**
$\mathbf{A}^2 = \begin{pmatrix} a^2 & 2a^2-17 & a^2-1 \\ 0 & 16 & 0 \\ 0 & -15 & 1 \end{pmatrix}$.	**B1**
Substitutes for $\mathbf{A}^2$ and $\mathbf{A}$ in correct equation.	**M1**
$\mathbf{A}^{-1} = \frac{1}{4a}\begin{pmatrix} 4 & 5a+8 & 4a+4 \\ 0 & -a & 0 \\ 0 & -3a & -4a \end{pmatrix}$.	**A1**
	Total: 16

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Q25

9231/21, 9231/22 • May/June 2021 • Question 1

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Mark scheme.

(a)	
Finds determinant: $\begin{vmatrix} a & 3 & 1 \\ 2 & 1 & 3 \\ -1 & 2 & -5 \end{vmatrix} = -11a + 26.$	**M1**
Determinant in terms of a.	**A1**
States that $\det \mathbf{A} \neq 0$ (since a is an integer) leads to unique solution.	**A1**
The three planes intersect at a single point.	**B1**
(b)	
$a = 4$.	**B1**
	Total: 5

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Q26

9231/21, 9231/22 • May/June 2021 • Question 6

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Mark scheme.

(a)	
Upper diagonal matrix / characteristic equation: eigenvalues of $\mathbf{A}$ are $5, -6, 1$.	**B1**
Uses vector product (or equations) to find corresponding eigenvectors.	**M1**
One mark for each correct eigenvector: $(1,0,0)^T$, $(2,3,0)^T$, $(2,0,-1)^T$.	**A1 A1 A1**
$\mathbf{P} = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 25 & 0 & 0 \\ 0 & 36 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$	**M1 A1**
(b)	
Finds characteristic equation: $(\lambda-5)(\lambda+6)(\lambda-1) = \lambda^3 - 31\lambda + 30 = 0$.	**B1**
Finds $\mathbf{A}^3$ in terms of $\mathbf{A}$: $\mathbf{A}^3 - 31\mathbf{A} + 30\mathbf{I} = \mathbf{0} \Rightarrow \mathbf{A}^3 = 31\mathbf{A} - 30\mathbf{I}$.	**M1**
Substitutes for $\mathbf{A}$: $\mathbf{A}^3 = \begin{pmatrix} 125 & -\frac{682}{3} & 248 \\ 0 & -216 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$	**M1 A1**
	Total: 11

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Q27

9231/23 • May/June 2021 • Question 8

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Mark scheme.

(a)	
Finds determinant: $\begin{vmatrix} 13 & 18 & -28 \\ -4 & -a & 8 \\ 2 & 6 & -5 \end{vmatrix} = 9a - 24.$	**M1**
$a = \frac{8}{3}$.	**A1**
(b)	
$\begin{pmatrix} 13 & 18 & -28 \\ -4 & -1 & 8 \\ 2 & 6 & -5 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} \Rightarrow \lambda = -1.$	**B1**
(c)	
Sets $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$.	**B1**
Expands determinant and factorises: $-\lambda^3 + 7\lambda^2 - 7\lambda - 15 = 0 \Rightarrow (\lambda+1)(\lambda-3)(\lambda-5) = 0$.	**M1**
$\lambda = -1, 3, 5$.	**A1**
Uses vector product (or equations) to find corresponding eigenvectors.	**M1**
Correct eigenvectors for $\lambda = 3$: $(1, 1, 1)^T$; for $\lambda = 5$: $(-1, 2, 1)^T$.	**A1 A1**
$\mathbf{P} = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}.$	**M1 A1**
(d)	
Substitutes $\mathbf{A}$ into characteristic equation and multiplies through by $\mathbf{A}^{-1}$: $-\mathbf{A}^3 + 7\mathbf{A}^2 - 7\mathbf{A} - 15\mathbf{I} = \mathbf{0}$.	**M1**
$\mathbf{A}^{-1} = \frac{1}{15}(-\mathbf{A}^2 + 7\mathbf{A} - 7\mathbf{I})$.	**A1**
	Total: 13

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Q28

9231/21, 9231/23 • October/November 2021 • Question 2

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Mark scheme.

$(\lambda+1)(\lambda-1)(\lambda-3)=\lambda^3-3\lambda^2-\lambda+3[=0]$. Finds characteristic equation.	**M1 A1**
$\mathbf{A}^3-3\mathbf{A}^2-\mathbf{A}+3\mathbf{I}=0 \Rightarrow \mathbf{A}^4-3\mathbf{A}^3-\mathbf{A}^2+3\mathbf{A}=0$. Substitutes $\mathbf{A}$ and multiplies through by $\mathbf{A}$.	**M1 A1**
$\mathbf{A}^4=3(3\mathbf{A}^2+\mathbf{A}-3\mathbf{I})+\mathbf{A}^2-3\mathbf{A}=10\mathbf{A}^2-9\mathbf{I}$. Substitutes $\mathbf{A}^3$.	**M1 A1**
	Total: 6

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Q29

9231/22 • October/November 2021 • Question 6

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Mark scheme.

$(\lambda-1)(\lambda-2)(\lambda+3)=\lambda^3-7\lambda+6[=0]$. Finds characteristic equation.	**M1 A1**
$\mathbf{P}^2-7\mathbf{I}+6\mathbf{P}^{-1}=0$. Substitutes $\mathbf{P}$ and multiplies through by $\mathbf{P}^{-1}$.	**M1**
$\mathbf{P}^2=\begin{pmatrix}1&18&24\\0&4&-6\\0&0&9\end{pmatrix}$ leading to $\mathbf{P}^{-1}=\begin{pmatrix}1&-3&-4\\0&\frac{1}{2}&1\\0&0&-\frac{1}{3}\end{pmatrix}$.	**M1 A1**
Applies $\mathbf{A}=\mathbf{P}\begin{pmatrix}4&0&0\\0&5&0\\0&0&6\end{pmatrix}\mathbf{P}^{-1}$.	**M1**
Multiplies two adjacent matrices.	**M1 A1**
$\mathbf{A}=\begin{pmatrix}4&3&2\\0&5&-2\\0&0&6\end{pmatrix}$.	**A1**
Eigenvectors of $\mathbf{A}^3$: $(1,0,0)^T$, $(6,2,0)^T$, $(6,6,-3)^T$ (or any non-zero multiples).	**B1**
Eigenvalues of $\mathbf{A}^3$: 64, 125, 216.	**B1**
	Total: 11

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Q30

9231/21, 9231/22 • May/June 2022 • Question 8

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Mark scheme.

Sets determinant equal to zero: $\begin{vmatrix} 3 & a & 0 \\ 5 & -1 & 0 \\ 1 & 3 & 2 \end{vmatrix} = 0 \Rightarrow -10a - 6 = 0$	**(a) M1**
$a = -\frac{3}{5}$	**A1**
Eigenvalues of $\mathbf{A}$ are $3, -1, 2$ (lower diagonal matrix or characteristic equation)	**(b) B1**
Uses vector product (or equations) to find corresponding eigenvectors: $\lambda = 3$: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -4 & 0 \\ 1 & 3 & -1 \end{vmatrix} = \begin{pmatrix} 4 \\ 5 \\ 19 \end{pmatrix}$	**M1 A1**
$\lambda = -1$: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 0 \\ 1 & 3 & 3 \end{vmatrix} = \begin{pmatrix} 0 \\ -12 \\ 12 \end{pmatrix} \sim \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$	**A1**
$\lambda = 2$: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 1 & 3 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	**A1**
Or correctly matched permutations of columns: $\mathbf{P} = \begin{pmatrix} 4 & 0 & 0 \\ 5 & -1 & 0 \\ 19 & 1 & 1 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$	**M1 A1**
Characteristic equation: $(\lambda - 3)(\lambda + 1)(\lambda - 2) = \lambda^3 - 4\lambda^2 + \lambda + 6 = 0$	**(c) B1**
Substitutes for $\mathbf{A}$ and makes $\mathbf{A} + 6\mathbf{I}$ the subject: $\mathbf{A} + 6\mathbf{I} = -\mathbf{A}^3 + 4\mathbf{A}^2$	**M1**
Squares and factorises: $(\mathbf{A} + 6\mathbf{I})^2 = (-\mathbf{A}^3 + 4\mathbf{A}^2)^2 = \mathbf{A}^4(\mathbf{A} - 4\mathbf{I})^2$	**M1 A1**
	Total: 13

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Q31

9231/23 • May/June 2022 • Question 3

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Mark scheme.

Sets determinant equal to zero: $\begin{vmatrix} 6-\lambda & -9 & 5 \\ 5 & -8-\lambda & 5 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$	**(a) M1**
Expands: $\lambda^3 - 7\lambda + 6 = 0$	**M1**
Award A1A0 for 2 correct solutions, A0A0 for 1 correct solution: $\lambda = 1, 2, -3$	**A1 A1**
States that $\mathbf{A}$ satisfies its characteristic equation: $\mathbf{A}^3 - 7\mathbf{A} + 6\mathbf{I} = 0$	**(b) B1**
Multiplies through by $\mathbf{A}^{-1}$: $\mathbf{A}^2 - 7\mathbf{I} + 6\mathbf{A}^{-1} = 0$	**M1**
$\mathbf{A}^{-1} = -\frac{1}{6}\mathbf{A}^2 + \frac{7}{6}\mathbf{I}$	**A1**
	Total: 7

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Q32

9231/21, 9231/23 • October/November 2022 • Question 2

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Mark scheme.

(a)	
$\begin{vmatrix} 1 & -1 & 2 \\ 1 & -1 & -3 \\ 1 & -1 & 7 \end{vmatrix} = -7 - 3 + 10 - 2(0) = 0.$ Shows determinant is zero.	**M1 A1**
(b)	
Row-reduce: $z = \frac{9}{5}$, $x - y = \frac{2}{5}$. (M1 for row operations/eliminating a variable, A1 for one correct equation in two unknowns.)	**M1 A1**
The three planes form a sheaf (all three planes have the same common line).	**B1**
(c)	
$3z + a = 4 - 2z \Rightarrow a = -5$ required for consistency; derives contradiction when $a \neq -5$.	**B1**
The three planes form a triangular prism.	**B1**
	Total: 7

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Q33

9231/21, 9231/23 • October/November 2022 • Question 6

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Mark scheme.

(a)	
Eigenvalues of $\mathbf{A}$ are $2, 5, -2$.	**B1**
$\lambda = 2$: eigenvector $(1, 0, 0)^T$.	**M1 A1**
$\lambda = 5$: eigenvector $(1, -1, 0)^T$.	**A1**
$\lambda = -2$: eigenvector $(1, -1, 1)^T$.	**A1**
$\mathbf{P} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 32 & 0 & 0 \\ 0 & 3125 & 0 \\ 0 & 0 & -32 \end{pmatrix}$.	**M1 A1**
(b)	
Characteristic equation $(\lambda - 2)(\lambda - 5)(\lambda + 2) = \lambda^3 - 5\lambda^2 - 4\lambda + 20 = 0$.	**B1**
$\mathbf{A}^3 = 5\mathbf{A}^2 + 4\mathbf{A} - 20\mathbf{I}$.	**M1**
Multiplies by $\mathbf{A}$ and substitutes for $\mathbf{A}^3$: $\mathbf{A}^4 = 5\mathbf{A}^3 + 4\mathbf{A}^2 - 20\mathbf{A} = 5(5\mathbf{A}^2 + 4\mathbf{A} - 20\mathbf{I}) + 4\mathbf{A}^2 - 20\mathbf{A}$.	**M1**
$\mathbf{A}^4 = 29\mathbf{A}^2 - 100\mathbf{I}$.	**A1**
	Total: 11

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Q34

9231/22 • October/November 2022 • Question 1

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Mark scheme.

(a)	
$\begin{vmatrix} 1 & 2 & 3 \\ k & 4 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 6k - 12$. Evaluates determinant (or solves system, eliminating at least one variable).	**M1 A1**
$k \neq 2$.	**A1**
(b)	
$k = 2$: two parallel planes, other plane not parallel.	**B1**
Parallel planes not identical (accept diagram).	**B1**
	Total: 5

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Q35

9231/22 • October/November 2022 • Question 7

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Mark scheme.

(a)	
$\mathbf{Ae} = \lambda\mathbf{e}$ leading to $\mathbf{e} = \lambda\mathbf{A}^{-1}\mathbf{e}$. Multiplies by $\mathbf{A}^{-1}$ on LHS.	**M1**
$\lambda^{-1}\mathbf{e} = \mathbf{A}^{-1}\mathbf{e}$.	**A1**
(b)	
$\lambda = -1$: eigenvector $(1, 4, -1)^T$ (via vector product or equations).	**M1 A1**
(c)	
$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$: eigenvalue $\lambda = -4$.	**B1**
$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$: eigenvalue $\lambda = 5$.	**B1**
(d)	
$\mathbf{P} = \begin{pmatrix} 1 & 0 & 1 \\ 4 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{5} \end{pmatrix}$ (or correctly matched permutation).	**M1 A1**
(e)	
Characteristic equation $(\lambda+1)(\lambda+4)(\lambda-5) = \lambda^3 - 21\lambda - 20 = 0$.	**B1**
$\mathbf{A}^3 - 21\mathbf{A} - 20\mathbf{I} = 0$.	**M1**
$20\mathbf{A}^{-1} = \mathbf{A}^2 - 21\mathbf{I}$ leading to $\mathbf{A}^{-1} = \frac{1}{20}\mathbf{A}^2 - \frac{21}{20}\mathbf{I}$.	**M1 A1**
	Total: 12

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Q36

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Mark scheme.

Shows that determinant is zero or uses row operations to obtain e.g. $x + 2y + 3z = 1$, $y + 2z = 1$.	**M1**
$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = (5\times9-8\times6)-2(4\times9-7\times6)+3(4\times8-7\times5) = -3+12-9 = 0.$	**A1**

(b)

Uses all three equations to reduce to one equation with two unknowns (reducing to two equations only scores M1 A0).	**M1**
$x + 2y + 3z = 1$, $4x + 5y + 6z = 1 \Rightarrow z = \frac{1}{3}(1 - x - 2y) \Rightarrow y = -2x - 1$, $7x + 8y + 9z = 1$.	**A1**
The three planes form a sheaf (accept clear sketch, or the three planes intersect along a common line).	**B1**
	Total: 5

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Q37

9231/21, 9231/22 • May/June 2023 • Question 5

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Mark scheme.

Sets determinant $\begin{vmatrix} 18-\lambda & 5 & -11 \\ 8 & 6-\lambda & -4 \\ 32 & 10 & -20-\lambda \end{vmatrix} = 0.$	**M1**
Expands determinant (can use other rows/columns).	**A1**
Obtains $\lambda^3 - 4\lambda^2 - 20\lambda + 48 = 0$ (AG).	**A1**
$(\lambda-2)(\lambda+4)(\lambda-6) = 0$ leading to $\lambda = 2, -4, 6$. **(b)**	**B1**
Uses vector product (or equations) to find corresponding eigenvectors; must attempt to solve equations for M1.	**M1 A1**
$\lambda = 2$: eigenvector $(1, -1, 1)^T$; $\lambda = -4$: eigenvector $(1, 0, 2)^T$; $\lambda = 6$: eigenvector $(1, 2, 2)^T$.	**A1 A1**
Thus $\mathbf{P} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \\ 1 & 2 & 2 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 32 & 0 & 0 \\ 0 & -1024 & 0 \\ 0 & 0 & 7776 \end{pmatrix}$ (or correctly matched permutations of columns; M0 if a column of zeros appears in $\mathbf{P}$).	**M1 A1**
	Total: 10

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Q38

9231/23 • May/June 2023 • Question 8

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Mark scheme.

Shows that determinant is non-zero or solves equations: $\begin{vmatrix} a & -6a & 2a+2 \\ 0 & 1-a & 0 \\ 0 & 2-a & -1 \end{vmatrix} = a(a-1) \neq 0$; or $x = 1 + \frac{12a}{1-a} - 2\frac{(a+1)^2}{1-a} = \frac{-2a^2+7a-1}{1-a}$, $y = \frac{2}{1-a}$, $z = \frac{2(2-a)}{1-a} - 3 = \frac{a+1}{1-a}$.	**M1 A1**
Three planes intersect at a single point (must be clear that there is just one point where all 3 planes intersect). **(b)**	**B1**
Equates determinant to zero or working to find characteristic equation: $\begin{vmatrix} a-\lambda & -6a & 2a+2 \\ 0 & 1-a-\lambda & 0 \\ 0 & 2-a & -1-\lambda \end{vmatrix} = 0$.	**M1**
AG (factorisation must be clear): $(a-\lambda)(1-a-\lambda)(-1-\lambda) = 0 \Rightarrow \lambda = a, 1-a, -1$. **(c)**	**A1**
Uses vector product (or equations) to find corresponding eigenvectors: $\lambda = a$: eigenvector $(1,0,0)^T$.	**M1 A1**
$\lambda = -1$: eigenvector $(-2,0,1)^T$.	**A1**
$\lambda = 1-a$: eigenvector $(2,1,1)^T$.	**A1**
Correctly matched permutations of columns (their eigenvectors must be non-zero for M1): $\mathbf{P} = \begin{pmatrix} 1 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} a^4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (1-a)^4 \end{pmatrix}$. **(d)**	**M1 A1**
Finds characteristic equation multiplied out: $(a-\lambda)(1-a-\lambda)(-1-\lambda) = \lambda^3 - (a^2-a+1)\lambda - (a^2-a) = 0$.	**B1**
Multiplies through by $\mathbf{A}$: $\mathbf{A}^4 = (a^2-a+1)\mathbf{A}^2 + (a^2-a)\mathbf{A}$.	**M1 A1**
	Total: 14

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Q39

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Mark scheme.

Evaluates determinant. Can expand along any row, e.g. $-(36-36)+(-126+126)+k(84-84)$. If using row operations, they must show an inconsistent system for M1. All their row operations must be correct for A1.	**M1**
$\begin{vmatrix} 14 & -4 & 6 \\ 1 & 1 & k \\ -21 & 6 & -9 \end{vmatrix} = 14(-9-6k)+4(-9+21k)+6(6+21)=0.$	**A1**
Two parallel planes, not identical.	**B1**
Other plane not parallel.	**B1**
	Total: 4

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Q40

9231/21, 9231/23 • October/November 2023 • Question 7

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Mark scheme.

(a)	
$\lambda = -6,\ \lambda = -2,\ \lambda = 8.$	**B1**
Uses vector product (or equations) to find eigenvector for $\lambda = -6$: $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.	**M1 A1**
Eigenvector for $\lambda = -2$: $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$.	**A1**
Eigenvector for $\lambda = 8$: $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$.	**A1**
$\mathbf{P} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} -\frac{1}{6} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{8} \end{pmatrix}$ (or correctly matched permutations of columns).	**M1 A1**
(b)	
Substitutes $\mathbf{A}$ into characteristic equation: $\mathbf{A}^3 - 52\mathbf{A} - 96\mathbf{I} = 0$.	**M1**
Multiplies through by $\mathbf{A}^{-1}$: $96\mathbf{A}^{-1} = \mathbf{A}^2 - 52\mathbf{I}$.	**M1**
$\mathbf{A}^2 = \begin{pmatrix} 36 & -16 & 36 \\ 0 & 4 & 30 \\ 0 & 0 & 64 \end{pmatrix}$.	**B1**
$\mathbf{A}^{-1} = \begin{pmatrix} -\frac{1}{6} & -\frac{1}{6} & \frac{3}{8} \\ 0 & -\frac{1}{2} & \frac{5}{16} \\ 0 & 0 & \frac{1}{8} \end{pmatrix}$.	**A1**
	Total: 11

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Q41

9231/22 • October/November 2023 • Question 6

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Mark scheme.

(a)	
$1,\ 2,\ -1.$	**B1**
(b)	
States that $\mathbf{P}$ satisfies its characteristic equation: $\mathbf{P}^3 - 2\mathbf{P}^2 - \mathbf{P} + 2\mathbf{I} = 0$.	**B1**
Multiplies through by $\mathbf{P}^{-1}$: $2\mathbf{P}^{-1} = \mathbf{I} + 2\mathbf{P} - \mathbf{P}^2$.	**M1**
$\mathbf{P}^2 = \begin{pmatrix} 1 & -3 & -1 \\ 0 & 4 & 1 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \mathbf{P}^{-1} = \begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 \end{pmatrix}$.	**M1 A1**
(c)	
$\mathbf{D} = \begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$.	**B1**
Applies $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$: $\mathbf{A}^{-1} = \mathbf{P}\begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}\mathbf{P}^{-1}$.	**M1**
Multiplies two adjacent matrices: $= \begin{pmatrix} \frac{1}{a} & -2 & \frac{1}{2} \\ 0 & 4 & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 \end{pmatrix}$.	**M1 A1**
$\mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{a} & \frac{1-2a}{2a} & \frac{3-3a}{2a} \\ 0 & 2 & \frac{3}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$.	**A1**
	Total: 10

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Q42

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Mark scheme.

(a)	
Finds determinant or solves equations: $\begin{vmatrix} 6 & a & 0 \\ 2 & -1 & 0 \\ 1 & 5 & 4 \end{vmatrix} = -8a - 24.$	**M1**
$a \neq -3$.	**A1**
(b)	
If $a \neq -3$ then system has unique solution (so consistent).	**B1FT**
Eliminates variable using all 3 equations. Or states there are two distinct equations and three unknowns. Or gives accurate geometrical description with $a = -3$: if $a = -3$ then $6x - 3y = 3$, $2x - y = 1 \Rightarrow 11x + 4z = 7$, $x + 5y + 4z = 2$.	**M1**
So the system has infinitely many solutions (so consistent). [Alternative: B1 sets $y = 0$; M1A1 $\left(\frac{1}{2}, 0, \frac{3}{8}\right)$ is a solution so system is consistent for all values of a.]	**A1**
(c)	
Lower diagonal matrix or characteristic equation. Eigenvalues of $\mathbf{A}$ are $6, -1, 4$.	**B1**
Uses vector product (or equations) to find corresponding eigenvectors: $\lambda = 6$: $(14, 4, 17)$.	**M1 A1**
$\lambda = -1$: $(0, -1, 1)$.	**A1**
$\lambda = 4$: $(0, 0, 1)$.	**A1**
Or correctly matched permutations of columns: $\mathbf{P} = \begin{pmatrix} 14 & 0 & 0 \\ 4 & -1 & 0 \\ 17 & 1 & 1 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 11664 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 6400 \end{pmatrix}$.	**M1 A1**
(d)	
Characteristic equation: $(\lambda - 6)(\lambda + 1)(\lambda - 4) = \lambda^3 - 9\lambda^2 + 14\lambda + 24 = 0$.	**B1**
Substitutes for $\mathbf{A}$ and makes $14\mathbf{A} + 24\mathbf{I}$ the subject: $14\mathbf{A} + 24\mathbf{I} = -\mathbf{A}^3 + 9\mathbf{A}^2$.	**M1**
Squares and factorises. CWO: $(14\mathbf{A} + 24\mathbf{I})^2 = (-\mathbf{A}^3 + 9\mathbf{A}^2)^2 = \mathbf{A}^4(\mathbf{A} - 9\mathbf{I})^2$, so $b = -9$.	**M1 A1**

Total: 16

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Q43

9231/23 • May/June 2024 • Question 8

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Mark scheme.

(a)	
Substitutes point into $x+2y+3z$: $1+2(6)+3(0)=d_1$ or $3+2(-6)+3(0)=d_2$.	**M1**
$x+2y+3z=13$.	**A1**
$x+2y+3z=-9$.	**A1**
(b)	
Forms vector equation of line: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -12 \\ 0 \end{pmatrix}$.	**M1**
AG, CWO: $x-1=\dfrac{y-6}{-6}$, $z=0 \Rightarrow \frac{1}{2}x+\frac{1}{12}y=1$, $z=0$.	**A1**
(c)	
Uses $\det(\mathbf{A}-\lambda\mathbf{I})$ (attempt an expansion).	**M1**
Expands determinant, expansion of brackets shown, AG. CWO: $-\lambda^3+3\lambda^2+\frac{7}{4}\lambda=0$.	**A1**
$\lambda=0,\frac{7}{2},-\frac{1}{2}$.	**B1**
(d)	
Uses vector product to find corresponding eigenvectors: $\lambda=0$: $(3,-18,11)$.	**M1 A1**
$\lambda=\frac{7}{2}$: $(6,6,1)$; $\lambda=-\frac{1}{2}$: $(-6,-6,7)$.	**A1 A1**
Or correctly matched permutations of columns: $\mathbf{P}=\begin{pmatrix} 3 & 6 & -6 \\ -18 & 6 & -6 \\ 11 & 1 & 7 \end{pmatrix}$, $\mathbf{D}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & (7/2)^n & 0 \\ 0 & 0 & (-1/2)^n \end{pmatrix}$.	**M1 A1**
	Total: 14

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Q44

9231/21, 9231/23 • October/November 2024 • Question 1

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Mark scheme.

Evaluates determinant: $\begin{vmatrix} 1 & 5 & 6 \\ k & 2 & 2 \\ -3 & 4 & 8 \end{vmatrix} = 8 - 5(8k+6) + 6(4k+6)$.	**M1A1**
$k \neq \frac{7}{8}$.	**A1**
Three planes intersect at a unique point.	**B1**
	Total: 4

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Q45

9231/21, 9231/23 • October/November 2024 • Question 4

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Mark scheme.

(a)	
Multiplies matrix with eigenvector: $\mathbf{A}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-2\\-2\\-2\end{pmatrix}$.	**M1**
$\lambda = -2$.	**A1**
(b)	
Sets determinant equal to zero.	**M1**
Expands determinant: $(-11-\lambda)(-2-\lambda)(13-\lambda)+128(-2-\lambda) = 0 \Rightarrow (-2-\lambda)(\lambda^2-2\lambda-15) = 0 \Rightarrow \lambda^3-19\lambda-30 = 0$ (AG).	**A1**
$\lambda = 5, -3$.	**B1**
(c)	
States that $\mathbf{A}$ satisfies its characteristic equation: $\mathbf{A}^3 - 19\mathbf{A} - 30\mathbf{I} = 0$.	**B1**
Multiplies through by $\mathbf{A}^{-1}$: $30\mathbf{A}^{-1} = \mathbf{A}^2 - 19\mathbf{I}$.	**M1**
Substitutes $\mathbf{A}^2 = \begin{pmatrix}-7 & -5 & 16\\0 & 4 & 0\\-32 & -5 & 41\end{pmatrix}$ to obtain $\mathbf{A}^{-1} = \frac{1}{30}\begin{pmatrix}-26 & -5 & 16\\0 & -15 & 0\\-32 & -5 & 22\end{pmatrix}$.	**M1A1**
	Total: 9

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Q46

9231/22 • October/November 2024 • Question 8

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Mark scheme.

(a)

Sets determinant equal to zero: $\begin{vmatrix} -2-\lambda & 0 & 0 \\ 0 & 7-\lambda & 9 \\ 4 & 1 & 7-\lambda \end{vmatrix} = 0.$ **B1**

Expands: $(-2-\lambda)((7-\lambda)^2 - 9) = 0 \Rightarrow \lambda^3 - 12\lambda^2 + 12\lambda + 80 = 0$ (AG). **M1A1**

$\lambda = -2, 4, 10.$ **B1**

(b)

Substitutes $\mathbf{A}$ and multiplies through by $\mathbf{A}$: $\mathbf{A}^3 - 12\mathbf{A}^2 + 12\mathbf{A} + 80\mathbf{I} = 0 \Rightarrow \mathbf{A}^4 - 12\mathbf{A}^3 + 12\mathbf{A}^2 + 80\mathbf{A} = 0.$ **M1A1**

Substitutes $\mathbf{A}^3$: $\mathbf{A}^4 = 12(12\mathbf{A}^2 - 12\mathbf{A} - 80\mathbf{I}) - 12\mathbf{A}^2 - 80\mathbf{A} = 132\mathbf{A}^2 - 224\mathbf{A} - 960\mathbf{I}.$ **M1A1**

(c)

Uses vector product (or equations) to find eigenvector for $\lambda = -2$: $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$. ***M1A1**

Eigenvectors for $\lambda = 4$: $\begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix}$; for $\lambda = 10$: $\begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$. **A1A1**

$\mathbf{P} = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 3 & 3 \\ -1 & -1 & 1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 625 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2401 \end{pmatrix}$ (or correctly matched permutation of columns). **DM1A1**

Total: 14

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Q47

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Mark scheme.

(a)	
$\mathbf{Ae} = \lambda\mathbf{e} \Rightarrow \mathbf{A}^3\mathbf{e} = \lambda\mathbf{A}(\mathbf{Ae})$. Multiplies by $\mathbf{A}^2$ on LHS. Trying to invert an eigenvector or missing eigenvector is M0.	**M1**
$\Rightarrow \mathbf{A}^3\mathbf{e} = \lambda\mathbf{A}(\lambda\mathbf{e}) = \lambda^2(\mathbf{Ae}) = \lambda^3\mathbf{e}$.	**A1**
(b)	
Forms $\det(\mathbf{A}-\lambda\mathbf{I}) = 0$. Do not accept checking each given value is an eigenvalue. Do not accept row operations on $\mathbf{A}$.	**M1**
$\Rightarrow (-1-\lambda)(1-\lambda)(5-\lambda) = 0 \Rightarrow \lambda = -1, 1, 5$. AG.	**A1**
(c)	
Uses vector product (or equations) of rows of $\mathbf{A} - \lambda\mathbf{I}$ to find corresponding eigenvectors. $\lambda = -1$: eigenvector $\begin{pmatrix}1\\0\\0\end{pmatrix}$.	**M1 A1**
$\lambda = 1$: eigenvector $\begin{pmatrix}5\\2\\1\end{pmatrix}$; $\lambda = 5$: eigenvector $\begin{pmatrix}2\\0\\3\end{pmatrix}$.	**A1 A1**
Thus $\mathbf{P} = \begin{pmatrix}1&5&2\\0&2&0\\0&1&3\end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix}-3&0&0\\0&-1&0\\0&0&3\end{pmatrix}$. Or correctly matched permutations of columns. A column of zeros is M0. Repeated eigenvalues in $\mathbf{D}$ is M0.	**M1 A1**
(d)	
Expands: $(\mathbf{A}-2\mathbf{I})^3 = \mathbf{A}^3 - 6\mathbf{A}^2 + 12\mathbf{A} - 8\mathbf{I}$.	**B1**
Substitutes $\mathbf{A}^3 = 5\mathbf{A}^2 + \mathbf{A} - 5\mathbf{I}$: $= 5\mathbf{A}^2 + \mathbf{A} - 5\mathbf{I} - 6\mathbf{A}^2 + 12\mathbf{A} - 8\mathbf{I}$.	**M1**
$= -\mathbf{A}^2 + 13\mathbf{A} - 13\mathbf{I}$.	**A1**
	Total: 13

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Q48

9231/23 • May/June 2025 • Question 8

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Mark scheme.

(a)	
Sets determinant equal to zero and forms 3-term quadratic equation: $\begin{vmatrix} \frac{3}{2} & 3 & 8 \\ a & 3 & 4 \\ 0 & a & -1 \end{vmatrix} = 0 \Rightarrow 8a^2 - 3a - \frac{9}{2} = 0.$	**M1 A1**
$a = \frac{3}{16}\left(1 \pm \sqrt{17}\right).$	**A1**
(b)	
Finds characteristic equation: $\left(\lambda - \frac{3}{2}\right)(\lambda - 3)(\lambda + 1) = \lambda^3 - \frac{7}{2}\lambda^2 + \frac{9}{2} = 0.$	**B1**
Uses Cayley-Hamilton theorem and replaces λ with $\mathbf{A}$: $\frac{9}{2}\mathbf{I} = \frac{7}{2}\mathbf{A}^2 - \mathbf{A}^3 \Rightarrow \frac{9}{2}\mathbf{B} = \frac{7}{2}\mathbf{A} - \mathbf{A}^2.$	**M1**
Multiplies by $\mathbf{B}^2 = \mathbf{A}^{-2}$, CAO: $\mathbf{B}^2 = \frac{7}{9}\mathbf{I} - \frac{2}{9}\mathbf{A}.$	**M1 A1**
(c)	
Eigenvalues of $\mathbf{A}$ are $\frac{3}{2}$, 3 and -1 (upper diagonal matrix or characteristic equation).	**B1**
Uses vector product (or equations) to find corresponding eigenvectors. $\lambda = \frac{3}{2}$: eigenvector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.	**M1 A1**
$\lambda = 3$: eigenvector $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$; $\lambda = -1$: eigenvector $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.	**A1 A1**
Thus $\mathbf{P} = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -1 \end{pmatrix}$. Or correctly matched permutations of columns. Column of zeros or repeated column in $\mathbf{P}$, or repeated eigenvalues in $\mathbf{D}$, is M0.	**M1 A1**
	Total: 14

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Q49

9231/24 • May/June 2025 • Question 1

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Mark scheme.

(a)

Expands determinant and forms quadratic expression: $\begin{vmatrix} 1 & 2 & 3 \\ k & 5 & 6 \\ 7 & 2k & 9 \end{vmatrix} = 45 - 12k - 2(9k - 42) + 3(2k^2 - 35)$.	**M1**
Solves quadratic: $6k^2 - 30k + 24 = 0 \Rightarrow k = 1, 4$. Need both values for A1.	**M1 A1**

(b)

For M1 must use all three equations and reduce to, e.g., $y = x$. Accept explicit solution using a parameter. Accept row echelon form (with clear operations on all three rows) $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 3 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$: $x + 2y + 3z = 1$, $x + 5y + 6z = 2 \Rightarrow 3y + 3z = 1$, $7x + 2y + 9z = 3 \Rightarrow 3x + 3z = 1$.	**M1 A1**
Three planes meeting in a common line (sheaf).	**B1**

Total: 6

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Q50

9231/24 • May/June 2025 • Question 7

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Mark scheme.

(a)

Eigenvalues of $\mathbf{A}$ are 1, 2 and -3 (upper diagonal matrix or characteristic equation).	**B1**
Uses vector product (or equations) to find corresponding eigenvectors. $\lambda = 1$: eigenvector $\begin{pmatrix}1\\0\\0\end{pmatrix}$.	**M1 A1**
$\lambda = 2$: eigenvector $\begin{pmatrix}7\\1\\0\end{pmatrix}$.	**A1**
$\lambda = -3$: eigenvector $\begin{pmatrix}1\\1\\-1\end{pmatrix}$.	**A1**
Thus $\mathbf{P} = \begin{pmatrix}1 & 7 & 1\\0 & 1 & 1\\0 & 0 & -1\end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix}1 & 0 & 0\\0 & 64 & 0\\0 & 0 & 729\end{pmatrix}$. Or correctly matched permutations of columns, OE. A column of zeros or a repeated column is M0.	**M1 A1**

(b)

Characteristic equation: $(\lambda-1)(\lambda-2)(\lambda+3) = \lambda^3 - 7\lambda + 6 = 0$. '$= 0$' can be implied later.	**B1**
Substitutes for $\mathbf{A}$ and makes $\mathbf{A}^3$ the subject (could substitute for $\mathbf{A}$ at the end): $\mathbf{A}^3 = 7\mathbf{A} - 6\mathbf{I}$.	**M1**
Expands $(7\mathbf{A}-6\mathbf{I})(7\mathbf{A}-6\mathbf{I})$: $\mathbf{A}^6 = (7\mathbf{A}-6\mathbf{I})(7\mathbf{A}-6\mathbf{I}) = 49\mathbf{A}^2 - 84\mathbf{A} + 36\mathbf{I}$.	**M1 A1**

Total: 11

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Q51

9231/21, 9231/23 • October/November 2025 • Question 1

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Mark scheme.

Evaluates determinant: $\begin{vmatrix} 1 & -1 & 1 \\ 1 & k & 3 \\ 1 & 2 & k \end{vmatrix} = k\begin{vmatrix} k & 3 \\ 2 & k \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & k \end{vmatrix} + \begin{vmatrix} 1 & k \\ 1 & 2 \end{vmatrix} = k^2 - 7.$	**M1**
$k^2 - 7$ obtained.	**A1**
$k = \pm\sqrt{7}$.	**A1**
	Total: 3

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Q52

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Mark scheme.

$a,\ 2a,\ -3a.$	**(a) B1**
$\mathbf{P}^2 = \begin{pmatrix} a^2 & 3a & -2a-1 \\ 0 & 4a^2 & a \\ 0 & 0 & 9a^2 \end{pmatrix}.$	**(b)(i) B1**
States that $\mathbf{P}$ satisfies its characteristic equation: $\mathbf{P}^3 - 7a^2\mathbf{P} + 6a^3\mathbf{I} = 0.$	**(ii) B1**
Multiplies through by $\mathbf{P}^{-1}$: $6a^3\mathbf{P}^{-1} = 7a^2\mathbf{I} - \mathbf{P}^2.$	**M1**
$\mathbf{P}^{-1} = \frac{1}{6a^3}\begin{pmatrix} 6a^2 & -3a & 2a+1 \\ 0 & 3a^2 & -a \\ 0 & 0 & -2a^2 \end{pmatrix}.$	**A1**
$\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$	**(c) B1**
Applies $\mathbf{A} = \mathbf{PDP}^{-1}$.	**M1**
Multiplies two adjacent matrices; P must be correct.	**M1**
Intermediate matrix product correct.	**A1**
$\mathbf{A} = \begin{pmatrix} 1 & \frac{1}{2a} & -\frac{4a+1}{6a^2} \\ 0 & 2 & \frac{1}{3a} \\ 0 & 0 & 3 \end{pmatrix}$ (OE).	**A1**
	Total: 10

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Q53

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Mark scheme.

Finds determinant and forms quadratic equation (condone one error in determinant calc): $\begin{vmatrix} 1 & -1 & 2k \\ k & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow 2k^2 + 3k + 1 = 0.$	**(a) M1**
$2k^2 + 3k + 1 = 0 \Rightarrow k = -\frac{1}{2}, -1$ (each value A1).	**A1 A1**
Derives contradiction clearly: $x - y - z = 1$, $-\frac{1}{2}x + y + 2z = 2 \Rightarrow \frac{1}{2}x + z = 3$, $2x - y + z = 3 \Rightarrow x + 2z = 2$, contradictory.	**(b) M1** **A1**
Triangular prism (accept diagram).	**B1**
Derives contradiction: $x - y - 2z = 1$, $-x + y + 2z = 2 \Rightarrow 1 = -2$.	**(c) M1** **A1**
Two parallel planes, other plane not parallel.	**B1**
Parallel planes not identical (accept diagram).	**B1**
Finds characteristic equation of $\mathbf{A}$: $\det\begin{pmatrix} 1-\lambda & -1 & -2 \\ -1 & 1-\lambda & 2 \\ 2 & -1 & 1-\lambda \end{pmatrix} = 0.$	**(d) M1**
$-\lambda^3 + 3\lambda^2 - 8\lambda = 0$ (must be simplified equation).	**A1**
Substitutes matrix and makes $\mathbf{A}^3$ the subject: $\mathbf{A}^3 = 3\mathbf{A}^2 - 8\mathbf{A}$.	**M1**
Multiplies by $\mathbf{A}$ and substitutes for $\mathbf{A}^3$: $\mathbf{A}^4 = 3\mathbf{A}^3 - 8\mathbf{A}^2 = 3(3\mathbf{A}^2 - 8\mathbf{A}) - 8\mathbf{A}^2$.	**M1**
$\mathbf{A}^4 = \mathbf{A}^2 - 24\mathbf{A}$.	**A1**
	Total: 15

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Mark scheme.

Finds determinant: $\begin{vmatrix} 5 & a & 0 \\ 20 & -5 & 0 \\ 2 & -3 & 1 \end{vmatrix} = -20a - 25.$	**(a) M1**
$a \neq -\frac{5}{4}$.	**A1**
Three planes intersect at a single point.	**B1**
Shows contradiction clearly: if $a = -\frac{5}{4}$ then $5x - \frac{5}{4}y = 3$, $20x - 5y = 2 \Rightarrow 2 = 12$.	**(b) B1**
Two distinct parallel planes.	**B1**
Other plane not parallel.	**B1**
Eigenvalues of $\mathbf{A}$ are $5, -5, 1$ (lower diagonal matrix or characteristic equation).	**(c) B1**
Uses vector product (or equations) to find eigenvectors: $\lambda = 5$: $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.	**M1 A1**
$\lambda = -5$: $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$.	**A1**
$\lambda = 1$: $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.	**A1**
$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 64 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 16 \end{pmatrix}$ (eigenvalues of $(\mathbf{A} + 3\mathbf{I})^2$: $8^2, (-2)^2, 4^2$). Or correctly matched permutations of columns.	**M1 A1**
	Total: 13

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Questions available in the companion questions booklet.

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