Further Math with Ali Hashir

Differential Equations

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/11, 9231/12 • May/June 2015 • Question 11

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Mark scheme.

$\frac{dv}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$	**B1**
$\frac{d^2v}{dx^2} = -\frac{1}{y^2}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \frac{d^2v}{dy^2} = -\frac{1}{y^2}\frac{d^2y}{dx^2} + \frac{2}{y^3}\left(\frac{dy}{dx}\right)^2$	**M1A1**
$\frac{2}{y^3}\left(\frac{dy}{dx}\right)^2 - \frac{1}{y^2}\frac{d^2y}{dx^2} - \frac{2}{y^2}\frac{dy}{dx} + \frac{5}{y} = 17+6x-5x^2 \sim \frac{d^2v}{dx^2} + 2\frac{dv}{dx} + 5v = 17+6x-5x^2$ (AG)	**A1**
$m^2 + 2m + 5 = 0 \Rightarrow m = -1 \pm 2i \Rightarrow CF : e^{-x}(A\cos 2x + B\sin 2x)$	**M1A1**
PI : $v = px^2 + qx + r \Rightarrow v' = 2px + q \Rightarrow v'' = 2p$	**M1**
Equate coefficients: $5p = -5$ $4p + 5q = 6$ $2p + 2q + 5r = 17$	**M1**
$p = -1$, $q = 2$, $r = 3$	**A1**
GS: $v = e^{-x}(A\cos 2x + B\sin 2x) + 3 + 2x - x^2$	**A1**
When $x = 0$ $y = \frac{1}{2} \Rightarrow v = 2$ and $\frac{dy}{dx} = -1 \Rightarrow \frac{dv}{dx} = 4$, so $\Rightarrow 2 = A + 3 \Rightarrow A = -1$	**B1**
$v' = e^{-x}(-2A\sin 2x + 2B\cos 2x) - e^{-x}(A\cos 2x + B\sin 2x)$	**M1**
$\Rightarrow 4 = 2B + 1 + 2 \Rightarrow 2B = 1 \Rightarrow B = \frac{1}{2}$	**A1**
$\Rightarrow y = \left[e^{-x}\left(\frac{1}{2}\sin 2x - \cos 2x\right) + 3 + 2x - x^2\right]^{-1}$	**A1**
	Total: 14

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Q2

9231/13 • May/June 2015 • Question 9

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Mark scheme.

$(m-5)(m+2)=0 \Rightarrow CF: x = A\mathrm{e}^{5t} + B\mathrm{e}^{-2t}$	**M1A1**
PI : $x = p\sin t + q\cos t \Rightarrow \dot{x} = p\cos t - q\sin t \Rightarrow \ddot{x} = -p\sin t - q\cos t$	**M1**
$\Rightarrow -p\sin t - q\cos t - 3p\cos t + 3q\sin t - 10p\sin t - 10q\cos t = 2\sin t - 3\cos t$	**A1**
$\Rightarrow -11p + 3q = 2$ and $3p + 11q = 3 \Rightarrow p = -0.1,\ q = 0.3$	**dM1A1**
GS: $x = A\mathrm{e}^{5t} + B\mathrm{e}^{-2t} + 0.3\cos t - 0.1\sin t$ CAO	**A1**
Initial conditions $\Rightarrow A + B + 0.3 = 3.3 \Rightarrow A + B = 3$	**B1**
and from $\dot{x} = 5A\mathrm{e}^{5t} - 2B\mathrm{e}^{-2t} - 0.3\sin t - 0.1\cos t$	**M1**
$5A - 2B - 0.1 = 0.9 \Rightarrow 5A - 2B = 1$	**A1**
$A = 1,\ B = 2 \Rightarrow x = \mathrm{e}^{5t} + 2\mathrm{e}^{-2t} + 0.3\cos t - 0.1\sin t$ CAO	**A1**
	Total: 11

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Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 2

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Mark scheme.

$m^2 + 4m + 4 = 0 \Rightarrow (m+2)^2 = 0 \Rightarrow m = -2$	**M1**
CF: $A\mathrm{e}^{-2t} + Bt\mathrm{e}^{-2t}$ soi	**A1**
PI: $x = pt^2 + qt + r \Rightarrow \dot{x} = 2pt + q \Rightarrow \ddot{x} = 2p$	**M1**
$\Rightarrow 2p + 8pt + 4q + 4pt^2 + 4qt + 4r = 7 - 2t^2$	**M1**
$\Rightarrow p = -\frac{1}{2}$, $q = 1$, $r = 1$	**A1**
GS: $x = A\mathrm{e}^{-2t} + Bt\mathrm{e}^{-2t} - \frac{1}{2}t^2 + t + 1$	**A1**
	Total: 6

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Q4

9231/11, 9231/12 • May/June 2016 • Question 9

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Mark scheme.

$y' = 2kxe^{2x} + 2kx^2e^{2x}$	**B1**
$y'' = 2ke^{2x} + 8kxe^{2x} + 4kx^2e^{2x}$	**B1**
$2ke^{2x} + 8kxe^{2x} + 4kx^2e^{2x} - 8kxe^{2x} - 8kx^2e^{2x} + 4kx^2e^{2x} = 4e^{2x}$	**M1**
$\Rightarrow 2k = 4 \Rightarrow k = 2$	**A1**
$m^2 - 4m + 4 = 0 \Rightarrow (m-2)^2 = 0 \Rightarrow m = 2$	**M1**
CF: $y = Ae^{2x} + Bxe^{2x}$	**A1**
$y = Ae^{2x} + Bxe^{2x} + 2x^2e^{2x}$	**A1**
$y = 3$ when $x = 0 \Rightarrow A = 3$	**B1**
$y' = 2Ae^{2x} + B(e^{2x} + 2xe^{2x}) + (4xe^{2x} + 4x^2e^{2x})$	**M1**
$y' = -2$ when $x = 0$ and $A = 3 \Rightarrow B = -8$	**A1**
$\Rightarrow y = 3e^{2x} - 8xe^{2x} + 2x^2e^{2x}$	**A1**
	Total: 11

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Q5

9231/13 • May/June 2016 • Question 10

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Mark scheme.

$x\dfrac{dy}{dx} = x\dfrac{dy}{du} \times \dfrac{du}{dx} = x\dfrac{dy}{du} \times \dfrac{1}{x} = \dfrac{dy}{du}$ (AG)	**B1**
$x^2\dfrac{d^2y}{dx^2} = x^2\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = x^2\dfrac{d}{dx}\left(\dfrac{1}{x}\dfrac{dy}{du}\right)$	**M1**
$= x^2\left(-\dfrac{1}{x^2}\dfrac{dy}{du} + \dfrac{1}{x}\dfrac{d}{du}\left[\dfrac{dy}{du}\right]\dfrac{du}{dx}\right)$	
$= -\dfrac{dy}{du} + x\dfrac{d^2y}{du^2}.\dfrac{1}{x} = \dfrac{d^2y}{du^2} - \dfrac{dy}{du}$ (AG)	**A1**
Substituting in differential equation:	
$\dfrac{d^2y}{du^2} - \dfrac{dy}{du} + 3\dfrac{dy}{du} + 17y = 34u + 21 \Rightarrow \dfrac{d^2y}{du^2} + 2\dfrac{dy}{du} + 17y = 34u + 21$ (AG)	**B1**
$m^2 + 2m + 17 = 0 \Rightarrow m = -1 \pm 4i \Rightarrow y_c = e^{-u}(A\cos 4u + B\sin 4u)$	**M1A1**
$y_p = cu + d \Rightarrow y' = c$ and $y'' = 0 \Rightarrow 2c + 17cu + 17d \equiv 34u + 21$	**M1**
$\Rightarrow c = 2, d = 1 \Rightarrow y_p = 2u + 1$	**A1**
$\Rightarrow y = \dfrac{1}{x}(A\cos[4\ln x] + B\sin[4\ln x]) + 2\ln x + 1$ (or in terms of u)	**A1**
$y = 0$ when $x = 1 \Rightarrow 0 = A + 1 \Rightarrow A = -1$ or $u = 0, y = 0$ etc	**B1**
$\dfrac{dy}{dx} = -\dfrac{1}{x^2}(A\cos[4\ln x] + B\sin[4\ln x]) + \dfrac{1}{x}\left(-A\sin[4\ln x].\dfrac{4}{x} + B\cos[4\ln x].\dfrac{4}{x}\right) + \dfrac{2}{x}$	**M1**
$\dfrac{dy}{dx} = -1$ when $x = 1 \Rightarrow -1 = 1 + 4B + 2 \Rightarrow B = -1$ or all in terms of u	**A1**
Hence $y = 2\ln x + 1 - \dfrac{1}{x}(\cos[4\ln x] + \sin[4\ln x])$ (oe)	**A1**
	Total: 13

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Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 6

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Mark scheme.

$m^2 + 7m + 10 = 0 \Rightarrow (m+2)(m+5) = 0 \Rightarrow m = -2$ or -5	**M1**
CF: $A\mathrm{e}^{-2t} + B\mathrm{e}^{-5t}$	**A1**
PI: $x = p\sin 2t + q\cos 2t \Rightarrow \dot{x} = 2p\cos 2t - 2q\sin 2t \Rightarrow \ddot{x} = -4p\sin 2t - 4q\cos 2t$	**M1A1**
Substituting: $14p + 6q = 0$	
$6p - 14q = 116$	**M1**
Solving: $p = 3$, $q = -7 \Rightarrow x = 3\sin 2t - 7\cos 2t$	**M1A1**
GS: $x = A\mathrm{e}^{-2t} + B\mathrm{e}^{-5t} + 3\sin 2t - 7\cos 2t$	**A1**
For large positive values of t, $x \approx 3\sin 2t - 7\cos 2t$.	**B1**
	Total: 9

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Q7

9231/11, 9231/12 • May/June 2017 • Question 10

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Mark scheme.

$\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} = 2t^{\frac{1}{2}}\dfrac{dy}{dt}$ (AG)	**B1**
$\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \times \dfrac{dt}{dx} = \left\{t^{-\frac{1}{2}}\dfrac{dy}{dt} + 2t^{\frac{1}{2}}\dfrac{d^2y}{dt^2}\right\} 2t^{\frac{1}{2}} = 2\dfrac{dy}{dt} + 4t\dfrac{d^2y}{dt^2}$ (AG)	**M1A1**
Substitute in $(*)$: $2\dfrac{dy}{dt} + 4t\dfrac{d^2y}{dt^2} - 16t\dfrac{dy}{dt} - 2\dfrac{dy}{dt} + 12ty = 4te^{-t}$	**B1**
$\Rightarrow \dfrac{d^2y}{dt^2} - 4\dfrac{dy}{dt} + 3y = e^{-t}$ (AG)	
CF: $(m-1)(m-3) = 0$	**M1**
$y = Ae^{t} + Be^{3t}$	**A1**
PI: $y = ke^{-t} \Rightarrow y' = -ke^{-t} \Rightarrow y'' = ke^{-t}$	**M1**
$ke^{-t} + 4ke^{-t} + 3ke^{-t} = e^{-t}$	**M1**
$\Rightarrow k = \dfrac{1}{8}$	**A1**
GS: $y = Ae^{t} + Be^{3t} + \dfrac{1}{8}e^{-t}$	**A1 FT**
$y = Ae^{x^2} + Be^{3x^2} + \dfrac{1}{8}e^{-x^2}$	**A1 FT**
	Total: 11

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Q8

9231/13 • May/June 2017 • Question 8

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Mark scheme.

$m^2 + 6m + 9 = 0 \Rightarrow m = -3$	**M1**
CF: $x = Ae^{-3t} + Bte^{-3t}$	**A1**
$x = pt^2 + qt + r \Rightarrow \dot{x} = 2pt + q$ and $\ddot{x} = 2p$	**M1**
$\Rightarrow 2p + 6(2pt + q) + 9\left(pt^2 + qt + r\right) = 18t^2 + 6t + 1$	**M1**
$\Rightarrow 9pt^2 + (9q + 12p)t + (2p + 6q + 9r) = 18t^2 + 6t + 1$	**A1**
$\Rightarrow p = 2,\ q = -2,\ r = 1$ whence PI: $2t^2 - 2t + 1$	
GS: $x = Ae^{-3t} + Bte^{-3t} + 2t^2 - 2t + 1$	**A1FT**
$x = 3$ when $t = 0 \Rightarrow A = 2$	**B1**
$\dot{x} = -3Ae^{-3t} - 3Bte^{-3t} + Be^{-3t} + 4t - 2$	**M1**
$\dot{x} = 0$ when $t = 0 \Rightarrow B = 8$	**A1**
Hence $x = 2e^{-3t} + 8te^{-3t} + 2t^2 - 2t + 1$	**A1**
	Total: 10

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Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 2

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Mark scheme.

CF: $m^2 + 2m + 5 = 0 \Rightarrow m = -1 \pm 2\mathrm{i}$	**M1**
$\mathrm{e}^{-t}(A\cos 2t + B\sin 2t)$	**A1**
PI: $x = pt^2 + qt + r \Rightarrow \dot{x} = 2pt + q \Rightarrow \ddot{x} = 2p$	**M1**
$2p + 4pt + 2q + 5pt^2 + 5qt + 5r = 4 - 5t^2$	**M1**
$\Rightarrow p = -1,\ q = \frac{4}{5},\ r = \frac{22}{25}$	**A1**
GS: $x = \mathrm{e}^{-t}(A\cos 2t + B\sin 2t) + \frac{22}{25} + \frac{4}{5}t - t^2$	**A1FT**
	Total: 6

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Q10

9231/11, 9231/12 • May/June 2018 • Question 7

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Mark scheme.

The auxiliary equation is $\left(n+\frac{1}{7}\right)^2=0 \Rightarrow n=-\frac{1}{7}$.	**M1**
CF: $y=(A+Bx)\,e^{-\frac{1}{7}x}$.	**A1**
PI: $y=px+q$, $y'=p$ and $y''=0$.	**M1**
Correct form of PI and derivatives	
$14p+px+q=49x+735 \Rightarrow p=49,\ q=49$.	**M1A1**
Substitutes in equation correctly	
Thus $y=(A+Bx)\,e^{-\frac{1}{7}x}+49(x+1)$.	**A1FT**
FT only on correct form of CF AEF	
$y=0$ when $x=0$ gives $A+49=0 \Rightarrow A=-49$.	**B1**
$y'=-\frac{1}{7}(A+Bx)\,e^{-\frac{1}{7}x}+Be^{-\frac{1}{7}x}+49$.	**M1**
Differentiating their y	
$y'=0$ when $x=0$ gives $-\frac{1}{7}A+B+49=0 \Rightarrow B=-56$	**A1**
AEF	
Thus $y=-(49+56x)e^{-\frac{1}{7}x}+49(x+1)$.	**A1**
	Total: 10

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Q11

9231/13 • May/June 2018 • Question 10

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Mark scheme.

$y' = x + x't$	**B1**
$y'' = x' + x' + tx''$	**B1**
Substitute correctly	**B1**
AG	
Auxiliary equation: $m^2 + 9 = 0 \Rightarrow m = \pm 3i$.	**M1**
Correct auxiliary	
CF $= A\cos 3t + B\sin 3t$.	**A1**
PI $y = at^2 + bt + c$ so $y' = 2at + b$ and $y'' = 2a$	**M1**
Differentiate twice and substitute	
$a = \frac{1}{3},\ b = 0,\ c = \frac{1}{27}$.	**A1**
$y = A\cos 3t + B\sin 3t + \frac{1}{3}t^2 + \frac{1}{27}$	**A1FT**
Their CF + their PI both in correct form	
$x = \frac{\pi}{9}$ when $t = \frac{\pi}{3}$ gives $A = \frac{1}{27}$.	**B1**
$y' = -3A\sin 3t + 3B\cos 3t + \frac{2}{3}t$	**M1**
Differentiating their y of equivalent difficulty	
$x' = \frac{2}{3}$ when $t = \frac{\pi}{3}$ gives $B = -\frac{\pi}{27}$.	**A1**
$x = \dfrac{\cos 3t - \pi\sin 3t + 9t^2 + 1}{27t}$	**A1**
AEF	

Total: 12

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Q12

9231/11, 9231/13 • October/November 2018 • Question 10

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Mark scheme.

$m^2 + 2m + 10 = 0 \Rightarrow m = -1 \pm 3i$. (Finds complementary function.)	**M1**
$x = e^{-t}(A\cos 3t + B\sin 3t)$	**A1**
$x = p\cos 3t + q\sin 3t$ (Finds particular integral.)	**M1**
$\Rightarrow \dot{x} = -3p\sin 3t + 3q\cos 3t \Rightarrow \ddot{x} = -9p\cos 3t - 9q\sin 3t$	**A1**
$(p + 6q)\cos 3t + (q - 6p)\sin 3t = 37\sin 3t$	**M1**
$\Rightarrow p = -6,\ q = 1$	**A1**
$x = e^{-t}(A\cos 3t + B\sin 3t) - 6\cos 3t + \sin 3t$	
$x = 3$ when $t = 0 \Rightarrow A - 6 = 3 \Rightarrow A = 9$ (Uses initial conditions.)	**B1**
$\dot{x} = e^{-t}(-3A\sin 3t + 3B\cos 3t) - e^{-t}(A\cos 3t + B\sin 3t) + 18\sin 3t + 3\cos 3t$	**M1A1**
$\dot{x} = 0$ when $t = 0 \Rightarrow 3B - A + 3 = 0 \Rightarrow B = 2$ (Obtains solution, AEF.)	**A1**
$x = e^{-t}(9\cos 3t + 2\sin 3t) - 6\cos 3t + \sin 3t$	
As $t \to \infty$, $e^{-t} \to 0 \Rightarrow x \approx -6\cos 3t + \sin 3t$ (Obtains limit.)	**B1**
$\sin 3t - 6\cos 3t = \sqrt{37}\left(\frac{1}{\sqrt{37}}\sin 3t - \frac{6}{\sqrt{37}}\cos 3t\right)$ (Converts to $R\sin(3t-\phi)$)	**M1**
$= \sqrt{37}\sin(3t - \tan^{-1} 6)$. (AG)	**A1**
	Total: 13

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Q13

9231/12 • October/November 2018 • Question 4

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Mark scheme.

$m^2 + 2m + 1 = 0 \Rightarrow (m+1)^2 = 0 \Rightarrow m = -1$ (Forms and solves auxiliary equation.)	**M1**
CF: $(A + Bt)\mathrm{e}^{-t}$ (States CF.)	**A1**
PI: $x = p\sin t + q\cos t$ (Uses correct form of PI and differentiates twice.)	**M1**
$\Rightarrow \dot{x} = p\cos t - q\sin t \Rightarrow \ddot{x} = -p\sin t - q\cos t$	**A1**
$-p\sin t - q\cos t + 2(p\cos t - q\sin t) + p\sin t + q\cos t = 4\sin t$ (Compares coefficients and attempts to solve)	**M1**
$2p = 0 \Rightarrow p = 0$. $-2q = 4 \Rightarrow q = -2$.	**A1**
GS: $x = (A + Bt)\mathrm{e}^{-t} - 2\cos t$ (States general solution. FT on correct form only)	**A1FT**
$x \approx -2\cos t$	**B1FT**
	Total: 8

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Q14

9231/11, 9231/12 • May/June 2019 • Question 7

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Mark scheme.

$10u^2 + 3u - 1 = 0 \Rightarrow (2u+1)(5u-1) = 0$	**M1**
CF: $x = Ae^{-\frac{1}{2}t} + Be^{\frac{1}{5}t}$	**A1**
PI: $x = p + qt \Rightarrow \dot{x} = q \Rightarrow \ddot{x} = 0$	**M1**
$3q - p - qt = t + 2 \Rightarrow q = -1, p = -5.$	**M1A1**
GS: $x = Ae^{-\frac{1}{2}t} + Be^{\frac{1}{5}t} - t - 5$	**A1**
$\dot{x} = -\frac{1}{2}Ae^{-\frac{1}{2}t} + \frac{1}{5}Be^{\frac{1}{5}t} - 1$	**M1**
$A + B = 5$	**M1**
$-\frac{1}{2}A + \frac{1}{5}B = 1$	
$A = 0, B = 5$	**A1**
$x = 5e^{\frac{1}{5}t} - t - 5$	**A1**
	Total: 10

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Q15

9231/13 • May/June 2019 • Question 8

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Mark scheme.

$9u^2 + 6u + 1 = 0 \Rightarrow (3u+1)^2 = 0$	**M1**
CF: $x = (At + B)\mathrm{e}^{-\frac{1}{3}t}$	**A1**
$x = p\sin t + q\cos t \Rightarrow \dot{x} = p\cos t - q\sin t \Rightarrow \ddot{x} = -p\sin t - q\cos t$	**M1**
$-9p - 6q + p = 50$	**M1**
$-9q + 6p + q = 0$	
$q = -3, p = -4.$	**A1**
$x = (At + B)\mathrm{e}^{-\frac{1}{3}t} - 4\sin t - 3\cos t$	**A1**
$\dot{x} = -\frac{1}{3}(At + B)\mathrm{e}^{-\frac{1}{3}t} + A\mathrm{e}^{-\frac{1}{3}t} - 4\cos t + 3\sin t$	**M1**
$B = 3$	**B1**
$-\frac{1}{3}B + A - 4 = 0 \Rightarrow A = 5$	**A1**
$x = (5t + 3)\mathrm{e}^{-\frac{1}{3}t} - 4\sin t - 3\cos t$	**A1**
	Total: 10

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Q16

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 11

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Mark scheme.

$w = \cos y \Rightarrow \dfrac{\mathrm{d}w}{\mathrm{d}x} = -\sin y \dfrac{\mathrm{d}y}{\mathrm{d}x}$	**B1**
$\dfrac{\mathrm{d}^2w}{\mathrm{d}x^2} = -\sin y \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - \cos y \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	**B1**
$\dfrac{\mathrm{d}^2w}{\mathrm{d}x^2} + 2\dfrac{\mathrm{d}w}{\mathrm{d}x} + w = -\sin y \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - \cos y \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 - 2\sin y \dfrac{\mathrm{d}y}{\mathrm{d}x} + \cos y$	**M1**
Uses substitution to obtain w-x equation, AG.	
$= -\cos y\left(\mathrm{e}^{-2x}\sec y\right) = -\mathrm{e}^{-2x}$	**A1**
$m^2 + 2m + 1 = 0 \Rightarrow m = -1$	**M1**
Finds CF.	
CF: $w = (Ax + B)\mathrm{e}^{-x}$	**A1**
PI: $w = k\mathrm{e}^{-2x} \Rightarrow w' = -2k\mathrm{e}^{-2x} \Rightarrow w'' = 4k\mathrm{e}^{-2x}$	**M1**
Forms PI and differentiates.	
$4k - 4k + k = -1 \Rightarrow k = -1$	**A1**
$w = (Ax + B)\mathrm{e}^{-x} - \mathrm{e}^{-2x}$	**A1**
States general solution.	
$x = 0,\ y = \frac{1}{3}\pi,\ w = \frac{1}{2} \Rightarrow B = \frac{3}{2}$	**B1**
Uses initial conditions to find constants.	
$w' = -(Ax + B)\mathrm{e}^{-x} + A\mathrm{e}^{-x} + 2\mathrm{e}^{-2x}$	**M1**
Differentiates general solution.	
$x = 0,\ y = \frac{1}{3}\pi,\ y' = \dfrac{\sqrt{3}}{3},\ w' = -\dfrac{1}{2} \Rightarrow -\dfrac{1}{2} = -\dfrac{3}{2} + A + 2 \Rightarrow A = -1$	**M1A1**
Substitutes initial conditions.	
$y = \cos^{-1}\left(\left(\dfrac{3}{2} - x\right)\mathrm{e}^{-x} - \mathrm{e}^{-2x}\right)$	**A1**
States particular solution for y in terms of x.	
	Total: 14

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Q17

9231/21, 9231/22 • May/June 2020 • Question 1

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Mark scheme.

$\mathrm{e}^{\int 5\,\mathrm{d}x} = \mathrm{e}^{5x}$	**M1**
integrating factor e^{5x}	**A1**
$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\mathrm{e}^{5x}\right) = \mathrm{e}^{-2x}$	**M1**
$y\mathrm{e}^{5x} = -\frac{1}{2}\mathrm{e}^{-2x} + C$	**A1**
$0 = -\frac{1}{2} + C$	**M1**
$y = \frac{1}{2}\mathrm{e}^{-5x} - \frac{1}{2}\mathrm{e}^{-7x}$	**A1**
	Total: 6

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Q18

9231/21, 9231/22 • May/June 2020 • Question 7

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Mark scheme.

(a)	
$\dfrac{\mathrm{d}x}{\mathrm{d}t} = t^3\dfrac{\mathrm{d}y}{\mathrm{d}t} + 3t^2y$	**B1**
$\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = t^3\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} + 6t^2\dfrac{\mathrm{d}y}{\mathrm{d}t} + 6ty$	**B1**
combine to obtain $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\dfrac{\mathrm{d}x}{\mathrm{d}t} + 13x = t^3\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} + 6t^2\dfrac{\mathrm{d}y}{\mathrm{d}t} + 6ty + 4t^3\dfrac{\mathrm{d}y}{\mathrm{d}t} + 12t^2y + 13t^3y = 61\mathrm{e}^{\frac{1}{2}t}$ (AG)	**M1 A1**
(b)	
auxiliary equation $m^2 + 4m + 13 = 0 \Rightarrow m = -2 \pm 3\mathrm{i}$	**M1**
complementary function $x = \mathrm{e}^{-2t}(A\cos 3t + B\sin 3t)$	**A1**
try $x = k\mathrm{e}^{\frac{1}{2}t} \Rightarrow \dot{x} = \frac{1}{2}k\mathrm{e}^{\frac{1}{2}t} \Rightarrow \ddot{x} = \frac{1}{4}k\mathrm{e}^{\frac{1}{2}t}$	**B1**
$\frac{1}{4}k + 2k + 13k = 61 \Rightarrow k = 4$	**M1 A1**
$t^3y = \mathrm{e}^{-2t}(A\cos 3t + B\sin 3t) + 4\mathrm{e}^{\frac{1}{2}t} \Rightarrow y = t^{-3}\mathrm{e}^{-2t}(A\cos 3t + B\sin 3t) + 4t^{-3}\mathrm{e}^{\frac{1}{2}t}$	**M1 A1**
	Total: 11

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Q19

9231/23 • May/June 2020 • Question 1

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Mark scheme.

$m^2 - 8m - 9 = 0 \Rightarrow m = -1, 9$	**M1**
$x = A\mathrm{e}^{-t} + B\mathrm{e}^{9t}$	**A1**
try $x = k\mathrm{e}^{8t} \Rightarrow \dot{x} = 8k\mathrm{e}^{8t} \Rightarrow \ddot{x} = 64k\mathrm{e}^{8t}$	**B1**
$64k\mathrm{e}^{8t} - 64k\mathrm{e}^{8t} - 9k\mathrm{e}^{8t} = 9\mathrm{e}^{8t} \Rightarrow -9k = 9$	**M1**
$k = -1$	**A1**
$x = A\mathrm{e}^{-t} + B\mathrm{e}^{9t} - \mathrm{e}^{8t}$	**A1**
	Total: 6

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Q20

9231/23 • May/June 2020 • Question 7

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Mark scheme.

(a)

$\dfrac{\mathrm{d}y}{\mathrm{d}x} + \dfrac{y}{\sqrt{x^2+1}} = \dfrac{x}{x^2+1}\left(x - \sqrt{x^2+1}\right)$	**B1**
$\mathrm{e}^{\int \frac{1}{\sqrt{x^2+1}}\,\mathrm{d}x} = \mathrm{e}^{\sinh^{-1} x}$	**M1 A1**
$= x + \sqrt{x^2+1}$ (AG)	**A1**

(b)

$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(y\left(x + \sqrt{x^2+1}\right)\right) = -\dfrac{x}{x^2+1}$	**M1 A1**
$y\left(x+\sqrt{x^2+1}\right) = -\int \dfrac{x}{x^2+1}\,\mathrm{d}x = -\frac{1}{2}\ln(x^2+1) + C$	**M1 A1**
$\ln 2 = C$	**M1**
$y = \dfrac{\ln 2 - \frac{1}{2}\ln(x^2+1)}{x+\sqrt{x^2+1}} = \left(x - \sqrt{x^2+1}\right)\ln\left(\frac{1}{2}\sqrt{\ldots}\right)$ i.e. $y = \left(x-\sqrt{x^2+1}\right)\ln\left(\dfrac{2}{\sqrt{x^2+1}}\right)$	**M1 A1**

Total: 11

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Q21

9231/21, 9231/23 • October/November 2020 • Question 2

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Mark scheme.

Auxiliary equation: $9m^2 + 6m + 1 = 0 \Rightarrow m = -\frac{1}{3}$.	**M1**
Complementary function: $y = \mathrm{e}^{-\frac{1}{3}x}(Ax + B)$.	**A1**
Particular integral and its derivatives: $y = p + qx + rx^2 \Rightarrow y' = q + 2rx \Rightarrow y'' = 2r$.	**B1**
Substitutes and equates coefficients: $18r + 6q + 12rx + p + qx + rx^2 = 3x^2 + 30x$.	**M1**
$r = 3$, $q = -6$, $p = -18$.	**A1**
$y = \mathrm{e}^{-\frac{1}{3}x}(Ax + B) + 3x^2 - 6x - 18$.	**A1**
(b) $y = 3x^2 - 6x - 18$.	**B1 FT**
	Total: 7

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Q22

9231/21, 9231/23 • October/November 2020 • Question 6(b)

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Mark scheme.

Finds integrating factor: $\mathrm{e}^{\int \cot\theta\,\mathrm{d}\theta} = \mathrm{e}^{\ln\sin\theta} = \sin\theta$.	**M1 A1**
Correct form on LHS and uses identity from part (a): $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(y\sin\theta) = \sin^4\theta = \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3)$.	**M1**
$y\sin\theta = \frac{1}{8}\left(\frac{1}{4}\sin 4\theta - 2\sin 2\theta + 3\theta\right) + C$.	**A1**
Substitutes initial conditions: $0 = \frac{1}{8}\left(\frac{3}{2}\pi\right) + C$.	**M1**
OE: $y\sin\theta = \frac{1}{8}\left(\frac{1}{4}\sin 4\theta - 2\sin 2\theta + 3\theta - \frac{3}{2}\pi\right)$.	**A1**
	Total: 6

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Q23

9231/22 • October/November 2020 • Question 4

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Mark scheme.

Divides through by x: $\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2}{x}y = \frac{1}{x}\mathrm{e}^x$.	**B1**
Finds integrating factor, must be integrating x^{-1}: $\mathrm{e}^{2\int x^{-1}\mathrm{d}x} = x^2$.	**M1 A1**
Correct form on LHS and attempt to integrate $\frac{1}{x}\mathrm{e}^x$ multiplied by their integrating factor: $\frac{\mathrm{d}}{\mathrm{d}x}(yx^2) = x\mathrm{e}^x$.	**M1**
$yx^2 = (x-1)\mathrm{e}^x + C$.	**A1**
Finds C: $C = 3$.	**M1**
Divides through by coefficient of y: $y = \frac{(x-1)\mathrm{e}^x + 3}{x^2}$.	**M1 A1**
	Total: 8

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Q24

9231/22 • October/November 2020 • Question 6

↑ Back to contents

Mark scheme.

Auxiliary equation: $m^2 + 8m + 15 = 0 \Rightarrow m = -3, -5$.	**M1**
Complementary function: $x = A\mathrm{e}^{-5t} + B\mathrm{e}^{-3t}$.	**A1**
Particular integral and its derivatives: $x = p\sin 3t + q\cos 3t \Rightarrow \dot{x} = 3p\cos 3t - 3q\sin 3t \Rightarrow \ddot{x} = -9p\sin 3t - 9q\cos 3t$.	**M1 A1**
Substitutes and equates coefficients: $-9p - 24q + 15p = 0 \Rightarrow 6p - 24q = 0$; $-9q + 24p + 15q = 102 \Rightarrow 4p + q = 17$.	**M1**
$p = 4,\ q = 1$.	**A1**
$x = A\mathrm{e}^{-5t} + B\mathrm{e}^{-3t} + 4\sin 3t + \cos 3t$.	**A1**
Differentiating the correct form: $\dot{x} = -5A\mathrm{e}^{-5t} - 3B\mathrm{e}^{-3t} + 12\cos 3t - 3\sin 3t$.	**M1**
Forms simultaneous equations using initial conditions: $1 = A + B + 1,\ 0 = -5A - 3B + 12 \Rightarrow A = 6,\ B = -6$.	**M1 A1**
$x = 6\mathrm{e}^{-5t} - 6\mathrm{e}^{-3t} + 4\sin 3t + \cos 3t$.	**A1**
	Total: 11

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Q25

9231/21, 9231/22 • May/June 2021 • Question 2

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Mark scheme.

(a)	
Auxiliary equation: $m^2 + 3m + 2 = 0$ leading to $m = -1, -2$.	**M1**
Complementary function: $y = A\mathrm{e}^{-x} + B\mathrm{e}^{-2x}$.	**A1**
Particular integral and its derivatives: $y = p + qx$ leading to $y' = q$ leading to $y'' = 0$.	**B1**
Substitutes and equates coefficients: $3q + 2(p + qx) = 2x + 1$.	**M1**
$q = 1$, $p = -1$.	**A1**
$y = A\mathrm{e}^{-x} + B\mathrm{e}^{-2x} + x - 1$ (must see "$y =$").	**A1**
(b)	
$y = x - 1$ (accept $y \approx x - 1$).	**B1 FT**
	Total: 7

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Q26

9231/21, 9231/22 • May/June 2021 • Question 4

↑ Back to contents

Mark scheme.

Divides through by $\sin\theta$: $\dfrac{dy}{d\theta} + \operatorname{cosec}\theta\, y = \dfrac{\tan\frac{1}{2}\theta}{\sin\theta}$.	**B1**
Finds integrating factor: $e^{\int \operatorname{cosec}\theta\, d\theta} = e^{\ln\tan\frac{1}{2}\theta} = \tan\frac{1}{2}\theta$.	**M1 A1**
Correct form on LHS and uses an appropriate identity: $\dfrac{d}{d\theta}\left(y\tan\frac{1}{2}\theta\right) = \dfrac{\tan^2\frac{1}{2}\theta}{\sin\theta} = \frac{1}{2}\tan\frac{1}{2}\theta\sec^2\frac{1}{2}\theta$.	**M1**
Integrates RHS: $y\tan\frac{1}{2}\theta = \frac{1}{2}\tan^2\frac{1}{2}\theta + C$.	**M1 A1**
Substitutes initial conditions: $1 = \frac{1}{2} + C$.	**M1**
Divides through by their integrating factor: $y = \frac{1}{2}\left(\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta\right)$ [$= \operatorname{cosec}\theta$].	**M1 A1**
	Total: 9

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Q27

9231/23 • May/June 2021 • Question 5

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Mark scheme.

(a)	
Differentiates particular integral: $y = kxe^{-x}$ leading to $y' = ke^{-x}(1-x)$ leading to $y'' = ke^{-x}(x-2)$.	**B1 B1**
Substitutes: $ke^{-x}(x-2) - 2ke^{-x}(1-x) - 3kxe^{-x} = 4e^{-x}$ leading to $-4k = 4$.	**M1**
$k = -1$.	**A1**
(b)	
Auxiliary equation: $m^2 - 2m - 3 = 0$ leading to $m = -1, 3$.	**M1**
Complementary function: $y = Ae^{-x} + Be^{3x}$.	**A1**
General solution: $y = Ae^{-x} + Be^{3x} - xe^{-x} = (A-x)e^{-x} + Be^{3x}$.	**A1 FT**
Derivative of general solution: $y' = -(A-x)e^{-x} - e^{-x} + 3Be^{3x}$.	**B1**
Substitutes initial conditions into general solution: $A+B = \frac{1}{2}$, $-A-1+3B = \frac{1}{2}$ leading to $A = 0$, $B = \frac{1}{2}$.	**M1**
$y = \frac{1}{2}e^{3x} - xe^{-x}$.	**A1**
	Total: 10

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Q28

9231/23 • May/June 2021 • Question 6

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Mark scheme.

(a)	
Writes in exponential form: $\sinh x = \frac{1}{2}(\mathrm{e}^x - \mathrm{e}^{-x})$, $\cosh x = \frac{1}{2}(\mathrm{e}^x + \mathrm{e}^{-x})$.	**B1**
Expands: $\left(\frac{1}{2}(\mathrm{e}^x - \mathrm{e}^{-x})\right)^2 = \frac{1}{2}(\mathrm{e}^{2x} - 2 + \mathrm{e}^{-2x})$.	**M1**
$= \cosh 2x - 1$ (AG).	**A1**
(b)	
Finds integrating factor: $\mathrm{e}^{\int \coth x \, \mathrm{d}x} = \mathrm{e}^{\ln \sinh x} = \sinh x$.	**M1 A1**
Correct form on both sides: $\frac{\mathrm{d}}{\mathrm{d}x}(y \sinh x) = 4 \sinh^2 x$.	**M1**
Integrates $\sinh^2 x$ using identity: $y \sinh x = 2\left(\frac{1}{2}\sinh 2x - x\right) + C = \sinh 2x - 2x + C$.	**M1 A1**
Substitutes initial conditions: $\sinh \ln 3 = \sinh \ln 9 - 2\ln 3 + C$ leading to $\frac{4}{3} = \frac{40}{9} - 2\ln 3 + C$.	**M1**
$y \sinh x = \sinh 2x - 2x + 2\ln 3 - \frac{28}{9}$.	**A1**
	Total: 10

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Q29

9231/21, 9231/23 • October/November 2021 • Question 5

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Mark scheme.

$m^2 - 2m + 1 = 0 \Rightarrow m = 1$. Auxiliary equation.	**M1**
$y = e^x(Ax + B)$. Complementary function (accept '$y =$' missing).	**A1**
$y = p\sin x + q\cos x$, $y' = p\cos x - q\sin x$, $y'' = -p\sin x - q\cos x$. Particular integral and its derivatives.	**M1 A1**
$-p + 2q + p = 0$, $-q - 2p + q = 4$. Substitutes and equates coefficients.	**M1**
$p = -2$, $q = 0$.	**A1**
$y = e^x(Ax + B) - 2\sin x$.	**A1**
$y' = Ae^x + e^x(Ax + B) - 2\cos x$. Differentiates.	**M1**
$B = -4$, $A + B - 2 = 3 \Rightarrow A = 9$. Forms simultaneous equations using initial conditions.	**M1 A1**
$y = e^x(9x - 4) - 2\sin x$.	**A1**
	Total: 11

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Q30

9231/21, 9231/23 • October/November 2021 • Question 7

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Mark scheme.

$\dfrac{\mathrm{d}y}{\mathrm{d}x} + \dfrac{1}{\sqrt{x^2-1}}y = \dfrac{x^2 - x\sqrt{x^2-1}}{\sqrt{x^2-1}}$. Divides through by $\sqrt{x^2-1}$.	**B1**
$\mathrm{e}^{\int \frac{1}{\sqrt{x^2-1}}\mathrm{d}x} = \mathrm{e}^{\cosh^{-1}x}$. Finds integrating factor (M1 for correct form $\mathrm{e}^{a\int \frac{1}{\sqrt{x^2-1}}\mathrm{d}x}$, $a \neq 0$ constant).	**M1 A1**
$x + \sqrt{x^2-1}$ (AG).	**A1**
$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(y(x+\sqrt{x^2-1})\right) = \dfrac{x}{\sqrt{x^2-1}}$. Correct form on LHS and simplifies RHS.	**M1 A1**
$y(x+\sqrt{x^2-1}) = \sqrt{x^2-1} + C$. Integrates RHS (needs non-zero multiple of $\frac{x}{\sqrt{x^2-1}}$).	**M1 A1**
$2 = \frac{3}{4} + C$ leading to $C = \frac{5}{4}$. Substitutes initial conditions.	**M1**
$y = \dfrac{\sqrt{x^2-1}+\frac{5}{4}}{x+\sqrt{x^2-1}}$. Divides through by coefficient of y.	**M1 A1**
	Total: 11

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Q31

9231/22 • October/November 2021 • Question 2

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Mark scheme.

$e^{\ln(x^4+5)} = x^4 + 5$. Finds integrating factor.	**M1 A1**
$\dfrac{d}{dx}\left(y(x^4+5)\right) = 6x^5 + 30x$. Writes in correct form.	**M1**
$y(x^4+5) = x^6 + 15x^2 + C$.	**A1**
$6 = 16 + C$. Substitutes initial conditions.	**M1**
$y = \dfrac{x^6 + 15x^2 - 10}{x^4 + 5}$. Division through by coefficient of y.	**M1 A1**
	Total: 7

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Q32

9231/22 • October/November 2021 • Question 7

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Mark scheme.

$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2\frac{\mathrm{d}w}{\mathrm{d}x} + 2xw.$	**B1**
$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x^2\frac{\mathrm{d}^2w}{\mathrm{d}x^2} + 4x\frac{\mathrm{d}w}{\mathrm{d}x} + 2w.$	**B1**
Uses substitution to find w-x equation: $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 5y = x^2\frac{\mathrm{d}^2w}{\mathrm{d}x^2} + 4x\frac{\mathrm{d}w}{\mathrm{d}x} + 2w + 4x^2\frac{\mathrm{d}w}{\mathrm{d}x} + 8xw + 5x^2w.$	**M1**
$5x^2\frac{\mathrm{d}^2w}{\mathrm{d}x^2} + (4x^2+4x)\frac{\mathrm{d}w}{\mathrm{d}x} + (5x^2+8x+2)w = 5x^2+4x+2$ (AG).	**A1**
$m^2+4m+5=0$ leading to $m=-2\pm\mathrm{i}$. Auxiliary equation.	**M1**
$y = \mathrm{e}^{-2x}(A\cos x + B\sin x)$. Complementary function.	**A1**
$y = px^2+qx+r$, $y' = 2px+q$, $y'' = 2p$. Particular integral and its derivatives.	**B1**
Substitutes and equates coefficients: $2p+8px+4q+5px^2+5qx+5r = 5px^2+(8p+5q)x+(2p+4q+5r)$; $5p=5$, $8p+5q=4$, $2p+4q+5r=2$.	**M1**
$p=1$, $q=-\frac{4}{5}$, $r=\frac{16}{25}$.	**A1**
Substitutes for y and finds w in terms of x: $x^2w = \mathrm{e}^{-2x}(A\cos x + B\sin x) + x^2 - \frac{4}{5}x + \frac{16}{25}$.	**M1**
$w = (x\mathrm{e}^x)^{-2}(A\cos x + B\sin x) + 1 - (\frac{5}{4}x)^{-1} + (\frac{5}{4}x)^{-2}.$	**A1**
	Total: 11

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Q33

9231/21, 9231/22 • May/June 2022 • Question 3

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Mark scheme.

Auxiliary equation: $z^2 + z + 1 = 0$ leading to $m = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$	**(a) M1**
Complementary function: $x = e^{-\frac{1}{2}t}\left(A\sin\frac{\sqrt{3}}{2}t + B\cos\frac{\sqrt{3}}{2}t\right)$	**A1**
Particular integral and its derivatives: $x = pt^2 + qt + r$ leading to $\dot{x} = 2pt + q$ leading to $\ddot{x} = 2p$	**B1**
Substitutes and equates coefficients (PI must have correct form): $2p + 2pt + q + pt^2 + qt + r = t^2 + 1$	**M1**
$r = 1,\ q = -2,\ p = 1$	**A1**
$x = e^{-\frac{1}{2}t}\left(A\sin\frac{\sqrt{3}}{2}t + B\cos\frac{\sqrt{3}}{2}t\right) + t^2 - 2t + 1$	**A1**
$e^{-\frac{1}{2}t} \to 0$ as $t \to \infty$	**(b) M1**
$\ddot{x} \to 2$	**A1 FT**
	Total: 8

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Q34

9231/21, 9231/22 • May/June 2022 • Question 6

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Mark scheme.

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$	**B1**
Substitutes and derives first order separable equation: $x\left(v + x\dfrac{dv}{dx}\right) = vx + \sqrt{9x^2 + x^2v^2}$	**M1**
$x\dfrac{dv}{dx} = \sqrt{v^2+9}$	**A1**
Separates variables and integrates both sides: $\displaystyle\int \frac{1}{\sqrt{v^2+9}}\,dv = \int \frac{1}{x}\,dx$	**M1**
$\sinh^{-1}\left(\dfrac{v}{3}\right) = \ln x + C$	**A1**
Substitutes initial conditions: $C = 0$	**B1**
Converts to logarithmic form: $\ln\left(\dfrac{v}{3} + \sqrt{\dfrac{v^2}{9}+1}\right) = \ln x$	**M1**
$y + \sqrt{y^2 + 9x^2} = 3x^2$	**A1**
Squares to eliminate radical: $9x^4 - 6x^2y + y^2 - y^2 - 9x^2 = 0$	**M1**
$y = \frac{3}{2}(x^2 - 1)$	**A1**
	Total: 10

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Q35

9231/23 • May/June 2022 • Question 5

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Mark scheme.

Divides through by $x(x+7)$: $\dfrac{dy}{dx} + \dfrac{7y}{x(x+7)} = \dfrac{1}{x+7}$	**B1**
Converts to partial fractions or completes the square: $\dfrac{7}{x(x+7)} = \dfrac{1}{x} - \dfrac{1}{x+7}$ or $\dfrac{7}{x(x+7)} = \dfrac{7}{\left(x+\frac{7}{2}\right)^2 - \frac{49}{4}}$	**B1**
Finds integrating factor (accept non-zero multiple of $\frac{x}{x+7}$): $e^{\int 7x^{-1}(x+7)^{-1}dx} = e^{\ln x - \ln(x+7)} = \dfrac{x}{x+7}$	**M1 A1**
Correct form on LHS and attempt to integrate RHS (RHS must be of the form $\frac{a}{x+7} + \frac{b}{(x+7)^2}$): $\dfrac{d}{dx}\left(\dfrac{yx}{x+7}\right) = \dfrac{x}{(x+7)^2} = \dfrac{1}{x+7} - \dfrac{7}{(x+7)^2}$	**M1**
$\dfrac{yx}{x+7} = \ln(x+7) + 7(x+7)^{-1}\ (+C)$	**A1**
Finds C, substitutes into their expression: $\frac{7}{8} = \ln 8 + \frac{7}{8} + C$	**M1**
Divides their expression through by coefficient of y: $y = \left(1 + \dfrac{7}{x}\right)\ln\left(\dfrac{x+7}{8}\right) + \dfrac{7}{x}$	**M1 A1**
	Total: 9

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Q36

9231/23 • May/June 2022 • Question 7

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Mark scheme.

Auxiliary equation: $4m^2 - 1 = 0$ leading to $m = \pm\frac{1}{2}$	**(a) M1**
Complementary function (allow "$y =$" missing): $y = Ae^{-\frac{1}{2}x} + Be^{\frac{1}{2}x}$	**A1**
Substitutes $y = k$: $-k = 3$ leading to $k = -3$	**M1 A1**
General solution (must have $y =$ or sensible equivalent): $y = Ae^{-\frac{1}{2}x} + Be^{\frac{1}{2}x} - 3$	**A1 FT**
Derivative of general solution: $y' = -\frac{1}{2}Ae^{-\frac{1}{2}x} + \frac{1}{2}Be^{\frac{1}{2}x}$	**B1**
Substitutes initial conditions, solves simultaneous equations: $A+B = 0$, $-\frac{1}{2}A + \frac{1}{2}B = 2$ leading to $A = -2, B = 2$	**M1**
Particular solution: $y = 2e^{\frac{1}{2}x} - 2e^{-\frac{1}{2}x} - 3$	**A1**
Makes x the subject (y must be of the form $a_1e^{bx} \pm a_2e^{-bx} + c$): $y = 0$ leading to $4\sinh\frac{1}{2}x = 3$ leading to $\frac{1}{2}x = \sinh^{-1}\frac{3}{4}$	**(b) M1**
Uses logarithmic form: $\sinh^{-1}\frac{3}{4} = \ln\left(\frac{3}{4} + \sqrt{\frac{9}{16} + 1}\right) = \ln 2$	**M1**
$x = 2\ln 2 = \ln 4$	**A1**
	Total: 11

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Q37

9231/21, 9231/23 • October/November 2022 • Question 5

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Mark scheme.

$2m^2 + 2m + 1 = 0 \Rightarrow m = -\frac{1}{2} \pm \frac{1}{2}\mathrm{i}$. Correct auxiliary equation.	**M1**
Complementary function $y = \mathrm{e}^{-\frac{1}{2}x}(A\cos\frac{1}{2}x + B\sin\frac{1}{2}x)$.	**A1**
Particular integral $y = px^2 + qx + r$ and derivatives $y' = 2px + q$, $y'' = 2p$.	**B1**
Substitutes and equates coefficients: $p = 4$, $4p + q = 3$, $4p + 2q + r = 3$.	**M1**
$q = -13$, $r = 13$.	**A1**
General solution $y = \mathrm{e}^{-\frac{1}{2}x}(A\cos\frac{1}{2}x + B\sin\frac{1}{2}x) + 4x^2 - 13x + 13$.	**A1**
Full use of product rule for y'.	**M1**
Uses initial conditions: $A + 13 = 0$, $\frac{1}{2}B - \frac{1}{2}A - 13 = 0 \Rightarrow A = -13, B = 13$.	**M1 A1**
$y = \mathrm{e}^{-\frac{1}{2}x}(-13\cos\frac{1}{2}x + 13\sin\frac{1}{2}x) + 4x^2 - 13x + 13$.	**A1**
	Total: 10

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Q38

9231/21, 9231/23 • October/November 2022 • Question 8

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Mark scheme.

(a)	
$\dfrac{\mathrm{d}u}{\mathrm{d}\theta} = -2(\theta-1)$ leading to $(\theta-1)\,\mathrm{d}\theta = -\frac{1}{2}\,\mathrm{d}u$.	**B1**
$\displaystyle\int \frac{\theta-1}{\sqrt{1-(\theta-1)^2}}\,\mathrm{d}\theta = -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,\mathrm{d}u = -\sqrt{u}+C = -\sqrt{1-(\theta-1)^2}+C.$	**M1 A1**
(b)	
$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} - \dfrac{y}{\theta} = \theta\sin^{-1}(\theta-1)$. Divides through by θ.	**B1**
Integrating factor $\mathrm{e}^{-\int \theta^{-1}\mathrm{d}\theta} = \mathrm{e}^{-\ln\theta} = \theta^{-1}$.	**M1 A1**
$\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\theta^{-1}y) = \sin^{-1}(\theta-1)$.	**M1**
$\theta^{-1}y = \theta\sin^{-1}(\theta-1) - \displaystyle\int \frac{\theta}{\sqrt{1-(\theta-1)^2}}\,\mathrm{d}\theta$. Integrates $\sin^{-1}(\theta-1)$.	**M1 A1**
Applies part (a) and $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x = \sin^{-1}x + C$.	**M1**
$\theta^{-1}y = \theta\sin^{-1}(\theta-1) + \sqrt{1-(\theta-1)^2} - \sin^{-1}(\theta-1) + C$.	**A1**
Substitutes initial conditions: $1 = 1 + C$.	**M1**
$y = \theta(\theta-1)\sin^{-1}(\theta-1) + \theta\sqrt{1-(\theta-1)^2}$.	**M1 A1**
	Total: 14

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Q39

9231/22 • October/November 2022 • Question 4

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Mark scheme.

$\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{4}{4t^2-1}x = 1$. Divides through by $4t^2-1$.	**B1**
$\frac{4}{4t^2-1} = \frac{2}{2t-1} - \frac{2}{2t+1}$. Writes as partial fractions.	**B1**
Integrating factor $\mathrm{e}^{\int 4(4t^2-1)^{-1}\mathrm{d}t} = \mathrm{e}^{\ln(2t-1)-\ln(2t+1)} = \frac{2t-1}{2t+1}$.	**M1 A1**
$\frac{\mathrm{d}}{\mathrm{d}t}\left(x\frac{2t-1}{2t+1}\right) = \frac{2t-1}{2t+1} = \frac{2t+1-2}{2t+1}$. Correct form on LHS.	**M1**
$x\frac{2t-1}{2t+1} = t - \ln(2t+1) + C$.	**A1**
Substitutes initial conditions: $1 = 1 - \ln 3 + C$.	**M1**
$x = \left(\frac{2t+1}{2t-1}\right)\left(t + \ln\left(\frac{3}{2t+1}\right)\right)$.	**M1 A1**
	Total: 9

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Q40

9231/22 • October/November 2022 • Question 8

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Mark scheme.

(a)	
$\dfrac{dy}{dx} = \sinh u \dfrac{du}{dx}$.	**B1**
$\dfrac{d^2y}{dx^2} = \sinh u \dfrac{d^2u}{dx^2} + \cosh u \left(\dfrac{du}{dx}\right)^2$.	**B1**
Uses substitution to find y-x equation, AG.	**M1**
$\sqrt{\cosh^2 u - 1}\left(\dfrac{d^2u}{dx^2} + \dfrac{du}{dx}\right) + \cosh u \left(\dfrac{du}{dx}\right)^2 - 2\cosh u = 4e^{-x}$. AG.	**A1**
(b)	
Auxiliary equation $m^2 + m - 2 = 0 \Rightarrow m = 1, -2$.	**M1**
$y = Ae^x + Be^{-2x}$.	**A1**
Particular integral $y = ke^{-x}$, $y' = -ke^{-x}$, $y'' = ke^{-x}$.	**B1**
Substitutes and equates coefficients: $ke^{-x} - ke^{-x} - 2ke^{-x} = 4e^{-x}$.	**M1**
$k = -2$.	**A1**
$y = \cosh u = Ae^x + Be^{-2x} - 2e^{-x}$.	**A1**
$y' = \sinh u \dfrac{du}{dx} = Ae^x - 2Be^{-2x} + 2e^{-x}$.	**B1**
Substitutes initial conditions: $A + B - 2 = \frac{5}{3}$, $A - 2B + 2 = 4 \Rightarrow A = \frac{28}{9}, B = \frac{5}{9}$.	**M1 A1**
$u = \cosh^{-1}\left(\frac{28}{9}e^x + \frac{5}{9}e^{-2x} - 2e^{-x}\right)$.	**A1**
	Total: 14

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Q41

9231/21, 9231/22 • May/June 2023 • Question 2

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Mark scheme.

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}z}{\mathrm{d}x} - 1.$	**B1**
Substitutes and derives first order separable equation: $\dfrac{\mathrm{d}z}{\mathrm{d}x} - 1 = \dfrac{1+3z}{3z-1}.$	**M1**
$\dfrac{\mathrm{d}z}{\mathrm{d}x} = \dfrac{1+3z}{3z-1} + 1 = \dfrac{6z}{3z-1}.$	**A1**
Separates variables and integrates both sides: $\int \frac{1}{2} - \frac{1}{6}z^{-1}\,\mathrm{d}z = \int 1\,\mathrm{d}x.$	**M1**
$\frac{1}{2}z - \frac{1}{6}\ln z = x + C \Rightarrow -\frac{1}{2}x + \frac{1}{2}y - \frac{1}{6}\ln(x+y) = C.$	**A1**
Substitutes initial conditions into their expression.	**M1**
$C = -\frac{1}{2}$, giving $\frac{1}{6}\ln(x+y) + \frac{1}{2}(x-y) - \frac{1}{2} = 0$ (OE).	**A1**
	Total: 7

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Q42

9231/21, 9231/22 • May/June 2023 • Question 6

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Mark scheme.

Auxiliary equation $m^2 - 12m + 36 = 0$.	**M1**
Complementary function $x = \mathrm{e}^{6t}(At + B)$ (allow "$x =$" missing).	**A1**
Particular integral $x = p\sin t + q\cos t \Rightarrow x' = p\cos t - q\sin t \Rightarrow x'' = -p\sin t - q\cos t$.	**M1 A1**
Substitutes and equates coefficients: $-p + 12q + 36p = 37$, $-q - 12p + 36q = 0$ (must have two unknowns).	**M1**
$p = \frac{35}{37}$, $q = \frac{12}{37}$.	**A1**
$x = \mathrm{e}^{6t}(At + B) + \frac{35}{37}\sin t + \frac{12}{37}\cos t$ (must have "$x =$"; FT on CF).	**A1 FT**
Differentiates using product rule: $x' = A\mathrm{e}^{6t} + 6\mathrm{e}^{6t}(At + B) + \frac{35}{37}\cos t - \frac{12}{37}\sin t$ (must have their PI differentiated).	**M1***
Forms simultaneous equations using initial conditions: $B = -\frac{12}{37}$, $A + 6B + \frac{35}{37} = 0 \Rightarrow A = 1$.	**DM1 A1**
$x = \mathrm{e}^{6t}\left(t - \frac{12}{37}\right) + \frac{35}{37}\sin t + \frac{12}{37}\cos t$ (must have "$x =$").	**A1**
	Total: 11

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Q43

9231/23 • May/June 2023 • Question 2

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Mark scheme.

Auxiliary equation $6m^2+5m+1=0 \Rightarrow m=-\frac{1}{2},-\frac{1}{3}$.	**M1**
Complementary function $x = Ae^{-\frac{1}{2}t}+Be^{-\frac{1}{3}t}$ (allow with "$x=$" missing).	**A1**
Particular integral and its derivatives: $x=p+qt+rt^2 \Rightarrow x'=q+2rt \Rightarrow x''=2r$.	**B1**
Substitutes and equates coefficients: $12r+5q+10rt+p+qt+rt^2=t^2+10t+13$.	**M1**
$r=1$, $q=0$, $p=1$.	**A1**
$x=Ae^{-\frac{1}{2}t}+Be^{-\frac{1}{3}t}+t^2+1$ (must have "$x=$"). **(b)**	**A1**
$x=t^2+1$ (must have "$x=$"; accept "x is approximately"; do not accept "$x\to$"; FT on their PI).	**B1FT**
	Total: 7

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Q44

9231/21, 9231/23 • October/November 2023 • Question 4

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Mark scheme.

Auxiliary equation: $m^2+2m+3=0$.	**M1**
Complementary function: $[y=]\mathrm{e}^{-x}\left(A\cos\sqrt{2}x+B\sin\sqrt{2}x\right)$.	**A1**
Particular integral and its derivatives: $y=px^2+qx+r \Rightarrow y'=2px+q \Rightarrow y''=2p$.	**B1**
Substitutes and equates coefficients: $3p=27$, $3q+4p=0$, $2p+2q+3r=0$.	**M1**
$p=9$, $q=-12$, $r=2$.	**A1**
General solution: $y=\mathrm{e}^{-x}\left(A\cos\sqrt{2}x+B\sin\sqrt{2}x\right)+9x^2-12x+2$.	**A1**
Differentiates (must use product rule): $y'=\mathrm{e}^{-x}\left(-\sqrt{2}A\sin\sqrt{2}x+\sqrt{2}B\cos\sqrt{2}x\right)-\mathrm{e}^{-x}\left(A\cos\sqrt{2}x+B\sin\sqrt{2}x\right)+18x-12$.	**M1***
Uses initial conditions: $A+2=2$, $\sqrt{2}B-A-12=-8 \Rightarrow A=0, B=2\sqrt{2}$.	**DM1 A1**
$y=2\sqrt{2}\,\mathrm{e}^{-x}\sin\sqrt{2}x+9x^2-12x+2$.	**A1**
	Total: 10

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Q45

9231/21, 9231/23 • October/November 2023 • Question 6

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Mark scheme.

(a)	
$\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\sinh x = \frac{1}{2}(e^x - e^{-x})$.	**B1**
$\frac{1}{2}(e^x - e^{-x})(e^x + e^{-x}) = \frac{1}{2}(e^{2x} - e^{-2x}) = \sinh 2x$ (AG).	**M1 A1**
(b)	
$u = \sinh x \Rightarrow \mathrm{d}u = \cosh x\,\mathrm{d}x$.	**B1**
Applies identities to find integral in terms of u: $\int \sinh^2 2x \cosh x\,\mathrm{d}x = 4\int \sinh^2 x \cosh^2 x\,\mathrm{d}u = 4\int \sinh^2 x(\sinh^2 x + 1)\,\mathrm{d}u$.	**M1**
$= 4\int u^2(u^2+1)\,\mathrm{d}u$.	**A1**
$= 4\left(\frac{1}{5}u^5 + \frac{1}{3}u^3\right)(+C) = 4\left(\frac{1}{5}\sinh^5 x + \frac{1}{3}\sinh^3 x\right)(+C)$.	**A1**
(c)	
Finds integrating factor: $e^{\int \tanh x\,\mathrm{d}x} = e^{\ln\cosh x} = \cosh x$.	**M1 A1**
Correct form on LHS and attempt to integrate RHS: $\frac{\mathrm{d}}{\mathrm{d}x}(y\cosh x) = \sinh^2 2x \cosh x$.	**M1**
Integrates RHS using part (b): $y\cosh x = 4\left(\frac{1}{5}\sinh^5 x + \frac{1}{3}\sinh^3 x\right) + C$.	**M1 A1**
Substitutes initial conditions: $4 = C$.	**M1**
$y = 4\,\mathrm{sech}\,x\left(\frac{1}{5}\sinh^5 x + \frac{1}{3}\sinh^3 x + 1\right)$.	**A1**
	Total: 14

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Q46

9231/22 • October/November 2023 • Question 4

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Mark scheme.

Finds integrating factor: $e^{\int 3\,dx} = e^{3x}$.	**M1 A1**
Correct form on LHS and attempt to integrate RHS: $\dfrac{d}{dx}(ye^{3x}) = e^{3x}\sin x$.	**M1**
Integrates by parts once or uses $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$ (EITHER): $\int e^{3x}\sin x\,dx = -e^{3x}\cos x + 3\int e^{3x}\cos x\,dx$ (OR $= \frac{1}{3}e^{3x}\sin x - \frac{1}{3}\int e^{3x}\cos x\,dx$).	**M1 A1**
Integrates by parts again or substitutes trig exponential forms: $\int e^{3x}\sin x\,dx = -e^{3x}\cos x + 3\left(e^{3x}\sin x - 3\int e^{3x}\sin x\,dx\right)$ (or equivalent).	**M1**
Must not see i: $ye^{3x} = \frac{1}{10}e^{3x}(3\sin x - \cos x) + C$.	**A1**
Finds C, substitutes into their expression (must be integrated): $1 = -\frac{1}{10} + C$.	**M1**
Divides through by coefficient of y: $y = \frac{3}{10}\sin x - \frac{1}{10}\cos x + \frac{11}{10}e^{-3x}$.	**A1**
	Total: 9

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Q47

9231/22 • October/November 2023 • Question 8

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Mark scheme.

(a)	
$(v = y^4)\ \dfrac{\mathrm{d}v}{\mathrm{d}x} = 4y^3\dfrac{\mathrm{d}y}{\mathrm{d}x}$.	**B1**
$\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2} = 4y^3\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 12y^2\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$.	**B1**
Uses substitution to find v-x equation (AG): $\dfrac{\mathrm{d}^2v}{\mathrm{d}x^2} + \dfrac{\mathrm{d}v}{\mathrm{d}x} + 4v = 4y^3\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 12y^2\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 4y^3\dfrac{\mathrm{d}y}{\mathrm{d}x} + 4y^4$.	**M1**
$(\mathrm{AG}) = 4\left(y^3\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 3y^2\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + y^3\dfrac{\mathrm{d}y}{\mathrm{d}x} + y^4\right) = 4\mathrm{e}^{-2x}$.	**A1**
(b)	
Auxiliary equation: $m^2 + m + 4 = 0$.	**M1**
Complementary function (allow v = missing): $[v =]\mathrm{e}^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{15}}{2}x + B\sin\frac{\sqrt{15}}{2}x\right)$.	**A1**
Particular integral and its derivatives: $v = k\mathrm{e}^{-2x} \Rightarrow v' = -2k\mathrm{e}^{-2x} \Rightarrow v'' = 4k\mathrm{e}^{-2x}$.	**B1**
Substitutes and equates coefficients: $4k\mathrm{e}^{-2x} - 2k\mathrm{e}^{-2x} + 4k\mathrm{e}^{-2x} = 4\mathrm{e}^{-2x}$.	**M1**
$k = \frac{2}{3}$.	**A1**
Substitutes for y and finds v in terms of x (must not see i): $v = y^4 = \mathrm{e}^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{15}}{2}x + B\sin\frac{\sqrt{15}}{2}x\right) + \frac{2}{3}\mathrm{e}^{-2x}$.	**A1**
$v' = 4y^3\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-\frac{1}{2}x}\left(-\frac{\sqrt{15}}{2}A\sin\frac{\sqrt{15}}{2}x + \frac{\sqrt{15}}{2}B\cos\frac{\sqrt{15}}{2}x\right) - \frac{1}{2}\mathrm{e}^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{15}}{2}x + B\sin\frac{\sqrt{15}}{2}x\right) - \frac{4}{3}\mathrm{e}^{-2x}$.	**B1**
Substitutes initial conditions and forms simultaneous equations (must have used product rule when differentiating their v): $1 = A + \frac{2}{3}$, $-\frac{3}{2} = \frac{\sqrt{15}}{2}B - \frac{1}{2}A - \frac{4}{3} \Rightarrow A = \frac{1}{3}, B = 0$.	**M1 A1**
$y = \left(\frac{1}{3}\mathrm{e}^{-\frac{1}{2}x}\cos\frac{\sqrt{15}}{2}x + \frac{2}{3}\mathrm{e}^{-2x}\right)^{\frac{1}{4}}$ (accept $\pm$).	**A1**
	Total: 14

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Q48

9231/21, 9231/22 • May/June 2024 • Question 6

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Mark scheme.

(a)	
Substitutes sinh and cosh in terms of exponentials: $(\cosh x + \sinh x)^{1/2} = \left(\frac{1}{2}(e^x + e^{-x}) + \frac{1}{2}(e^x - e^{-x})\right)^{1/2}$.	**M1**
AG: $(e^x)^{1/2} = e^{x/2}$.	**A1**
(b)	
Auxiliary equation: $m^2 + m + 3 = 0 \Rightarrow m = -\frac{1}{2} \pm \frac{1}{2}\mathrm{i}\sqrt{11}$.	**M1**
Complementary function: $y = e^{-x/2}\left(A\cos\frac{\sqrt{11}}{2}x + B\sin\frac{\sqrt{11}}{2}x\right)$.	**A1**
Particular integral and its derivatives: $y = ke^{x/2} \Rightarrow y' = \frac{1}{2}ke^{x/2} \Rightarrow y'' = \frac{1}{4}ke^{x/2}$.	**B1**
Substitutes and equates coefficients: $\frac{1}{4}k + \frac{1}{2}k + 3k = 5$.	**M1**
$k = \frac{4}{3}$.	**A1**
Must have '$y =$'. FT on their CF: $y = e^{-x/2}\left(A\cos\frac{\sqrt{11}}{2}x + B\sin\frac{\sqrt{11}}{2}x\right) + \frac{4}{3}e^{x/2}$.	**A1FT**
Differentiates to find $\frac{dy}{dx}$.	**B1**
Substitutes initial conditions and forms simultaneous equations. For M1, CF must be correct form: $A = -\frac{1}{3}$, $B = \frac{1}{\sqrt{11}}$.	**M1 A1**
Must have '$y =$': $y = e^{-x/2}\left(-\frac{1}{3}\cos\frac{\sqrt{11}}{2}x + \frac{1}{\sqrt{11}}\sin\frac{\sqrt{11}}{2}x\right) + \frac{4}{3}e^{x/2}$.	**A1**
	Total: 12

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Q49

9231/21, 9231/22 • May/June 2024 • Question 7

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Mark scheme.

(a)

Applies substitution: $\int \frac{x}{\sqrt{1+x^2}}\,\mathrm{d}x = \frac{1}{2}\int \frac{1}{\sqrt{u}}\,\mathrm{d}u = \sqrt{u}\ [+C] = \sqrt{1+x^2}\ [+C]$.	**M1 A1**

(b)

Divides through by x: $\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{y}{x} = x\sinh^{-1}x$.	**B1**
Finds integrating factor: $\mathrm{e}^{-\int x^{-1}\,\mathrm{d}x} = \mathrm{e}^{-\ln x} = x^{-1}$.	**M1 A1**
Correct form on LHS: $\frac{\mathrm{d}}{\mathrm{d}x}(x^{-1}y) = \sinh^{-1}x$.	**M1**
Integrates RHS. RHS must be of the form $c\sinh^{-1}x$: $x^{-1}y = x\sinh^{-1}x - \int \frac{x}{\sqrt{1+x^2}}\,\mathrm{d}x$.	**M1 A1**
$x^{-1}y = x\sinh^{-1}x - \sqrt{1+x^2} + C$.	**A1**
Substitutes initial conditions: $1 = \ln(1+\sqrt{2}) - \sqrt{2} + C$.	***M1**
Divides through by their integrating factor: $y = x^2\sinh^{-1}x - x\sqrt{1+x^2} - x\ln(1+\sqrt{2}) + x(1+\sqrt{2})$.	**DM1 A1**
	Total: 12

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Q50

9231/23 • May/June 2024 • Question 5

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Mark scheme.

(a)	
Auxiliary equation: $m^2 + 10m + 25 = 0 \Rightarrow m = -5$.	**M1**
Complementary function. Allow '$x =$' missing: $x = \mathrm{e}^{-5t}(At + B)$.	**A1**
Particular integral and its derivatives. For M1, both derivatives must be present and of the correct form (only allow sign errors): $x = p\sin t + q\cos t \Rightarrow x' = p\cos t - q\sin t \Rightarrow x'' = -p\sin t - q\cos t$.	**M1 A1**
Substitutes and equates coefficients: $-p - 10q + 25p = 338$, $-q + 10p + 25q = 0$.	**M1**
$p = 12$, $q = -5$.	**A1**
Must have '$x =$': $x = \mathrm{e}^{-5t}(At + B) + 12\sin t - 5\cos t$.	**A1**
(b)	
Considering PI only: $x \approx 12\sin t - 5\cos t = 13\left(\frac{12}{13}\sin t - \frac{5}{13}\cos t\right)$.	**M1**
A1 for R and A1 for ϕ. Condone 22.6°. Withhold final A1 mark if working with incorrect PI. CWO: $12\sin t - 5\cos t = 13\sin(t - 0.395\ldots)$.	**A1 A1**
	Total: 10

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Q51

9231/23 • May/June 2024 • Question 7

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Mark scheme.

(a)

Applies product rule and $\frac{\mathrm{d}}{\mathrm{d}x}\left(\cosh^{-1}\frac{x}{3}\right) = \frac{1}{\sqrt{x^2-9}}$.	**M1**
OE: $\frac{x^2}{2\sqrt{x^2-9}} + \frac{1}{2}\sqrt{x^2-9} - \frac{9}{2\sqrt{x^2-9}}$.	**A1**
Puts over common denominator, AG: $\frac{x^2-9}{2\sqrt{x^2-9}} + \frac{1}{2}\sqrt{x^2-9} = \sqrt{x^2-9}$.	**A1**

(b)

Divides through by x: $\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{y}{x} = x\sqrt{x^2-9}$.	**B1**
Finds integrating factor: $\mathrm{e}^{-\int x^{-1}\,\mathrm{d}x} = \mathrm{e}^{-\ln x} = x^{-1}$.	**M1 A1**
Correct form on LHS: $\frac{\mathrm{d}}{\mathrm{d}x}(x^{-1}y) = \sqrt{x^2-9}$.	**M1**
Integrates RHS. For M1, RHS must be of the form $c\sqrt{x^2-9}$: $x^{-1}y = \frac{x}{2}\sqrt{x^2-9} - \frac{9}{2}\cosh^{-1}\frac{x}{3} + C$.	**M1 A1**
Substitutes initial conditions: $\frac{1}{3} = C$.	**M1**
Divides through by their integrating factor: $y = x\left(\frac{x}{2}\sqrt{x^2-9} - \frac{9}{2}\cosh^{-1}\frac{x}{3} + \frac{1}{3}\right)$.	**M1 A1**

Total: 12

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Q52

9231/21, 9231/23 • October/November 2024 • Question 5

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Mark scheme.

Auxiliary equation, two distinct real roots: $6m^2 - 5m + 1 = 0 \Rightarrow m = \frac{1}{2}, \frac{1}{3}$.	**M1**
Complementary function: $x = Ae^{t/2} + Be^{t/3}$.	**A1**
Particular integral and its derivatives: $x = pt^2 + qt + r \Rightarrow x' = 2pt + q \Rightarrow x'' = 2p$.	**B1**
Substitutes and equates coefficients: $p = 1$, $-10p + q = 1$, $12p - 5q + r = 1$.	**M1**
$p = 1$, $q = 11$, $r = 44$.	**A1**
General solution: $x = Ae^{t/2} + Be^{t/3} + t^2 + 11t + 44$ (FT on CF).	**A1**
Differentiates: $x' = \frac{1}{2}Ae^{t/2} + \frac{1}{3}Be^{t/3} + 2t + 11$.	**M1**
Uses initial conditions: $A+B+44 = 12$, $\frac{1}{2}A+\frac{1}{3}B+11 = -6 \Rightarrow A = -38, B = 6$.	**M1A1**
$x = -38e^{t/2} + 6e^{t/3} + t^2 + 11t + 44$.	**A1**
	Total: 10

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Q53

9231/21, 9231/23 • October/November 2024 • Question 7

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Mark scheme.

(a)	
Divides through by $\sqrt{x^2+16}$: $\dfrac{dy}{dx} + \dfrac{y}{\sqrt{x^2+16}} = x$.	**B1**
Finds integrating factor $e^{\int \frac{1}{\sqrt{x^2+16}} dx} = e^{\sinh^{-1}(x/4)}$.	**M1A1**
$= \frac{1}{4}x + \frac{1}{4}\sqrt{x^2+16}$ (AG).	**A1**
(b)	
Correct form on LHS and RHS: $\dfrac{d}{dx}\left(y\left(x+\sqrt{x^2+16}\right)\right) = x^2 + x\sqrt{x^2+16}$.	**M1A1**
Integrates RHS: $y\left(x+\sqrt{x^2+16}\right) = \frac{1}{3}x^3 + \frac{1}{3}(x^2+16)^{3/2} + C$.	**M1A1**
Substitutes initial conditions: $6(3+\sqrt{25}) = \frac{27}{3} + \frac{25}{3}\sqrt{25} + C$.	**M1**
$y\left(x+\sqrt{x^2+16}\right) = \frac{1}{3}x^3 + \frac{1}{3}(x^2+16)^{3/2} - \frac{8}{3}$.	**A1**
	Total: 10

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Q54

9231/22 • October/November 2024 • Question 5

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Mark scheme.

Auxiliary equation. M1 for correct type of roots (complex): $3m^2 + 2m + 1 = 0 \Rightarrow m = -\frac{1}{3} \pm \frac{\sqrt{2}}{3}i$.	**M1**
Complementary function: $y = e^{-x/3}\left(A\cos\frac{\sqrt{2}}{3}x + B\sin\frac{\sqrt{2}}{3}x\right)$.	**A1**
Particular integral and its derivatives: $y = px^2 + qx + r \Rightarrow y' = 2px + q \Rightarrow y'' = 2p$.	**B1**
Substitutes and equates coefficients: $p = 1$, $4p + q = 0$, $6p + 2q + r = 0$.	**M1**
$q = -4$, $r = 2$.	**A1**
General solution: $y = e^{-x/3}\left(A\cos\frac{\sqrt{2}}{3}x + B\sin\frac{\sqrt{2}}{3}x\right) + x^2 - 4x + 2$ (must have 'y='. FT on CF).	**A1FT**
Differentiates. GS must be in correct form.	***M1**
Uses initial conditions: $A + 2 = 0$, $\frac{\sqrt{2}}{3}B - \frac{1}{3}A - 4 = 0 \Rightarrow A = -2, B = 5\sqrt{2}$.	**DM1A1**
$y = e^{-x/3}\left(-2\cos\frac{\sqrt{2}}{3}x + 5\sqrt{2}\sin\frac{\sqrt{2}}{3}x\right) + x^2 - 4x + 2$.	**A1**
	Total: 10

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Q55

9231/22 • October/November 2024 • Question 7

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Mark scheme.

(a)	
Applies chain rule: $\dfrac{d}{dx}(\ln(\tanh x)) = \dfrac{\operatorname{sech}^2 x}{\tanh x}$.	**M1A1**
$= \dfrac{1}{\sinh x \cosh x} = \dfrac{2}{\sinh 2x}$ (AG).	**A1**
(b)	
Divides through by $\sinh(2x)$: $\dfrac{dy}{dx} + \dfrac{2y}{\sinh(2x)} = 1$.	**B1**
Correct form on LHS and RHS: $\dfrac{d}{dx}(y\tanh x) = \tanh x$.	**M1A1**
Integrates RHS: $y\tanh x = \ln(\cosh x) + C$.	**M1A1**
Substitutes initial conditions: $5\left(\frac{3}{5}\right) = \ln\frac{5}{4} + C$.	**M1**
$y\tanh x = \ln(\cosh x) + 3 - \ln\frac{5}{4}$ (accept equivalent exact form).	**A1**
	Total: 10

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Q56

9231/21, 9231/22 • May/June 2025 • Question 5

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Mark scheme.

Auxiliary equation $6m^2+3m+6=0 \Rightarrow m=-\frac{1}{4}\pm\frac{1}{4}\mathrm{i}\sqrt{15}$. M1 for correct type (complex) of roots.	**M1**
Complementary function $x=\mathrm{e}^{-\frac{1}{4}t}\left(A\cos\frac{\sqrt{15}}{4}t+B\sin\frac{\sqrt{15}}{4}t\right)$. Allow '$x=$' missing. Using x instead of t is A0.	**A1**
Particular integral and its derivatives: $x=k\mathrm{e}^{-t}\Rightarrow x'=-k\mathrm{e}^{-t}\Rightarrow x''=k\mathrm{e}^{-t}$.	**B1**
Substitutes and equates coefficients: $6k\mathrm{e}^{-t}-3k\mathrm{e}^{-t}+6k\mathrm{e}^{-t}=\mathrm{e}^{-t}$.	**M1**
$k=\frac{1}{9}$.	**A1**
General solution $x=\mathrm{e}^{-\frac{1}{4}t}\left(A\cos\frac{\sqrt{15}}{4}t+B\sin\frac{\sqrt{15}}{4}t\right)+\frac{1}{9}\mathrm{e}^{-t}$. FT on CF and their value of k. Must have '$x=$'.	**A1FT**
Differentiates using product rule.	***M1**
Substitutes initial conditions and forms simultaneous equations: $A=-\frac{1}{9}$, $B=\frac{1}{3\sqrt{15}}$.	**DM1,A1**
Final answer must have '$x=$'.	**A1**
	Total: 10

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Q57

9231/21, 9231/22 • May/June 2025 • Question 7

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Mark scheme.

Finds integrating factor using $\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert$: $\int \frac{x+5}{x^2+10x+61}dx = \frac{1}{2}\ln(x^2+10x+61)$.	**M1 A1**
$e^{-\frac{1}{2}\ln(x^2+10x+61)} = \dfrac{1}{\sqrt{x^2+10x+61}}$.	**A1**
Correct form on LHS using their integrating factor: $\dfrac{d}{dx}\left(\dfrac{y}{\sqrt{x^2+10x+61}}\right) = \dfrac{1}{\sqrt{x^2+10x+61}}$.	**M1**
Completes the square: $x^2+10x+61 = (x+5)^2+36$.	**B1**
Integrates RHS: $\int \dfrac{1}{\sqrt{(x+5)^2+6^2}}dx = \sinh^{-1}\left(\dfrac{x+5}{6}\right) + C$. Attempting to integrate by parts is M0.	***M1 A1**
Substitutes initial conditions: $C = -\sinh^{-1}\left(\frac{8}{6}\right) = -\ln\left(\frac{8}{6}+\frac{5}{3}\right) = -\ln 3$.	**DM1 A1**
Final answer.	**A1**
	Total: 10

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Q58

9231/23 • May/June 2025 • Question 4

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Mark scheme.

Auxiliary equation $m^2+m-2=0 \Rightarrow m=1,-2$. M1 for correct type of roots.	**M1**
Complementary function $x = Ae^t + Be^{-2t}$. Allow '$x =$' missing. Using x instead of t is A0.	**A1**
Particular integral and its derivatives: $x = pt^2+qt+r \Rightarrow x' = 2pt+q \Rightarrow x'' = 2p$.	**B1**
Substitutes and equates coefficients: $p=-1$, $2p-2q=1$, $2p+q-2r=-1$.	**M1**
$p=-1$, $q=-\frac{3}{2}$, $r=-\frac{5}{4}$.	**A1**
General solution $x = Ae^t + Be^{-2t} - t^2 - \frac{3}{2}t - \frac{5}{4}$. FT on CF and their values of p, q, r. Must have '$x =$'.	**A1FT**
Differentiates: $x' = Ae^t - 2Be^{-2t} - 2t - \frac{3}{2}$.	***M1**
Uses initial conditions: $A+B-\frac{5}{4}=0$, $A-2B-\frac{3}{2}=0 \Rightarrow A=\frac{4}{3}$, $B=-\frac{1}{12}$.	**DM1 A1**
Final answer must have '$x =$'.	**A1**
	Total: 10

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Q59

9231/23 • May/June 2025 • Question 7

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Mark scheme.

Finds integrating factor using $\int \frac{f'(x)}{f(x)} dx = \ln\lvert f(x)\rvert$: $\int \frac{2x+6}{x^2+6x+5} dx = \ln(x^2+6x+5)$.	**M1 A1**
$e^{-\ln(x^2+6x+5)} = \dfrac{1}{x^2+6x+5}$.	**A1**
Correct form on LHS: $\dfrac{d}{dx}\left(\dfrac{y}{x^2+6x+5}\right) = \dfrac{4}{x^2+6x+5}$.	**M1**
Partial fractions or completing the square: $\dfrac{4}{x^2+6x+5} = \dfrac{1}{x+1} - \dfrac{1}{x+5}$.	**B1**
Integrates RHS (RHS must be of the correct form): $\int \dfrac{4}{x^2+6x+5} dx = \ln\left(\dfrac{x+1}{x+5}\right) + C$.	***M1 A1**
Substitutes initial conditions: $0 = \ln\frac{1}{5} + C$.	**DM1**
$\dfrac{y}{x^2+6x+5} = \ln\left(\dfrac{x+1}{x+5}\right) - \ln\frac{1}{5} = \ln\left(\dfrac{5x+5}{x+5}\right)$. Accept $y = (x^2+6x+5)\ln\left(\dfrac{5x+5}{x+5}\right)$.	**A1**
	Total: 9

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Q60

9231/24 • May/June 2025 • Question 3

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Mark scheme.

Auxiliary equation $m^2 + 4m + 5 = 0 \Rightarrow m = -2 \pm \mathrm{i}$. M1 for correct type of roots.	**M1**
Complementary function $y = \mathrm{e}^{-2x}(A\cos x + B\sin x)$. Allow '$y =$' missing. Accept $y = \mathrm{e}^{-2x}(A\cos x + B\mathrm{i}\sin x)$.	**A1**
Particular integral and its derivatives: $y = k\mathrm{e}^{3x} \Rightarrow y' = 3k\mathrm{e}^{3x} \Rightarrow y'' = 9k\mathrm{e}^{3x}$.	**B1**
Substitutes and cancels e^{3x}: $9k + 12k + 5k = 13 \Rightarrow k = \frac{1}{2}$.	**M1 A1**
General solution. Must have '$y =$'. FT on their CF (the PI must be correct): $y = \mathrm{e}^{-2x}(A\cos x + B\sin x) + \frac{1}{2}\mathrm{e}^{3x}$.	**A1FT**
Uses $y(0) = 1$, FT on their $1 - k$; their CF must be correct: $A + k = 1 \Rightarrow A = 1 - k = \frac{1}{2}$.	**B1FT**
Finds y'; for M1 their CF must have the correct form: $y' = \mathrm{e}^{-2x}(-A\sin x + B\cos x) - 2\mathrm{e}^{-2x}(A\cos x + B\sin x) + \frac{3}{2}\mathrm{e}^{3x}$.	**M1**
Uses $y'(0) = 0$: $B - 2A + \frac{3}{2} = 0 \Rightarrow B = -\frac{1}{2}$.	**A1**
Must have '$y =$'.	**A1**
	Total: 10

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Q61

9231/24 • May/June 2025 • Question 6

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Mark scheme.

Divides through by x: $\dfrac{\mathrm{d}y}{\mathrm{d}x} - \dfrac{1}{x}y = 2x\tan^{-1}x$.	**B1**
Finds integrating factor: $\mathrm{e}^{-\int x^{-1}\mathrm{d}x} = x^{-1}$.	**M1 A1**
Correct form on LHS and attempt to integrate RHS by parts (treating $\tan^{-1}x$ as $\cot x$ is M0): $\dfrac{\mathrm{d}}{\mathrm{d}x}(yx^{-1}) = 2\tan^{-1}x$.	***M1**
$yx^{-1} = 2x\tan^{-1}x - 2\int \dfrac{x}{1+x^2}\mathrm{d}x = 2x\tan^{-1}x - \ln(1+x^2) + C$. Note $-\ln(1+x^2) = \ln\cos(\tan^{-1}x)$. SC B1 for $x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C$.	**A1 A1**
Finds C, substitutes given conditions into their solution: $\frac{1}{2}\pi = 2\left(\frac{1}{4}\pi\right) - \ln 2 + C \Rightarrow C = \ln 2$.	**DM1**
Divides through by their coefficient of y: $y = 2x^2\tan^{-1}x - x\ln(1+x^2) + x\ln 2$.	**DM1 A1**
	Total: 9

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Q62

9231/21, 9231/23 • October/November 2025 • Question 4

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Mark scheme.

Auxiliary equation $5m^2 + 2m + 1 = 0 \Rightarrow m = -\frac{1}{5} \pm \frac{2}{5}\mathrm{i}$. Correct type of roots for M1.	**M1**
Complementary function $y = \mathrm{e}^{-\frac{1}{5}x}\left(A\cos\frac{2}{5}x + B\sin\frac{2}{5}x\right)$.	**A1**
Particular integral and derivatives: $y = px^2 + qx + r \Rightarrow y' = 2px + q \Rightarrow y'' = 2p$.	**B1**
Substitutes and equates coefficients: $p = 1$, $4p + q = 5$, $10p + 2q + r = 3$.	**M1**
$q = 1$, $r = -9$.	**A1**
General solution $y = \mathrm{e}^{-\frac{1}{5}x}\left(A\cos\frac{2}{5}x + B\sin\frac{2}{5}x\right) + x^2 + x - 9$. FT on CF.	**A1 (FT)**
Differentiates using product rule, CF must be of correct form.	**M1***
Uses initial conditions: $A - 9 = 0$, $\frac{2}{5}B - \frac{1}{5}A + 1 = 0 \Rightarrow A = 9, B = 2$.	**M1 (dep)**
Correct values.	**A1**
$y = \mathrm{e}^{-\frac{1}{5}x}\left(9\cos\frac{2}{5}x + 2\sin\frac{2}{5}x\right) + x^2 + x - 9$. Must have '$y =$'.	**A1**
	Total: 10

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Q63

9231/21, 9231/23 • October/November 2025 • Question 7(b)

↑ Back to contents

Mark scheme.

Divides through by x: $\dfrac{dy}{dx} - \dfrac{y}{x} = x\tanh^{-1} x$.	**B1**
Finds integrating factor: $e^{-\int x^{-1}dx} = e^{-\ln x} = x^{-1}$.	**M1**
Correct integrating factor.	**A1**
Correct form on LHS and RHS: $\dfrac{d}{dx}(x^{-1}y) = \tanh^{-1} x$.	**M1**
Integrates correct function on RHS: $x^{-1}y = x\tanh^{-1} x - \displaystyle\int \frac{x}{1-x^2}dx$.	**M1***
Correct RHS integral form.	**A1**
$x^{-1}y = x\tanh^{-1} x + \frac{1}{2}\ln(1-x^2) + C$. '+C' not required.	**A1**
Substitutes initial conditions: $0 = \frac{1}{2}\tanh^{-1}\frac{1}{2} + \frac{1}{2}\ln\frac{3}{4} + C \Rightarrow C = -\frac{1}{4}\ln 3 - \frac{1}{2}\ln\frac{3}{4}$.	**M1 (dep)**
$x^{-1}y = x\tanh^{-1} x + \frac{1}{2}\ln(1-x^2) - \frac{1}{4}\ln\frac{27}{16}$.	**A1**
	Total: 9

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Q64

9231/22 • October/November 2025 • Question 5

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Mark scheme.

Auxiliary equation $m^2+4m+3=0 \Rightarrow m=-1,-3$ (may be implied by correct CF).	**(a) M1**
$y = Ae^{-x} + Be^{-3x}$.	**A1**
Particular integral and derivatives: $y = p\sin x + q\cos x \Rightarrow y' = p\cos x - q\sin x \Rightarrow y'' = -p\sin x - q\cos x$.	**M1 A1**
Substitutes and equates coefficients: $-p-4q+3p=0$, $-q+4p+3q=5$.	**M1**
$p=1$, $q=\frac{1}{2}$.	**A1**
$y = Ae^{-x} + Be^{-3x} + \sin x + \frac{1}{2}\cos x$.	**A1**
$e^{-x} \to 0$; sets up Pythagoras and compound angle formula: $y \approx \sin x + \frac{1}{2}\cos x = \frac{\sqrt{5}}{2}\left(\frac{2}{\sqrt{5}}\sin x + \frac{1}{\sqrt{5}}\cos x\right)$.	**(b) M1**
$\frac{\sqrt{5}}{2}\sin(x+0.464\ldots)$. A1 for R, A1 for ϕ (condone $26.6°$; allow $\phi = \tan^{-1}\frac{1}{2}$).	**A1 A1**
	Total: 10

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Q65

9231/22 • October/November 2025 • Question 7(b)

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Mark scheme.

Finds integrating factor: $e^{\frac{1}{2}\int(2+x)^{-1}dx} = \sqrt{2+x}$. Allow a sign error in the integrating factor for M1.	**M1 A1**
Correct form on LHS and simplifies RHS: $\frac{d}{dx}\left(y\sqrt{2+x}\right) = \sqrt{2+x}\sqrt{2-x} = \sqrt{4-x^2}$.	**M1 A1**
Integrates RHS: $y\sqrt{2+x} = \frac{1}{2}x\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{1}{2}x\right)$ $[+C]$.	**M1 A1**
Substitutes initial conditions: $\frac{1}{2}\sqrt{3} = \frac{1}{2}\sqrt{3} + \frac{1}{3}\pi + C$.	**M1**
$y\sqrt{2+x} = \frac{1}{2}x\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{1}{2}x\right) - \frac{1}{3}\pi$.	**A1**
	Total: 8

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Q66

9231/24 • October/November 2025 • Question 5

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Mark scheme.

Auxiliary equation $m^2+m+1=0 \Rightarrow m=-\frac{1}{2}\pm\frac{1}{2}\sqrt{3}$. Must give correct type of roots $a \pm b\sqrt{3}$.	**(a) M1**
$y = \mathrm{e}^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right)$. x missing/mislabelled is A0.	**A1**
Particular integral and derivatives: $y = p\sin 2x + q\cos 2x \Rightarrow y' = 2p\cos 2x - 2q\sin 2x \Rightarrow y'' = -4p\sin 2x - 4q\cos 2x$.	**M1 A1**
Substitutes and equates coefficients: $-4p-2q+p=1$, $-4q+2p+q=1$.	**M1**
$p=-\frac{1}{13}$, $q=-\frac{5}{13}$.	**A1**
$y = \mathrm{e}^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right) - \frac{1}{13}\sin 2x - \frac{5}{13}\cos 2x$. Must have "$y=$".	**A1**
$\mathrm{e}^{-\frac{1}{2}x} \to 0$ and correct use of compound angle identity: $y \approx -\frac{1}{13}\sin 2x - \frac{5}{13}\cos 2x = \sqrt{\frac{2}{13}}\left(-\frac{1}{\sqrt{26}}\sin 2x - \frac{5}{\sqrt{26}}\cos 2x\right)$.	**(b) M1**
$\sqrt{\frac{2}{13}}\sin(2x-1.77)$. A1 for R, A1 for ϕ (condone $101.3°$; allow $R=0.392$, $\phi=\cos^{-1}(-\frac{1}{\sqrt{26}})$).	**A1 A1**
	Total: 10

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Q67

9231/24 • October/November 2025 • Question 6

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Mark scheme.

Finds integrating factor using $\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert$: $\int \frac{1+x}{1-2x-x^2}dx = -\frac{1}{2}\ln(1-2x-x^2)$.	**M1 A1**
$e^{-\frac{1}{2}\ln(1-2x-x^2)} = \frac{1}{\sqrt{1-2x-x^2}}$.	**A1**
Correct form on LHS and RHS: $\frac{d}{dx}\left(\frac{y}{\sqrt{1-2x-x^2}}\right) = \frac{1}{\sqrt{1-2x-x^2}}$.	**M1**
Completes the square: $1-2x-x^2 = 2-(x+1)^2$.	**B1**
Integrates RHS (must be of the form $1/\sqrt{ax^2+bx+c}$): $\int \frac{1}{\sqrt{2-(x+1)^2}}dx = \sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + C$.	**M1* A1**
Substitutes initial conditions: $\pi = \sin^{-1}\frac{1}{\sqrt{2}} + C \Rightarrow C = \frac{3}{4}\pi$.	**M1 (dep) A1**
$\frac{y}{\sqrt{1-2x-x^2}} = \sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + \frac{3}{4}\pi$.	**A1**
	Total: 10

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Written by Ali Hashir.

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