Further Math with Ali Hashir

Differentiation

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/11, 9231/12 • May/June 2015 • Question 6

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Mark scheme.

$2x - 6\left(x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y\right) + 50y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	**M1A1A1**
At $(3,1)$ $\ 6 - 18y' - 6 + 50y' = 0 \Rightarrow y' = 0$ $\ (\text{AG})$	**A1**
$2 - 6(xy'' + y' + y') + 50[(y')^2 + yy''] = 0$	**B1B1**
At $(3,1)$ $\ 2 - 18y'' + 50y'' = 0 \Rightarrow y'' = -\dfrac{1}{16}$	**M1A1**
$\Rightarrow$ maximum. (All previous marks required for final mark, i.e. CSO)	**A1**
	Total: 9

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Q2

9231/13 • May/June 2015 • Question 7

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Mark scheme.

$2x + 2\left(x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y\right) - 8y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ (1st B1 for 1st and 3rd terms, 2nd B1 for rest including $= 0$).	**B1B1**
$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}(x - 4y) = -(x + y) \Rightarrow y = -x$ when $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	**B1**
Substituting for $y \Rightarrow x^2 - 2x^2 - 4x^2 + 20 = 0 \Rightarrow 5x^2 = 20 \Rightarrow x \pm 2$	**M1**
Coordinates are $(2, -2)$ and $(-2, 2)$	**A1**
$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}(x - 4y) + \dfrac{\mathrm{d}y}{\mathrm{d}x}\left(1 - 4\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) = -1 - \dfrac{\mathrm{d}y}{\mathrm{d}x}$ (OE)	**M1A1**
Substituting either $(2, -2)$ or $(-2, 2)$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	**dM1**
At $(2, -2)$ $y''(2) = -\dfrac{1}{10} \Rightarrow$ max.	**A1**
At $(-2, 2)$ $y''(2) = \dfrac{1}{10} \Rightarrow$ min.	**A1**
	Total: 10

Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 1

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Mark scheme.

$\dot{x} = -6\cos^2 t \sin t,\ \dot{y} = 6\sin^2 t \cos t$	**B1**
$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = -\tan t$ (OE)	**B1**
$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\sec^2 t \times \dfrac{-1}{6\cos^2 t \sin t} = \dfrac{1}{6}\sec^4 t \operatorname{cosec} t$ AG	**M1A1**
	Total: 4

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Q4

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 8

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Mark scheme.

$2x + 4(xy' + y) - 2yy' = 0 \quad (*)$	**M1A1**
$y' = 0 \Rightarrow 2x + 4y = 0 \Rightarrow x = -2y$ (AG)	**A1**
At stationary points $4y^2 - 8y^2 - y^2 + 20 = 0 \Rightarrow 5y^2 = 20 \Rightarrow y = \pm 2$	**M1A1**
Coordinates of stationary points are $(4, -2)$ and $(-4, 2)$	**A1**
From $(*)$: $x + 2(xy' + y) - yy' = 0$	
Differentiating: $1 + 2(xy'' + y' + y') - \left(yy'' + [y']^2\right) = 0$ (or by quotient rule)	**M1A1**
At $(4, -2)$ with $y' = 0$: $1 + 8y'' + 2y'' = 0 \Rightarrow y'' = -\frac{1}{10} \Rightarrow$ maximum.	**M1A1**
At $(-4, 2)$ with $y' = 0$: $1 - 8y'' - 2y'' = 0 \Rightarrow y'' = \frac{1}{10} \Rightarrow$ minimum.	**A1**
	Total: 11

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Q5

9231/11, 9231/12 • May/June 2017 • Question 3

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Mark scheme.

$\left(1+x^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=1$ or $\sec^2 y\dfrac{\mathrm{d}y}{\mathrm{d}x}=1\Rightarrow\left(1+x^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=1$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos^2 y$	**M1**
$\Rightarrow 2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+\left(1+x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=0\Rightarrow\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2x\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ (AG)	**M1A1**
Alt method: $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos^2 y\Rightarrow\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\cos y(-\sin y)\dfrac{\mathrm{d}y}{\mathrm{d}x}=-2x\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	**M1A1**
$y'(1)=\cos^2\left(\dfrac{\pi}{4}\right)\Rightarrow y'(1)=\dfrac{1}{2}$	**B1**
$\Rightarrow y''(1)=-\dfrac{1}{2}$	**B1 FT**
	Total: 5

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Q6

9231/13 • May/June 2017 • Question 4

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Mark scheme.

$3x^2 - \left(3y + 3x\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) + 2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	**B1B1**
At $(0,2)$: $-6 + 4\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3}{2}$	**B1**
$6x - 3\dfrac{\mathrm{d}y}{\mathrm{d}x} - \left(3\dfrac{\mathrm{d}y}{\mathrm{d}x} + 3x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)$	**M1A1**
$+2\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 2y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$	**A1**
At $(0,2)$: $-\dfrac{9}{2} - \dfrac{9}{2} + \dfrac{9}{2} + 4\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0 \Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{9}{8}$	**A1**
	Total: 7

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Q7

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 5

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Mark scheme.

$6x^2 + 6xy + 3x^2y' - 9y^2y' = 0 \ (*) \Rightarrow 2x(x+y) = (3y^2 - x^2)y'$	**M1A1**
$y' = 0$ and $x \neq 0 \Rightarrow x = -y$	**M1A1**
$\Rightarrow 2x^3 - 3x^3 + 3x^3 = 16 \Rightarrow A$ is $(2, -2)$	**A1**
$12x + 6xy' + 6y + 6xy' + 3x^2y'' - \left[18y(y')^2 + 9y^2y''\right] = 0$	***M1**
$x = 2 \ \ y = -2 \ \ y' = 0 \Rightarrow 8 - 4 + 4y'' - 12y'' = 0$	**DM1**
$\Rightarrow y'' = \frac{1}{2}$	**A1**
	Total: 8

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Q8

9231/13 • May/June 2018 • Question 1

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Mark scheme.

$\left(0+\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3=(-1)^2+0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=1$	**B1**
$3\left(x+\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\left(1+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)$	**M1A1**
Method mark for good attempt at implicit differentiation of LHS.	
$=2y\frac{\mathrm{d}y}{\mathrm{d}x}+1$	**B1**
Note: may expand bracket before differentiation but M1 is still for implicit differentiation	
$\Rightarrow 3\left(1+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)=-2+1\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{4}{3}$	**A1**
	Total: 5

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Q9

9231/12 • October/November 2018 • Question 11

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Mark scheme.

$\frac{\mathrm{d}x}{\mathrm{d}t} = 18 - 2t,\ \frac{\mathrm{d}y}{\mathrm{d}t} = 12t^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x} = \frac{12t^{\frac{1}{2}}}{18-2t} = \frac{6t^{\frac{1}{2}}}{9-t}$ (AEF)	**B1**
$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) \times \frac{\mathrm{d}t}{\mathrm{d}x}$ (Uses chain rule again)	**M1**
$= \frac{3t^{-\frac{1}{2}}(9-t) + 6t^{\frac{1}{2}}}{(9-t)^2(18-2t)}$ (Uses quotient rule)	**DM1**
$= \frac{3t^{-\frac{1}{2}}(9-t+2t)}{2(9-t)^3} = \frac{3(9+t)}{2t^{\frac{1}{2}}(9-t)^3}$ (AG)	**A1**

Total: 4

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Q10

9231/11, 9231/12 • May/June 2019 • Question 1

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Mark scheme.

$-\sin y\dfrac{\mathrm{d}y}{\mathrm{d}x}=1\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=-(\sin y)^{-1}$	**M1A1**
$\dfrac{\mathrm{d}^2y}{\mathrm{d}^2x}=(\sin y)^{-2}\cos y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=-\cot y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	**M1A1**
$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\left(\sin\dfrac{\pi}{3}\right)^{-1}=-\dfrac{2}{\sqrt{3}}$	**B1**
$\dfrac{\mathrm{d}^2y}{\mathrm{d}^2x}=-\dfrac{1}{\sqrt{3}}\left(-\dfrac{2}{\sqrt{3}}\right)^2=-\dfrac{4}{3\sqrt{3}}=-\dfrac{4}{9}\sqrt{3}$	**B1**
	Total: 6

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Q11

9231/21, 9231/22 • May/June 2020 • Question 2

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Mark scheme.

(a)

$\ln y = x \ln 2 \Rightarrow \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \ln 2$	**M1 A1**
$\frac{\mathrm{d}y}{\mathrm{d}x} = y \ln 2 = 2^x \ln 2$ (AG)	**A1**

(b)

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2^x(\ln 2)^2$	**B1**

(c)

$y(0) = 1,\ y'(0) = \ln 2,\ y''(0) = (\ln 2)^2$	**B1**
$2^x = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \cdots = 1 + (\ln 2)x + \frac{(\ln 2)^2}{2}x^2 + \ldots$	**M1 A1**

Total: 7

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Q12

9231/21, 9231/23 • October/November 2020 • Question 1

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Mark scheme.

$\mathrm{f}'(x) = -2x\mathrm{e}^{-x^2}$.	**B1**
$\mathrm{f}''(x) = 4x^2\mathrm{e}^{-x^2} - 2\mathrm{e}^{-x^2}$.	**B1**
Evaluates derivatives at zero: $\mathrm{f}(0) = 1$, $\mathrm{f}'(0) = 0$, $\mathrm{f}''(0) = -2$.	**M1**
$\mathrm{e}^{-x^2} = 1 - x^2$.	**M1 A1**
(b) Substitutes $1 - x^2$ or better: $\int_0^{\frac{1}{5}} 1 - x^2 \,\mathrm{d}x = \left[x - \frac{1}{3}x^3\right]_0^{\frac{1}{5}} = \frac{74}{375}$.	**M1 A1**

Total: 7

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Q13

9231/21, 9231/23 • October/November 2020 • Question 5

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Mark scheme.

Differentiates both sides with respect to t: $-\sin y\dfrac{\mathrm{d}y}{\mathrm{d}t}=1$.	**M1 A1**
Applies $\sin^2 y+\cos^2 y=1$ with $0<y<\pi \Rightarrow \sin y>0$: $-\sqrt{1-\cos^2 y}\,\dfrac{\mathrm{d}y}{\mathrm{d}t}=1$.	**M1**
AG, justifies taking positive square root: $\dfrac{\mathrm{d}y}{\mathrm{d}t}=-\dfrac{1}{\sqrt{1-t^2}}$.	**A1**
$\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{1}{\sqrt{1+t^2}}$.	**B1**
Finds first derivative: $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\sqrt{\dfrac{1+t^2}{1-t^2}}$.	**B1**
Differentiates $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ with respect to t.	**M1**
Applies chain rule: $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{\mathrm{d}}{\mathrm{d}t}\left(-\sqrt{\dfrac{1+t^2}{1-t^2}}\right)\times\dfrac{\mathrm{d}t}{\mathrm{d}x}$.	**M1**
OE (simplified): $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\dfrac{2t}{(1-t^2)^{3/2}}$.	**A1**
	Total: 9

Q14

9231/22 • October/November 2020 • Question 1

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Mark scheme.

$f'(x) = \sec^2\left(x + \frac{1}{4}\pi\right)$.	**B1**
$f''(x) = 2\sec^2\left(x + \frac{1}{4}\pi\right)\tan\left(x + \frac{1}{4}\pi\right)$.	**B1**
Evaluates derivatives at zero: $f(0) = 1$, $f'(0) = 2$, $f''(0) = 4$.	**M1**
$\tan\left(x + \frac{1}{4}\pi\right) = 1 + 2x + 2x^2$.	**M1 A1**
	Total: 5

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Q15

9231/22 • October/November 2020 • Question 5

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Mark scheme.

Differentiates y^2 correctly: $\dfrac{\mathrm{d}}{\mathrm{d}x}(y^2) = 2yy'$.	**B1**
Differentiates $(xy+1)^2$ correctly: $\dfrac{\mathrm{d}}{\mathrm{d}x}\left((xy+1)^2\right) = 2(xy+1)(xy'+y)$.	**B1**
Substitutes $(1,1)$, AG: $(1)y' + ((1)+1)(y'+1) = 0 \Rightarrow y' = -\frac{2}{3}$.	**B1**
Differentiates yy': $yy'' + (y')^2$.	**B1**
Differentiates $(xy+1)(xy'+y)$: $+(xy+1)(xy''+y'+y')+(xy'+y)(xy'+y) = 0$.	**B1 B1**
Substitutes $(1,1)$ and $y' = -\frac{2}{3}$: $y'' + \left(-\frac{2}{3}\right)^2 + 2\left(y'' - \frac{4}{3}\right) + \left(-\frac{2}{3}+1\right)\left(\frac{1}{3}\right) = 0$.	**M1**
$y'' = \dfrac{19}{27}$.	**A1**
	Total: 8

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Q16

9231/21, 9231/23 • October/November 2021 • Question 1

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Mark scheme.

$e^x(\tan x + \sec^2 x)$. Finds first derivative.	**B1**
$e^x(\tan x + 2\sec^2 x + 2\sec^2 x \tan x)$. Finds second derivative.	**M1 A1**
$y(0) = 0$, $y'(0) = 1$, $y''(0) = 2$. Evaluates derivatives at $x = 0$.	**M1**
$y = x + x^2$.	**A1**
	Total: 5

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Q17

9231/21, 9231/23 • October/November 2021 • Question 3

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Mark scheme.

$3xy^2y' + y^3$. Differentiates xy^3.	**B1**
$-4(x^3y' + 3x^2y) = 0$. Differentiates x^3y.	**B1**
$3(-1)(1)y' + 1^3 - 4((-1)^3y' + 3(-1)^2(1)) = 0$ leading to $y' = 11$ (AG).	**B1**
Differentiates $(3xy^2 - 4x^3)y'$.	**B1 B1**
$+3y^2y' - 12(x^2y' + 2xy) = 0$. Differentiates $y^3 - 12x^2y$.	**B1**
Substitutes $(-1, 1)$: $y''(-75) + 3(11) - 12(9) = 0$.	**M1**
$y'' = 900$.	**A1**
	Total: 8

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Q18

9231/22 • October/November 2021 • Question 1

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Mark scheme.

Combines using correct power series (or sensible attempt at all four derivatives of y): $\sinh(x^2) + \cosh(x^2)$.	**M1**
$1 + x^2 + \frac{1}{2}x^4$.	**A1**
$\frac{1}{2} \times 4! = 12$.	**B1**
Substitutes their power series (must be at least $a + bx^2$): $\int_0^{\frac{1}{2}} 1 + x^2 + \frac{1}{2}x^4 \, \mathrm{d}x$.	**M1**
$\left[x + \frac{1}{3}x^3 + \frac{1}{10}x^5\right]_0^{\frac{1}{2}} = \frac{523}{960}$.	**A1**

Total: 5

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Q19

9231/22 • October/November 2021 • Question 5(b)

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Mark scheme.

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\dot{y}}{\dot{x}} = \frac{4+\frac{3}{2}t^{-2}}{3-2t^{-2}}$. Finds $\frac{\mathrm{d}y}{\mathrm{d}x}$.	**B1**
$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{4+\frac{3}{2}t^{-2}}{3-2t^{-2}}\right) = \frac{(3-2t^{-2})(-3t^{-3})-(4+\frac{3}{2}t^{-2})(4t^{-3})}{(3-2t^{-2})^2}$. Differentiates $\frac{\mathrm{d}y}{\mathrm{d}x}$ with respect to t.	**B1**
$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{4+\frac{3}{2}t^{-2}}{3-2t^{-2}}\right) \times \frac{\mathrm{d}t}{\mathrm{d}x} = \frac{-25t^{-3}}{(3-2t^{-2})^3} = -25$ when $t=1$. Applies chain rule.	**M1 A1**
	Total: 4

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Q20

9231/21, 9231/22 • May/June 2022 • Question 5

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Mark scheme.

Differentiates $(x+1)y$ correctly: $\frac{d}{dx}\big((x+1)y\big) = (x+1)y' + y$	**(a) B1**
Differentiates $(x+y+1)^3$ correctly: $\frac{d}{dx}\big((x+y+1)^3\big) = 3(x+y+1)^2(y'+1)$	**B1**
Substitutes $(0,0)$, AG: $(1)y' + 3(y'+1) = 0 \Rightarrow y' = -\frac{3}{4}$	**B1**
Differentiates $(x+1)y' + y$: $(x+1)y'' + 2y'$	**(b) B1**
Differentiates $3(x+y+1)^2(1+y')$: $+3(x+y+1)^2y'' + 6(1+y')^2(x+y+1) = 0$ (Expect $6(x+y+1)(1+y')+6(x+y+1)(1+y')y'$ instead of $6(1+y')^2(x+y+1)$.)	**B1 B1**
Substitutes $(0,0)$: $y'' + 2(-\frac{3}{4}) + 3y'' + 6\left(\frac{1}{4}\right)^2 = 0$	**M1**
$y'' = \frac{9}{32}$	**A1**
Finds Maclaurin's series: $y = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 = -\frac{3}{4}x + \frac{9}{64}x^2$	**M1 A1**
	Total: 10

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Q21

9231/23 • May/June 2022 • Question 2(a)

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Mark scheme.

Finds first derivative: $\frac{d}{dx}(-\ln\cos x) = \tan x$	**B1**
Finds second derivative: $\frac{d}{dx}(\tan x) = \sec^2 x$	**B1**
Evaluates second derivative at zero (accept $\frac{1}{2}x^2$): $\frac{y''(0)}{2!} = \frac{1}{2}$	**M1 A1**
	Total: 4

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Q22

9231/23 • May/June 2022 • Question 4

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Mark scheme.

$\frac{dy}{dt} = 1 + \frac{1}{\sqrt{1+t^2}}$	**(a) B1**
$\frac{dx}{dt} = -1 + \frac{1}{1+t^2}$	**B1**
Uses chain rule: $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{\sqrt{1+t^2}+1}{\sqrt{1+t^2}} \times \frac{1+t^2}{-t^2}$	**M1**
AG: $-\sqrt{1+t^2}\left(\frac{\sqrt{1+t^2}+1}{t^2}\right) = -\frac{1+t^2+\sqrt{1+t^2}}{t^2}$	**A1**
Applies quotient/product rule: $\frac{d}{dt}\left(\frac{dy}{dx}\right) = -\frac{t^2\left(2t + t(1+t^2)^{-\frac{1}{2}}\right) - 2t\left(1+t^2+\sqrt{1+t^2}\right)}{t^4}$	**(b) M1 A1**
Uses chain rule (simplification not required for A1): $\frac{d^2y}{dx^2} = \frac{t^2(1+t^2)^{-\frac{1}{2}} - 2\left(1+\sqrt{1+t^2}\right)}{t^5} \times (1+t^2) = -\frac{\left(t^2+2+2\sqrt{t^2+1}\right)\sqrt{1+t^2}}{t^5}$	**M1 A1**
$= -\frac{80}{3}$ when $t = \frac{3}{4}$	**A1**
	Total: 9

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Q23

9231/21, 9231/23 • October/November 2022 • Question 1

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Mark scheme.

$y' = \dfrac{e^x}{1+e^x}$.	**B1**
$y'' = \dfrac{e^x}{(1+e^x)^2}$.	**B1**
$y(0) = \ln 2$, $y'(0) = \frac{1}{2}$, $y''(0) = \frac{1}{4}$.	**B1**
$y = y(0) + y'(0)x + \frac{1}{2!}y''(0)x^2 + \ldots$ (allow 2! missing).	**M1**
$\ln 2 + \frac{1}{2}x + \frac{1}{8}x^2$ (decimal used for $\ln 2$ scores A0).	**A1**
	Total: 5

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Q24

9231/22 • October/November 2022 • Question 2

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Mark scheme.

(a)

$\frac{\mathrm{d}}{\mathrm{d}x}((x+1)y) = (x+1)y' + y.$	**B1**
$\frac{\mathrm{d}}{\mathrm{d}x}(y^2) = 2yy'.$	**B1**
$(1)y' - 2 + 2(-2)y' = 0$ leading to $y' = -\frac{2}{3}$. AG.	**B1**

(b)

$(x+1)y'' + 2y'.$	**B1**
$+2yy'' + 2(y')^2 = 0.$	**B1**
Substitutes $(0,-2)$ into expression with y'': $y'' + 2(-\frac{2}{3}) + 2(-2)y'' + 2(-\frac{2}{3})^2 = 0.$	**M1**
$y'' = -\frac{4}{27}.$	**A1**

Total: 7

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Q25

9231/22 • October/November 2022 • Question 3(b)

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Mark scheme.

Uses expansion of e^x (or finds first/second derivatives $e^x - \frac{1}{4}e^{-x}$ and $e^x + \frac{1}{4}e^{-x}$): $\left(1 + x + \frac{x^2}{2!} + \ldots\right) + \frac{1}{4}\left(1 - x + \frac{(-x)^2}{2!} + \ldots\right)$.	**M1**
$\frac{5}{4} + \frac{3}{4}x + \frac{5}{8}x^2$.	**A1**
	Total: 2

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Q26

9231/23 • May/June 2023 • Question 1

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Mark scheme.

Correct first derivative: $f'(x) = (1-x^2)^{-\frac{1}{2}}$.	**B1**
Correct second derivative: $f''(x) = x(1-x^2)^{-\frac{3}{2}}$.	**B1**
Differentiates their $f''(x)$ using product rule: $f'''(x) = 3x^2(1-x^2)^{-\frac{5}{2}} + (1-x^2)^{-\frac{3}{2}}$.	**M1**
Evaluates their derivatives at $x = 0$ (must have attempted all three derivatives): $f(0) = 0$, $f'(0) = 1$, $f''(0) = 0$, $f'''(0) = 1$.	**M1**
$\sin^{-1} x = x + \frac{1}{6}x^3$ (CWO). (Alternative: differentiate $\sin y = x$ implicitly to obtain $f'(x) = \sec y$, $f''(x) = \tan y \sec^2 y$, $f'''(x) = 3\sec^5 y - 2\sec^3 y$, then evaluate at $x = 0$ — same mark allocation.) **(b)**	**A1**
$\frac{151}{750}$.	**B1**
	Total: 6

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Q27

9231/23 • May/June 2023 • Question 4

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Mark scheme.

Differentiates y^3 correctly: $\dfrac{\mathrm{d}}{\mathrm{d}x}(4y^3) = 12y^2y'$.	**B1**
Differentiates $(x+y)^6$ correctly: $\dfrac{\mathrm{d}}{\mathrm{d}x}\big((x+y)^6\big) = 6(x+y)^5(1+y')$.	**B1**
Substitutes $(-4, 3)$ (AG): $12(3^2)y' + 6(-1)(1+y') = 0 \Rightarrow y' = \frac{1}{17}$. **(b)**	**B1**
Differentiates y^2y': $2y^2y'' + 4y(y')^2$.	**B1**
Differentiates $(x+y)^5(1+y')$: $+(x+y)^5y'' + 5(x+y)^4(1+y')^2 = 0$.	**B1 B1**
Substitutes $(-4, 3)$: $17y'' + 12\left(\frac{1}{17}\right)^2 + 5 \times (-1)^4 \times \left(\frac{18}{17}\right)^2 = 0 \Rightarrow 17y'' + \frac{96}{17} = 0$.	**M1**
$y'' = -\dfrac{96}{289}$.	**A1**
	Total: 8

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Q28

9231/21, 9231/23 • October/November 2023 • Question 5(b)

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Mark scheme.

Finds first derivative: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\dot{y}}{\dot{x}} = \dfrac{2}{t^{\frac{1}{2}} - t^{-\frac{1}{2}}}$.	**B1**
Differentiates $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ with respect to t: $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{2}{t^{\frac{1}{2}} - t^{-\frac{1}{2}}}\right) = \dfrac{-2\left(\frac{1}{2}t^{-\frac{1}{2}} + \frac{1}{2}t^{-\frac{3}{2}}\right)}{\left(t^{\frac{1}{2}} - t^{-\frac{1}{2}}\right)^2}$.	**B1**
Applies chain rule (OE, does not have to be simplified for A1): $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{2}{t^{\frac{1}{2}} - t^{-\frac{1}{2}}}\right) \times \dfrac{\mathrm{d}t}{\mathrm{d}x} = -\dfrac{t+1}{(t-1)^3}$.	**M1 A1**
$0 < t < 1$ (accept $-1 < t < 1$). CWO.	**A1**
	Total: 5

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Q29

9231/22 • October/November 2023 • Question 1

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Mark scheme.

	EITHER Solution 1
Changes $\ln(x+2)$ or $\ln(x^2+5)$ so that the formula given in the list of formulae (MF19) can be applied: $\ln(x+2) = \ln\left(2\left(1+\frac{1}{2}x\right)\right) = \ln 2 + \ln\left(1+\frac{1}{2}x\right)$.	**M1**
$\ln(x+2) = \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots$	**A1**
$\ln(x^2+5) = \ln\left(5\left(1+\frac{1}{5}x^2\right)\right) = \ln 5 + \frac{1}{5}x^2 + \ldots$	**A1**
Sums power series: $\left(\ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots\right) + \left(\ln 5 + \frac{1}{5}x^2 + \ldots\right)$.	**M1**
	OR Solution 2
Finds first derivative: $f'(x) = (x+2)^{-1} + 2x(x^2+5)^{-1}$.	**(M1 A1)**
Finds second derivative: $f''(x) = -(x+2)^{-2} - (2x)^2(x^2+5)^{-2} + 2(x^2+5)^{-1}$.	**(A1)**
Evaluates derivatives at zero: $f(0) = \ln 10$, $f'(0) = \frac{1}{2}$, $f''(0) = \frac{3}{20}$.	**(M1)**
$\ln(x+2) + \ln(x^2+5) = \ln 10 + \frac{1}{2}x + \frac{3}{40}x^2$. Accept $\ln 10$ written as $\ln 2 + \ln 5$ but do not accept decimals. WWW.	**A1**
	Total: 5

Q30

9231/22 • October/November 2023 • Question 2

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Mark scheme.

(a)

$\frac{\mathrm{d}y}{\mathrm{d}t} = \mathrm{e}^t(t+1)$, $\frac{\mathrm{d}x}{\mathrm{d}t} = -t^{-2}$.	**B1**
Uses chain rule (AG): $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x} = -t^2\mathrm{e}^t(t+1) = -\mathrm{e}^t(t^3+t^2)$.	**M1 A1**

(b)

Applies product rule: $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = -\mathrm{e}^t(3t^2+2t) - \mathrm{e}^t(t^3+t^2) = -\mathrm{e}^t(t^3+4t^2+2t)$.	**M1 A1**
Uses chain rule, multiplies by their $\frac{\mathrm{d}t}{\mathrm{d}x}$ for M1: $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = t^3\mathrm{e}^t(t^2+4t+2)$.	**M1 A1**

Total: 7

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Q31

9231/21, 9231/22 • May/June 2024 • Question 2

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Mark scheme.

Changes e^{1+x^2} or e^{1-x} so that the formula given in the list of formulae (MF19) can be applied: $e^{1+x^2} = e(1 + x^2 + \dots)$.	**B1**
$e^{1-x} = e(1 - x + \frac{1}{2}x^2 + \dots)$.	**B1**
Sums power series.	**M1**
$e^{1+x^2} + e^{1-x} = 2e - ex + \frac{3}{2}ex^2 + \dots$	**A1**

Total: 4

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Q32

9231/21, 9231/22 • May/June 2024 • Question 3

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Mark scheme.

(a)

$\dfrac{\mathrm{d}y}{\mathrm{d}t} = \cos^{-1} t - \dfrac{t}{\sqrt{1-t^2}}, \quad \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{1}{\sqrt{1-t^2}}.$	**B1**
Uses chain rule, AG: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -t + \sqrt{1-t^2}\cos^{-1} t.$	**M1 A1**

(b)

Applies product rule: $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) = -2 - \dfrac{t\cos^{-1} t}{\sqrt{1-t^2}}.$	**M1 A1**
Uses chain rule: $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2\sqrt{1-t^2} - t\cos^{-1} t.$	**M1 A1**

Total: 7

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Q33

9231/23 • May/June 2024 • Question 3

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Mark scheme.

(a)	
Differentiates $8y^3$ correctly: $24y^2y'$.	**B1**
Differentiates $2xy$ correctly: $2(xy' + y)$.	**B1**
Substitutes $(-2,-1)$ into $3x^2 + 2xy' + 2y + 24y^2y' = 0 \Rightarrow 10 + 20y' = 0 \Rightarrow y' = -\frac{1}{2}$. AG.	**B1**
(b)	
Differentiates $3x^2$: $6x + 2xy'' + 2y'$.	**B1**
Differentiates $2y + 24y^2y'$: $2y' + 24y^2y'' + 48y(y')^2 = 0$.	**B1 B1**
Substitutes $(-2,-1)$ into their expression (must have derivatives y' and y''): $6(-2) + 2(-2)y'' + 4(-\frac{1}{2}) + 24(-1)^2y'' + 48(-1)(-\frac{1}{2})^2 = 0 \Rightarrow 20y'' - 26 = 0$.	**M1**
$y'' = \dfrac{13}{10}$.	**A1**
	Total: 8

Q34

9231/21, 9231/23 • October/November 2024 • Question 2

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Mark scheme.

(a)

Uses chain rule: $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -\frac{1}{\sqrt{1-t^2}} \times (-t^2) = \frac{t^2}{\sqrt{1-t^2}}$ (AG).	**M1A1**

(b)

Applies quotient rule: $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{2t\sqrt{1-t^2} + t^3(1-t^2)^{-1/2}}{1-t^2}$.	**M1A1**
Uses chain rule: $\frac{d^2y}{dx^2} = \frac{2t\sqrt{1-t^2} + t^3(1-t^2)^{-1/2}}{1-t^2}(-t^2) = -t^3(1-t^2)^{-3/2}(2-t^2)$.	**M1A1**

Total: 6

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Q35

9231/22 • October/November 2024 • Question 2

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Mark scheme.

(a)

Differentiates y^2 correctly: $\dfrac{d}{dx}(4y^2) = 8yy'$.	**B1**
Differentiates $\ln(xy)$ correctly: $\dfrac{d}{dx}(4\ln(xy)) = 4y'y^{-1} + 4x^{-1}$.	**B1**
Substitutes $\left(2, \frac{1}{2}\right)$ into $8y'y + 4y'y^{-1} + 4x^{-1} = 0$ to give $y' = -\dfrac{1}{6}$ (AG).	**B1**
EITHER (Solution 1):	**(b)**
Differentiates $8y'y + 4y'y^{-1} + 4x^{-1} = 0$: $8(y')^2 + 8y''y - 4(y')^2y^{-2} + 4y^{-1}y'' - 4x^{-2} = 0$.	**B1B1**
Substitutes $\left(2, \frac{1}{2}\right)$ and $y' = -\frac{1}{6}$. OR (Solution 2):	**M1**
$y' = -x^{-1}(2y + y^{-1})^{-1}$; $y'' = x^{-1}(2y + y^{-1})^{-2}(2y' - y^{-2}y') + x^{-2}(2y + y^{-1})^{-1}$.	**B1B1**
Substitutes $\left(2, \frac{1}{2}\right)$ and $y' = -\frac{1}{6}$.	**M1**
$12y'' - \dfrac{11}{9} = 0 \Rightarrow y'' = \dfrac{11}{108}$.	**A1**
	Total: 7

Q36

9231/23 • May/June 2025 • Question 1

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Mark scheme.

Finds first derivative: $f'(x) = -(x+2)^{-2}\mathrm{e}^{(x+2)^{-1}}$.	**B1**
Finds second derivative: $f''(x) = (x+2)^{-4}\mathrm{e}^{(x+2)^{-1}} + 2(x+2)^{-3}\mathrm{e}^{(x+2)^{-1}} = \dfrac{2x+5}{(x+2)^4}\mathrm{e}^{(x+2)^{-1}}$.	**B1**
Evaluates derivatives at zero: $f(0) = \mathrm{e}^{1/2}$, $f'(0) = -\frac{1}{4}\mathrm{e}^{1/2}$, $f''(0) = \frac{5}{16}\mathrm{e}^{1/2}$. Must have attempted second derivative using the product rule.	***M1**
$\mathrm{e}^{(x+2)^{-1}} = \mathrm{e}^{1/2} - \frac{1}{4}\mathrm{e}^{1/2}x + \frac{5}{32}\mathrm{e}^{1/2}x^2$.	**DM1 A1**
	Total: 5

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Q37

9231/23 • May/June 2025 • Question 3

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Mark scheme.

(a)

Differentiates y^2 correctly: $\dfrac{\mathrm{d}}{\mathrm{d}x}(9y^2) = 18yy'$.	**B1**
Differentiates $\sinh^{-1}(xy)$ correctly: $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(-3\sinh^{-1}(xy)\right) = -3\dfrac{xy'+y}{\sqrt{1+x^2y^2}}$.	**M1 A1**
Substitutes $\left(4, \frac{1}{3}\right)$ and shows at least one intermediate step: $\frac{18}{3}y' - 3\dfrac{4y'+\frac{1}{3}}{\frac{5}{3}} = 0 \Rightarrow y' = -\frac{1}{2}$, AG.	**A1**

(b)

Differentiates $18yy'$: $18yy'' + 18(y')^2$.	**B1**
Differentiates $-3\dfrac{xy'+y}{\sqrt{1+x^2y^2}}$.	**B1 B1**
Substitutes $\left(4, \frac{1}{3}\right)$ and $y' = -\frac{1}{2}$: $\frac{18}{3}y'' + \frac{18}{4} - 3\dfrac{\frac{5}{3}(4y''-1) - \left(\frac{5}{3}\right)^2\left(\frac{3}{5}\right)\left(\frac{4}{3}\right)}{\frac{25}{9}} = 0$.	**M1**
$-\frac{6}{5}y'' + \frac{87}{10} = 0 \Rightarrow y'' = \frac{29}{4}$.	**A1**

Total: 9

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Q38

9231/24 • May/June 2025 • Question 4

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Mark scheme.

(a)

Factorises or considers other roots of $x = 0$: $x = 0 \Rightarrow (t-1)(t^2+1) = 0$.	**B1**

(b)

$\dfrac{\mathrm{d}x}{\mathrm{d}t} = 3t^2 - 2t + 1$, $\dfrac{\mathrm{d}y}{\mathrm{d}t} = (t+1)\mathrm{e}^t$.	**B1**
Applies chain rule, AG: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} \times \dfrac{\mathrm{d}t}{\mathrm{d}x} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(t+1)\mathrm{e}^t}{3t^2-2t+1}$.	**M1 A1**

(c)

Differentiates $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ with respect to t.	***M1**
Applies chain rule: $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{(3t^2-2t+1)(t+2)\mathrm{e}^t - (t+1)(6t-2)\mathrm{e}^t}{(3t^2-2t+1)^3}$.	**M1 A1**
Evaluates derivatives at $t = 1$: $y(0) = \mathrm{e}$, $y'(0) = \mathrm{e}$, $y''(0) = -\frac{1}{4}\mathrm{e}$.	**A1**
Finds Maclaurin's series (for M1, must have attempted second derivative): $y = y(0) + y'(0)x + \dfrac{y''(0)}{2!}x^2 = \mathrm{e} + \mathrm{e}x - \frac{1}{8}\mathrm{e}x^2$.	**DM1 A1**

Total: 10

Q39

9231/22 • October/November 2025 • Question 3

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Mark scheme.

Differentiates x^2y correctly: $\frac{d}{dx}(x^2y) = x^2y' + 2xy$.	**(a) B1**
Differentiates $(x+y)^3$ correctly: $\frac{d}{dx}((x+y)^3) = 3(x+y)^2(1+y')$.	**M1**
Some evidence of substituting $(-1, 2)$ into an equation set to zero: $y' - 4 + 3(1 + y') = 0 \Rightarrow y' = \frac{1}{4}$. AG.	**A1**
Differentiates $x^2y' + 2xy$: $x^2y'' + 2xy' + 2xy' + 2y$.	**(b) B1** **B1**
Differentiates $3(x+y)^2(1+y')$: $+3(x+y)^2y'' + 6(1+y')^2(x+y)[= 0]$.	**B1 B1**
Substitutes $(-1, 2)$ and $y' = \frac{1}{4}$ into their expression with at least a y'' term: $(-1)^2y'' + 4(-1)(\frac{1}{4}) + 2(2) + 3y'' + 6(\frac{5}{4})^2[= 0]$.	**M1**
$y'' = -\frac{99}{32}$.	**A1**
	Total: 9

Q40

9231/22 • October/November 2025 • Question 7(a)

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Mark scheme.

Applies product rule and formula from MF19 for the derivative of $\sin^{-1} x$: $= -\frac{1}{2}x^2(4-x^2)^{-\frac{1}{2}} + \frac{1}{2}(4-x^2)^{\frac{1}{2}} + 2(4-x^2)^{-\frac{1}{2}}$.	**M1 A1**
As a minimum, working of $\frac{1}{2}(4-x^2)(4-x^2)^{-\frac{1}{2}} + \frac{1}{2}(4-x^2)^{\frac{1}{2}}$ must be seen before the answer $= (4-x^2)^{\frac{1}{2}}$. AG.	**A1**

Total: 3

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Q41

9231/24 • October/November 2025 • Question 7(b)

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Mark scheme.

Finds first derivative: $\dfrac{\dot{y}}{\dot{x}} = \dfrac{6\operatorname{cosec} t - 4\cot^2 t}{8\operatorname{cosec} t + 3\cot^2 t} = \dfrac{6\sin t - 4\cos^2 t}{8\sin t + 3\cos^2 t} = \dfrac{6\sec t - 4\cot t}{8\sec t + 3\cot t}$.	**B1**
Differentiates $\dfrac{dy}{dx}$ with respect to t (enough terms for M1).	**M1**
$\dfrac{100\cos t\sin^2 t + 50\cos^3 t}{(8\sin t + 3\cos^2 t)^2} = \dfrac{50\cos t(\sin^2 t + 1)}{(8\sin t + 3\cos^2 t)^2} = \dfrac{50\cot t\operatorname{cosec} t(\operatorname{cosec}^2 t + 1)}{(8\operatorname{cosec} t + 3\cot^2 t)^2}$.	**A1**
Applies chain rule: $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \times \dfrac{dt}{dx} = \dfrac{50\cot t\operatorname{cosec} t(\operatorname{cosec}^2 t + 1)}{(8\operatorname{cosec} t + 3\cot^2 t)^3}$.	**M1 A1**
	Total: 5

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Written by Ali Hashir.

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