



Further Math with Ali Hashir

Differential Equations

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at vectorspacepk@gmail.com.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/11, 9231/12 • May/June 2015 • Question 11

Show that the substitution $v = \frac{1}{y}$ reduces the differential equation

$$\frac{2}{y^3} \left(\frac{dy}{dx} \right)^2 - \frac{1}{y^2} \frac{d^2y}{dx^2} - \frac{2}{y^2} \frac{dy}{dx} + \frac{5}{y} = 17 + 6x - 5x^2$$

to the differential equation

$$\frac{d^2v}{dx^2} + 2\frac{dv}{dx} + 5v = 17 + 6x - 5x^2.$$

[4]

Hence find y in terms of x , given that when $x = 0$, $y = \frac{1}{2}$ and $\frac{dy}{dx} = -1$.

[10]

Q2

9231/13 • May/June 2015 • Question 9

Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} - 10x = 2 \sin t - 3 \cos t,$$

given that, when $t = 0$, $x = 3.3$ and $\frac{dx}{dt} = 0.9$. [11]

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Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 2

Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 7 - 2t^2.$$

[6]

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Q4

9231/11, 9231/12 • May/June 2016 • Question 9

Find the value of the constant k such that $y = kx^2e^{2x}$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 4e^{2x}. \quad (*)$$

[4]

Hence find the general solution of (*).

[3]

Find the particular solution of (*) such that $y = 3$ and $\frac{dy}{dx} = -2$ when $x = 0$.

[4]

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Q5

9231/13 • May/June 2016 • Question 10

Given that y is a function of x and that $x = e^u$, show that

$$x \frac{dy}{dx} = \frac{dy}{du} \quad \text{and} \quad x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{du^2} - \frac{dy}{du}. \quad [3]$$

Given also that

$$x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + 17y = 34 \ln x + 21,$$

deduce that

$$\frac{d^2y}{du^2} + 2 \frac{dy}{du} + 17y = 34u + 21. \quad [1]$$

Find y in terms of x given that $y = 0$ and $\frac{dy}{dx} = -1$ when $x = 1$. [9]

Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 6

Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 10x = 116 \sin 2t.$$

[8]

State an approximate solution for large positive values of t .

[1]

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Q7

9231/11, 9231/12 • May/June 2017 • Question 10

It is given that $x = t^{\frac{1}{2}}$, where $x > 0$ and $t > 0$, and y is a function of x .

(i) Show that $\frac{dy}{dx} = 2t^{\frac{1}{2}} \frac{dy}{dt}$ and $\frac{d^2y}{dx^2} = 2 \frac{dy}{dt} + 4t \frac{d^2y}{dt^2}$. [3]

(ii) Hence show that the differential equation

$$\frac{d^2y}{dx^2} - \left(8x + \frac{1}{x}\right) \frac{dy}{dx} + 12x^2y = 4x^2e^{-x^2} \quad (*)$$

reduces to the differential equation

$$\frac{d^2y}{dt^2} - 4 \frac{dy}{dt} + 3y = e^{-t}. \quad [1]$$

(iii) Find the general solution of (*), giving y in terms of x . [7]

Q8

9231/13 • May/June 2017 • Question 8

Find the solution of the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 18t^2 + 6t + 1,$$

given that, when $t = 0$, $x = 3$ and $\frac{dx}{dt} = 0$.

[10]

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Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 2

Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 4 - 5t^2.$$

[6]

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Q10

9231/11, 9231/12 • May/June 2018 • Question 7

Find the particular solution of the differential equation

$$49\frac{d^2y}{dx^2} + 14\frac{dy}{dx} + y = 49x + 735,$$

given that when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 0$.

[10]

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0 1
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Q11

9231/13 • May/June 2018 • Question 10

It is given that $t \neq 0$ and

$$t \frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 9tx = 3t^2 + 1.$$

(i) Show that if $y = tx$ then

$$\frac{d^2y}{dt^2} + 9y = 3t^2 + 1.$$

[3]

(ii) Find x in terms of t , given that $x = \frac{1}{9}\pi$ and $\frac{dx}{dt} = \frac{2}{3}$ when $t = \frac{1}{3}\pi$.

[9]

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Q12

9231/11, 9231/13 • October/November 2018 • Question 10

- (i) Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 37 \sin 3t,$$

given that $x = 3$ and $\frac{dx}{dt} = 0$ when $t = 0$. [10]

- (ii) Show that, for large positive values of t and for any initial conditions,

$$x \approx \sqrt{37} \sin(3t - \phi),$$

where the constant ϕ is such that $\tan \phi = 6$. [3]

Q13

9231/12 • October/November 2018 • Question 4

- (i) Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 4\sin t.$$

[7]

- (ii) State an approximate solution for large positive values of t .

[1]

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Q14

9231/11, 9231/12 • May/June 2019 • Question 7

Find the particular solution of the differential equation

$$10\frac{d^2x}{dt^2} + 3\frac{dx}{dt} - x = t + 2,$$

given that when $t = 0$, $x = 0$ and $\frac{dx}{dt} = 0$.

[10]

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Q15

9231/13 • May/June 2019 • Question 8

Find the particular solution of the differential equation

$$9\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + x = 50 \sin t,$$

given that when $t = 0$, $x = 0$ and $\frac{dx}{dt} = 0$.

[10]

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Q16

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 11

It is given that $w = \cos y$ and

$$\tan y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 + 2 \tan y \frac{dy}{dx} = 1 + e^{-2x} \sec y.$$

(i) Show that

$$\frac{d^2 w}{dx^2} + 2 \frac{dw}{dx} + w = -e^{-2x}.$$

[4]

(ii) Find the particular solution for y in terms of x , given that when $x = 0$, $y = \frac{1}{3}\pi$ and $\frac{dy}{dx} = \frac{1}{\sqrt{3}}.$

[10]

Q17

9231/21, 9231/22 • May/June 2020 • Question 1

Find the solution of the differential equation

$$\frac{dy}{dx} + 5y = e^{-7x}$$

for which $y = 0$ when $x = 0$. Give your answer in the form $y = f(x)$.

[6]

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Q18

9231/21, 9231/22 • May/June 2020 • Question 7

It is given that $x = t^3y$ and

$$t^3 \frac{d^2y}{dt^2} + (4t^3 + 6t^2) \frac{dy}{dt} + (13t^3 + 12t^2 + 6t)y = 61e^{\frac{1}{2}t}.$$

(a) Show that

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 61e^{\frac{1}{2}t}.$$

[4]

(b) Find the general solution for y in terms of t .

[7]

Q19

9231/23 • May/June 2020 • Question 1

Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} - 8\frac{dx}{dt} - 9x = 9e^{8t}.$$

[6]

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Q20

9231/23 • May/June 2020 • Question 7

(a) Show that an appropriate integrating factor for

$$(x^2 + 1) \frac{dy}{dx} + y\sqrt{x^2 + 1} = x^2 - x\sqrt{x^2 + 1}$$

is $x + \sqrt{x^2 + 1}$.

[4]

(b) Hence find the solution of the differential equation

$$(x^2 + 1) \frac{dy}{dx} + y\sqrt{x^2 + 1} = x^2 - x\sqrt{x^2 + 1}$$

for which $y = \ln 2$ when $x = 0$. Give your answer in the form $y = f(x)$.

[7]

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Q21

9231/21, 9231/23 • October/November 2020 • Question 2

The variables x and y are related by the differential equation

$$9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + y = 3x^2 + 30x.$$

- (a) Find the general solution for y in terms of x . [6]
- (b) State an approximate solution for large positive values of x . [1]

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Q22

9231/21, 9231/23 • October/November 2020 • Question 6(b)

Find the solution of the differential equation

$$\frac{dy}{d\theta} + y \cot \theta = \sin^3 \theta$$

for which $y = 0$ when $\theta = \frac{1}{2}\pi$.

[6]

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Q23

9231/22 • October/November 2020 • Question 4

Find the solution of the differential equation

$$x \frac{dy}{dx} + 2y = e^x$$

for which $y = 3$ when $x = 1$. Give your answer in the form $y = f(x)$.

[8]

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Q24

9231/22 • October/November 2020 • Question 6

Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 15x = 102 \cos 3t,$$

given that, when $t = 0$, $x = 1$ and $\frac{dx}{dt} = 0$.

[11]

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Q25

9231/21, 9231/22 • May/June 2021 • Question 2

The variables x and y are related by the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2x + 1.$$

- (a) Find the general solution for y in terms of x . [6]
- (b) State an approximate solution for large positive values of x . [1]

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Q26

9231/21, 9231/22 • May/June 2021 • Question 4

Find the solution of the differential equation

$$\sin \theta \frac{dy}{d\theta} + y = \tan \frac{1}{2}\theta,$$

where $0 < \theta < \pi$, given that $y = 1$ when $\theta = \frac{1}{2}\pi$. Give your answer in the form $y = f(\theta)$. [9]

[You may use without proof the result that $\int \operatorname{cosec} \theta \, d\theta = \ln \tan \frac{1}{2}\theta$.]

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Q27

9231/23 • May/June 2021 • Question 5

The variables x and y are related by the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 4e^{-x}.$$

(a) Find the value of the constant k such that $y = kxe^{-x}$ is a particular integral of the differential equation. [4]

(b) Find the solution of the differential equation for which $y = \frac{dy}{dx} = \frac{1}{2}$ when $x = 0$. [6]

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Q28

9231/23 • May/June 2021 • Question 6

(a) Starting from the definitions of $\sinh$ and $\cosh$ in terms of exponentials, prove that

$$2 \sinh^2 x = \cosh 2x - 1.$$

[3]

(b) Find the solution to the differential equation

$$\frac{dy}{dx} + y \coth x = 4 \sinh x$$

for which $y = 1$ when $x = \ln 3$.

[7]

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Q29

9231/21, 9231/23 • October/November 2021 • Question 5

Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\cos x,$$

given that, when $x = 0$, $y = -4$ and $\frac{dy}{dx} = 3$. [11]

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Q30

9231/21, 9231/23 • October/November 2021 • Question 7

(a) Show that an appropriate integrating factor for

$$\sqrt{x^2 - 1} \frac{dy}{dx} + y = x^2 - x\sqrt{x^2 - 1}$$

is $x + \sqrt{x^2 - 1}$.

[4]

(b) Hence find the solution of the differential equation

$$\sqrt{x^2 - 1} \frac{dy}{dx} + y = x^2 - x\sqrt{x^2 - 1}$$

for which $y = 1$ when $x = \frac{5}{4}$. Give your answer in the form $y = f(x)$.

[7]

Q31

9231/22 • October/November 2021 • Question 2

Find the solution of the differential equation

$$\frac{dy}{dx} + \frac{4x^3y}{x^4 + 5} = 6x$$

for which $y = 1$ when $x = 1$. Give your answer in the form $y = f(x)$.

[7]

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Q32

9231/22 • October/November 2021 • Question 7

It is given that $y = x^2w$ and

$$x^2 \frac{d^2w}{dx^2} + 4x(x+1) \frac{dw}{dx} + (5x^2 + 8x + 2)w = 5x^2 + 4x + 2.$$

(a) Show that

$$\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 5y = 5x^2 + 4x + 2.$$

[4]

(b) Find the general solution for w in terms of x .

[7]

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Q33

9231/21, 9231/22 • May/June 2022 • Question 3

The variables t and x are related by the differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = t^2 + 1.$$

(a) Find the general solution for x in terms of t . [6]

(b) Deduce an approximate value of $\frac{d^2x}{dt^2}$ for large positive values of t . [2]

Q34

9231/21, 9231/22 • May/June 2022 • Question 6

Use the substitution $y = vx$ to find the solution of the differential equation

$$x \frac{dy}{dx} = y + \sqrt{9x^2 + y^2}$$

for which $y = 0$ when $x = 1$. Give your answer in the form $y = f(x)$, where $f(x)$ is a polynomial in x . [10]

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Q35

9231/23 • May/June 2022 • Question 5

Find the solution of the differential equation

$$x(x+7)\frac{dy}{dx} + 7y = x$$

for which $y = 7$ when $x = 1$. Give your answer in the form $y = f(x)$.

[9]

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Q36

9231/23 • May/June 2022 • Question 7

The variables x and y are related by the differential equation

$$4\frac{d^2y}{dx^2} - y = 3.$$

It is given that, when $x = 0$, $y = -3$ and $\frac{dy}{dx} = 2$.

- (a) Find y in terms of x . [8]
- (b) Deduce the exact value of x for which $y = 0$. Give your answer in logarithmic form. [3]

Q37

9231/21, 9231/23 • October/November 2022 • Question 5

Find the particular solution of the differential equation

$$2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 4x^2 + 3x + 3,$$

given that, when $x = 0$, $y = \frac{dy}{dx} = 0$.

[10]

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Q38

9231/21, 9231/23 • October/November 2022 • Question 8

(a) Use the substitution $u = 1 - (\theta - 1)^2$ to find

$$\int \frac{\theta - 1}{\sqrt{1 - (\theta - 1)^2}} d\theta.$$

[3]

(b) Find the solution of the differential equation

$$\theta \frac{dy}{d\theta} - y = \theta^2 \sin^{-1}(\theta - 1),$$

where $0 < \theta < 2$, given that $y = 1$ when $\theta = 1$. Give your answer in the form $y = f(\theta)$. [11]

Q39

9231/22 • October/November 2022 • Question 4

Find the solution of the differential equation

$$(4t^2 - 1)\frac{dx}{dt} + 4x = 4t^2 - 1$$

for which $x = 3$ when $t = 1$. Give your answer in the form $x = f(t)$.

[9]

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Q40

9231/22 • October/November 2022 • Question 8

It is given that $y = \cosh u$, where $u > 0$, and

$$\sqrt{\cosh^2 u - 1} \left(\frac{d^2 u}{dx^2} + \frac{du}{dx} \right) + \cosh u \left(\frac{du}{dx} \right)^2 - 2 \cosh u = 4e^{-x}.$$

(a) Show that

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = 4e^{-x}. \quad [4]$$

(b) Find u in terms of x , given that, when $x = 0$, $u = \ln 3$ and $\frac{du}{dx} = 3$. [10]

Q41

9231/21, 9231/22 • May/June 2023 • Question 2

Use the substitution $z = x + y$ to find the solution of the differential equation

$$\frac{dy}{dx} = \frac{1 + 3x + 3y}{3x + 3y - 1}$$

for which $y = 0$ when $x = 1$. Give your answer in the form $a \ln(x + y) + b(x - y) + c = 0$, where a , b and c are constants to be determined. [7]

Q42

9231/21, 9231/22 • May/June 2023 • Question 6

Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 36x = 37 \sin t,$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$.

[11]

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Q43

9231/23 • May/June 2023 • Question 2

The variables x and t are related by the differential equation

$$6\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + x = t^2 + 10t + 13.$$

- (a) Find the general solution for x in terms of t . [6]
- (b) State an approximate solution for large positive values of t . [1]

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Q44

9231/21, 9231/23 • October/November 2023 • Question 4

Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = 27x^2,$$

given that, when $x = 0$, $y = 2$ and $\frac{dy}{dx} = -8$. [10]

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Q45

9231/21, 9231/23 • October/November 2023 • Question 6

- (a) Starting from the definitions of $\cosh$ and $\sinh$ in terms of exponentials, prove that

$$\sinh 2x = 2 \sinh x \cosh x.$$

[3]

- (b) Using the substitution $u = \sinh x$, find $\int \sinh^2 2x \cosh x \, dx$.

[4]

- (c) Find the particular solution of the differential equation

$$\frac{dy}{dx} + y \tanh x = \sinh^2 2x,$$

given that $y = 4$ when $x = 0$. Give your answer in the form $y = f(x)$.

[7]

Q46

9231/22 • October/November 2023 • Question 4

Find the solution of the differential equation

$$\frac{dy}{dx} + 3y = \sin x$$

for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.

[9]

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Q47

9231/22 • October/November 2023 • Question 8

It is given that $v = y^4$ and

$$y^3 \frac{d^2 y}{dx^2} + 3y^2 \left(\frac{dy}{dx} \right)^2 + y^3 \frac{dy}{dx} + y^4 = e^{-2x}.$$

(a) Show that

$$\frac{d^2 v}{dx^2} + \frac{dv}{dx} + 4v = 4e^{-2x}.$$

[4]

(b) Find y in terms of x , given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = -\frac{3}{8}$.

[10]

Q48

9231/21, 9231/22 • May/June 2024 • Question 6

(a) Show that $(\cosh x + \sinh x)^{1/2} = e^{x/2}$. [2]

(b) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5(\cosh x + \sinh x)^{1/2},$$

given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$. [10]

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Q49

9231/21, 9231/22 • May/June 2024 • Question 7

(a) Use the substitution $u = 1 + x^2$ to find $\int \frac{x}{\sqrt{1+x^2}} dx$. [2]

(b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \sinh^{-1} x,$$

given that $y = 1$ when $x = 1$. Give your answer in the form $y = f(x)$. [10]

Q50

9231/23 • May/June 2024 • Question 5

(a) Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 25x = 338 \sin t.$$

[7]

(b) Show that, for large positive values of t and for any initial conditions,

$$x \approx R \sin(t - \phi),$$

where the constants R and ϕ are to be determined.

[3]

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Q51

9231/23 • May/June 2024 • Question 7

(a) Show that

$$\frac{d}{dx} \left(\frac{x}{2} \sqrt{x^2 - 9} - \frac{9}{2} \cosh^{-1} \frac{x}{3} \right) = \sqrt{x^2 - 9}.$$

[3]

(b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \sqrt{x^2 - 9},$$

given that $y = 1$ when $x = 3$. Give your answer in the form $y = f(x)$.

[9]

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Q52

9231/21, 9231/23 • October/November 2024 • Question 5

Find the particular solution of the differential equation

$$6\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + x = t^2 + t + 1,$$

given that, when $t = 0$, $x = 12$ and $\frac{dx}{dt} = -6$. [10]

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Q53

9231/21, 9231/23 • October/November 2024 • Question 7

(a) Show that an appropriate integrating factor for

$$\sqrt{x^2 + 16} \frac{dy}{dx} + y = x\sqrt{x^2 + 16}$$

is $\frac{1}{4}x + \frac{1}{4}\sqrt{x^2 + 16}$. [4]

(b) Hence find the solution of the differential equation

$$\sqrt{x^2 + 16} \frac{dy}{dx} + y = x\sqrt{x^2 + 16}$$

for which $y = 6$ when $x = 3$. [6]

Q54

9231/22 • October/November 2024 • Question 5

Find the particular solution of the differential equation

$$3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^2,$$

given that, when $x = 0$, $y = \frac{dy}{dx} = 0$.

[10]

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Q55

9231/22 • October/November 2024 • Question 7

(a) Show that $\frac{d}{dx} (\ln(\tanh x)) = 2 \operatorname{cosech} 2x$. [3]

(b) Find the solution of the differential equation

$$\sinh 2x \frac{dy}{dx} + 2y = \sinh 2x$$

for which $y = 5$ when $x = \ln 2$. Give your answer in an exact form. [7]

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Q56

9231/21, 9231/22 • May/June 2025 • Question 5

Find the particular solution of the differential equation

$$6\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 6x = e^{-t},$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$.

[10]

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Q57

9231/21, 9231/22 • May/June 2025 • Question 7

Find the solution of the differential equation

$$\frac{dy}{dx} - \frac{x+5}{x^2+10x+61}y = 1,$$

given that $y = 0$ when $x = 3$. Give your answer in an exact form.

[10]

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Q58

9231/23 • May/June 2025 • Question 4

Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 2t^2 + t - 1,$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$.

[10]

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Q59

9231/23 • May/June 2025 • Question 7

Find the solution of the differential equation

$$\frac{dy}{dx} - \frac{2x+6}{x^2+6x+5}y = 4,$$

given that $y = 0$ when $x = 0$. Give your answer in an exact form.

[9]

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Q60

9231/24 • May/June 2025 • Question 3

Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 13e^{3x}$$

given that $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$.

[10]

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Q61

9231/24 • May/June 2025 • Question 6

Find the solution of the differential equation

$$x \frac{dy}{dx} - y = 2x^2 \tan^{-1} x$$

for which $y = \frac{1}{2}\pi$ when $x = 1$. Give your answer in the form $y = f(x)$. [9]

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Q62

9231/21, 9231/23 • October/November 2025 • Question 4

Find the particular solution of the differential equation

$$5\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^2 + 5x + 3,$$

given that, when $x = 0$, $y = \frac{dy}{dx} = 0$.

[10]

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Q63

9231/21, 9231/23 • October/November 2025 • Question 7(b)

Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \tanh^{-1} x,$$

for $0 < x < 1$, given that $y = 0$ when $x = \frac{1}{2}$. Give your answer in an exact form. [9]

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Q64

9231/22 • October/November 2025 • Question 5

(a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 5\cos x.$$

[7]

(b) For large positive values of x and for any initial conditions, show that the solution to part (a) can be approximated by

$$y \approx R\sin(x + \phi),$$

where the constants R and ϕ are to be determined.

[3]

Q65

9231/22 • October/November 2025 • Question 7(b)

Find the solution of the differential equation

$$2\frac{dy}{dx} + \frac{y}{2+x} = 2\sqrt{2-x}$$

for which $y = \frac{1}{2}$ when $x = 1$. Give your answer in an exact form.

[8]

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Q66

9231/24 • October/November 2025 • Question 5

(a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x + \cos 2x.$$

[7]

(b) For large positive values of x and for any initial conditions, show that the solution to part (a) can be approximated by

$$y \approx R \sin(2x - \phi),$$

where R and ϕ are positive constants to be determined.

[3]

Q67

9231/24 • October/November 2025 • Question 6

Find the particular solution of the differential equation

$$\frac{dy}{dx} + \frac{1+x}{1-2x-x^2}y = 1$$

given that $y = \pi$ when $x = 0$. Give your answer in an exact form. [10]

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Answers available in the companion mark scheme booklet.

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