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Further Math with Ali Hashir

Complex Numbers

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/11, 9231/12 • May/June 2015 • Question 8

By considering $\sum_{r=1}^{n} z^{2r-1}$, where $z = \cos\theta + \mathrm{i}\sin\theta$, show that, if $\sin\theta \neq 0$,

$$\sum_{r=1}^{n} \sin(2r-1)\theta = \frac{\sin^2 n\theta}{\sin\theta}.$$

[7]

Deduce that

$$\sum_{r=1}^{n} (2r-1)\cos\left[\frac{(2r-1)\pi}{2n}\right] = -\operatorname{cosec}\left(\frac{\pi}{2n}\right)\cot\left(\frac{\pi}{2n}\right).$$

[4]

Q2

9231/13 • May/June 2015 • Question 6

Let $z = \cos\theta + \mathrm{i}\sin\theta$. Use the binomial expansion of $(1+z)^n$, where n is a positive integer, to show that

$$\binom{n}{1}\cos\theta + \binom{n}{2}\cos 2\theta + \cdots + \binom{n}{n}\cos n\theta = 2^n \cos^n\left(\tfrac{1}{2}\theta\right)\cos\left(\tfrac{1}{2}n\theta\right) - 1.$$

[7]

Find

$$\binom{n}{1}\sin\theta + \binom{n}{2}\sin 2\theta + \cdots + \binom{n}{n}\sin n\theta.$$

[2]

Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 10

Using de Moivre's theorem, show that

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}.$$

[5]

Hence show that the equation $x^2 - 10x + 5 = 0$ has roots $\tan^2\left(\frac{1}{5}\pi\right)$ and $\tan^2\left(\frac{2}{5}\pi\right)$. [4]

Deduce a quadratic equation, with integer coefficients, having roots $\sec^2\left(\frac{1}{5}\pi\right)$ and $\sec^2\left(\frac{2}{5}\pi\right)$. [3]

Q4

9231/11, 9231/12 • May/June 2016 • Question 6

Use de Moivre's theorem to express $\cot 7\theta$ in terms of $\cot \theta$. [4]

Use the equation $\cot 7\theta = 0$ to show that the roots of the equation

$$x^6 - 21x^4 + 35x^2 - 7 = 0$$

are $\cot\left(\frac{1}{14}k\pi\right)$ for $k = 1, 3, 5, 9, 11, 13$, and deduce that

$$\cot^2\left(\tfrac{1}{14}\pi\right)\cot^2\left(\tfrac{3}{14}\pi\right)\cot^2\left(\tfrac{5}{14}\pi\right) = 7.$$

[5]

Q5

9231/13 • May/June 2016 • Question 9

Use de Moivre's theorem to show that $\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$. [4]

Find the corresponding expression for $\sin^4\theta$ in terms of $\cos 4\theta$ and $\cos 2\theta$. [4]

Hence find the exact value of $\int_0^{\frac{1}{8}\pi} (\cos^4\theta + \sin^4\theta)\, d\theta$. [3]

Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 10

Let $z = \cos\theta + \mathrm{i}\sin\theta$. Show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta \qquad \text{and} \qquad z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta.$$

[2]

By considering $\left(z - \frac{1}{z}\right)^4 \left(z + \frac{1}{z}\right)^2$, show that

$$\sin^4\theta\cos^2\theta = \tfrac{1}{32}\left(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2\right).$$

[7]

Hence find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} \sin^4\theta\cos^2\theta\,\mathrm{d}\theta$. [3]

Q7

9231/11, 9231/12 • May/June 2017 • Question 8

(i) Let $z = \cos\theta + \mathrm{i}\sin\theta$. Show that $z - \frac{1}{z} = 2\mathrm{i}\sin\theta$ and hence express $16\sin^5\theta$ in the form $\sin 5\theta + p\sin 3\theta + q\sin\theta$, where p and q are integers to be determined. [6]

(ii) Hence find the exact value of $\int_0^{\frac{1}{3}\pi} 16\sin^5\theta\,\mathrm{d}\theta$. [3]

Q8

9231/13 • May/June 2017 • Question 7

(i) Use de Moivre's theorem to prove that

$$\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}.$$

[5]

(ii) Hence find the solutions of the equation

$$t^4 - 4t^3 - 6t^2 + 4t + 1 = 0,$$

giving your answers in the form $\tan k\pi$, where k is a rational number. [5]

Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 10

(i) Use de Moivre's theorem to show that

$$\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta.$$

[5]

(ii) Hence explain why the roots of the equation $16x^4 - 20x^2 + 5 = 0$ are $x = \pm\sin\frac{1}{5}\pi$ and $x = \pm\sin\frac{2}{5}\pi$. [3]

(iii) Without using a calculator, find the exact values of

$$\sin\tfrac{1}{5}\pi\sin\tfrac{2}{5}\pi\sin\tfrac{3}{5}\pi\sin\tfrac{4}{5}\pi \quad \text{and} \quad \sin^2\left(\tfrac{1}{5}\pi\right) + \sin^2\left(\tfrac{2}{5}\pi\right).$$

[4]

Q10

9231/11, 9231/12 • May/June 2018 • Question 11

(i) Show that if $z = \mathrm{e}^{\mathrm{i}\theta}$ and $z \neq -1$ then

$$\frac{z-1}{z+1} = \mathrm{i}\tan\tfrac{1}{2}\theta.$$

[3]

(ii) Hence, or otherwise, show that if z is a cube root of unity then

$$\frac{z^3-1}{z^3+1} + \frac{z^2-1}{z^2+1} + \frac{z-1}{z+1} = 0.$$

[5]

(iii) Hence write down three roots of the equation

$$(z^3-1)(z^2+1)(z+1) + (z^2-1)(z^3+1)(z+1) + (z-1)(z^3+1)(z^2+1) = 0$$

and find the other three roots. Give your answers in an exact form. [6]

Q11

9231/13 • May/June 2018 • Question 3

(i) Use de Moivre's theorem to show that

$$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta.$$

[3]

(ii) Hence find all the roots of the equation

$$x^4 - 6x^2 + 1 = 0$$

in the form $\tan q\pi$, where q is a positive rational number. [5]

Q12

9231/11, 9231/13 • October/November 2018 • Question 7

(i) Use de Moivre's theorem to show that

$$\sin 8\theta = 8\sin\theta\cos\theta(1 - 10\sin^2\theta + 24\sin^4\theta - 16\sin^6\theta).$$

[6]

(ii) Use the equation $\dfrac{\sin 8\theta}{\sin 2\theta} = 0$ to find the roots of

$$16x^6 - 24x^4 + 10x^2 - 1 = 0$$

in the form $\sin k\pi$, where k is rational. [4]

Q13

9231/12 • October/November 2018 • Question 8

(i) By considering the binomial expansion of $\left(z+\dfrac{1}{z}\right)^6$, where $z=\cos\theta+\mathrm{i}\sin\theta$, express $\cos^6\theta$ in the form

$$\tfrac{1}{32}(p+q\cos 2\theta+r\cos 4\theta+s\cos 6\theta),$$

where p, q, r and s are integers to be determined. [6]

(ii) Hence find the exact value of

$$\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi}\cos^6\left(\tfrac{1}{2}x\right)\,\mathrm{d}x.$$

[4]

Q14

9231/13 • May/June 2019 • Question 3

(i) Write down the fifth roots of unity. [2]

(ii) Find all the roots of the equation

$$z^{10} + z^5 + 1 = 0,$$

giving each root in the form $\mathrm{e}^{\mathrm{i}\theta}$. [5]

Q15

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 9

(i) Use de Moivre's theorem to show that

$$\sec 6\theta = \frac{\sec^6\theta}{32 - 48\sec^2\theta + 18\sec^4\theta - \sec^6\theta}.$$

[6]

(ii) Hence obtain the roots of the equation

$$3x^6 - 36x^4 + 96x^2 - 64 = 0$$

in the form $\sec q\pi$, where q is rational. [5]

Q16

9231/21, 9231/22 • May/June 2020 • Question 3

(a) Find the roots of the equation $z^3 = -1 - \mathrm{i}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. [5]

Let $w = z_1^{3k} + z_2^{3k} + z_3^{3k}$, where k is a positive integer and z_1, z_2, z_3 are the roots of $z^3 = -1 - \mathrm{i}$.

(b) Express w in the form $R\mathrm{e}^{\mathrm{i}\alpha}$, where $R > 0$, giving R and α in terms of k. [3]

Q17

9231/23 • May/June 2020 • Question 8

(a) Use de Moivre's theorem to show that

$$\sin^6\theta = -\tfrac{1}{32}\left(\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10\right).$$

[6]

It is given that $\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10)$.

(b) Find the exact value of $\displaystyle\int_0^{\frac{1}{3}\pi} \left(\cos^6(\tfrac{1}{4}x) + \sin^6(\tfrac{1}{4}x)\right) \mathrm{d}x$. [4]

(c) Express each root of the equation $16c^6 + 16\left(1 - c^2\right)^3 - 13 = 0$ in the form $\cos k\pi$, where k is a rational number. [5]

Q18

9231/21, 9231/23 • October/November 2020 • Question 6(a)

Use de Moivre's theorem to show that $\sin^4\theta = \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3)$. [5]

Q19

9231/22 • October/November 2020 • Question 3

Find all the roots of the equation $(w+1)^6 = 1$, giving your answers in the form $x+\mathrm{i}y$ where x and y are real and exact. [4]

Q20

9231/22 • October/November 2020 • Question 7

(a) Show that $\sum_{r=1}^{n} z^{2r} = \dfrac{z^{2n+1} - z}{z - z^{-1}}$, for $z \neq 0, 1, -1$. [2]

(b) By letting $z = \cos\theta + \mathrm{i}\sin\theta$, show that, if $\sin\theta \neq 0$,

$$1 + 2\sum_{r=1}^{n} \cos(2r\theta) = \frac{\sin(2n+1)\theta}{\sin\theta}.$$

[5]

Q21

9231/21, 9231/22 • May/June 2021 • Question 5

(a) State the sum of the series $z + z^2 + z^3 + \cdots + z^n$, for $z \neq 1$. [1]

(b) Given that z is an nth root of unity and $z \neq 1$, deduce that $1 + z + z^2 + \cdots + z^{n-1} = 0$. [2]

(c) Given instead that $z = \frac{1}{3}(\cos\theta + \mathrm{i}\sin\theta)$, use de Moivre's theorem to show that

$$\sum_{m=1}^{\infty} 3^{-m} \cos m\theta = \frac{3\cos\theta - 1}{10 - 6\cos\theta}.$$

[7]

Q22

9231/23 • May/June 2021 • Question 1

(a) Find a and b such that

$$z^8 - \mathrm{i}z^5 - z^3 + \mathrm{i} = (z^5 - a)(z^3 - b).$$

[1]

(b) Hence find the roots of

$$z^8 - \mathrm{i}z^5 - z^3 + \mathrm{i} = 0,$$

giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 \leq \theta < 2\pi$. [6]

Q23

9231/23 • May/June 2021 • Question 4

By considering the binomial expansions of $\left(z+\frac{1}{z}\right)^5$ and $\left(z-\frac{1}{z}\right)^5$, where $z=\cos\theta+\mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\tan^5\theta = \frac{\sin 5\theta - a\sin 3\theta + b\sin\theta}{\cos 5\theta + a\cos 3\theta + b\cos\theta},$$

where a and b are integers to be determined. [7]

Q24

9231/21, 9231/23 • October/November 2021 • Question 6

(a) Use de Moivre's theorem to show that

$$\operatorname{cosec} 5\theta = \frac{\operatorname{cosec}^5 \theta}{5\operatorname{cosec}^4 \theta - 20\operatorname{cosec}^2 \theta + 16}.$$

[6]

(b) Hence obtain the roots of the equation

$$x^5 - 10x^4 + 40x^2 - 32 = 0$$

in the form $\operatorname{cosec}(q\pi)$, where q is rational. [4]

Q25

9231/22 • October/November 2021 • Question 4

(a) Write down all the roots of the equation $x^5 - 1 = 0$. [2]

(b) Use de Moivre's theorem to show that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$. [4]

(c) Use the results of parts (a) and (b) to express each real root of the equation

$$8x^9 - 8x^7 + x^5 - 8x^4 + 8x^2 - 1 = 0$$

in the form $\cos k\pi$, where k is a rational number. [4]

Q26

9231/21, 9231/22 • May/June 2022 • Question 7

(a) Use de Moivre's theorem to show that

$$\operatorname{cosec} 7\theta = \frac{\operatorname{cosec}^7 \theta}{7\operatorname{cosec}^6 \theta - 56\operatorname{cosec}^4 \theta + 112\operatorname{cosec}^2 \theta - 64}.$$

[6]

(b) Hence obtain the roots of the equation

$$x^7 - 14x^6 + 112x^4 - 224x^2 + 128 = 0$$

in the form $\operatorname{cosec} q\pi$, where q is rational. [5]

Q27

9231/23 • May/June 2022 • Question 1

Find the roots of the equation $z^3 = 7\sqrt{3} - 7\mathrm{i}$, giving your answers in the form $re^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$. [5]

Q28

9231/21, 9231/23 • October/November 2022 • Question 7

(a) State the sum of the series $1 + w + w^2 + w^3 + \cdots + w^{n-1}$, for $w \neq 1$. [1]

(b) Show that $(1 + \mathrm{i}\tan\theta)^k = \sec^k\theta(\cos k\theta + \mathrm{i}\sin k\theta)$, where θ is not an integer multiple of $\frac{1}{2}\pi$. [2]

(c) By considering $\displaystyle\sum_{k=0}^{n-1}(1 + \mathrm{i}\tan\theta)^k$, show that

$$\sum_{k=0}^{n-1}\sec^k\theta\sin k\theta = \cot\theta\left(1 - \sec^n\theta\cos n\theta\right),$$

provided θ is not an integer multiple of $\frac{1}{2}\pi$. [5]

(d) Hence find $\displaystyle\sum_{k=0}^{6m-1} 2^k\sin\left(\tfrac{1}{3}k\pi\right)$ in terms of m. [2]

Q29

9231/22 • October/November 2022 • Question 5

(a) Write down the fourth roots of unity. [1]

(b) Use de Moivre's theorem to show that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1.$$

[4]

(c) Hence obtain the real roots of the equation

$$16(8x^4 - 8x^2 + 1)^4 - 9 = 0$$

in the form $\cos(q\pi)$, where q is a rational number. [5]

Q30

9231/21, 9231/22 • May/June 2023 • Question 3

(a) By considering the binomial expansion of $(z + z^{-1})^4$, where $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that $\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$. [5]

(b) Use the substitution $x = \sin\theta$ to find the exact value of $\int_0^{\frac{1}{2}} (1 - x^2)^{\frac{3}{2}}\,\mathrm{d}x$. [3]

Q31

9231/23 • May/June 2023 • Question 3

By considering the binomial expansions of $\left(z+\frac{1}{z}\right)^4$ and $\left(z-\frac{1}{z}\right)^4$, where $z=\cos\theta+\mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\cot^4\theta = \frac{\cos 4\theta + a\cos 2\theta + b}{\cos 4\theta - a\cos 2\theta + b},$$

where a and b are integers to be determined. [7]

Q32

9231/21, 9231/23 • October/November 2023 • Question 2

Find the roots of the equation $(z + 5\mathrm{i})^3 = 4 + 4\sqrt{3}\,\mathrm{i}$, giving your answers in the form $r\cos\theta + \mathrm{i}(r\sin\theta - 5)$, where $r > 0$ and $0 < \theta < 2\pi$. [5]

Q33

9231/22 • October/November 2023 • Question 3

(a) Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

[4]

(b) Hence obtain the roots of the equation

$$32x^5 - 40x^3 + 10x - \sqrt{2} = 0$$

in the form $\cos(q\pi)$, where q is a rational number. [4]

Q34

9231/21, 9231/22 • May/June 2024 • Question 1

Find the roots of the equation $z^3 = -108\sqrt{3} + 108\text{i}$, giving your answers in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r > 0$ and $0 < \theta < 2\pi$. [5]

Q35

9231/23 • May/June 2024 • Question 6

(a) Show that

$$\sum_{r=1}^{n} z^{4r} = \frac{z^{4n+2} - z^2}{z^2 - z^{-2}}, \quad \text{for } z^2 \neq z^{-2}.$$

[2]

(b) By letting $z = \cos\theta + \mathrm{i}\sin\theta$, show that, if $\sin 2\theta \neq 0$,

$$\sum_{r=1}^{n} \sin(4r\theta) = \frac{\cos 2\theta - \cos(4n+2)\theta}{2\sin 2\theta}.$$

[5]

Q36

9231/21, 9231/23 • October/November 2024 • Question 8

(a) By considering the binomial expansion of $\left(z + \frac{1}{z}\right)^7$, where $z = \cos\theta + i\sin\theta$, use de Moivre's theorem to show that

$$\cos^7\theta = a\cos 7\theta + b\cos 5\theta + c\cos 3\theta + d\cos\theta,$$

where a, b, c and d are constants to be determined. [5]

Let $I_n = \int_0^{\pi/4} \cos^n\theta \, d\theta$.

(b) Show that

$$nI_n = 2^{-n/2} + (n-1)I_{n-2}.$$

[4]

(c) Using the results given in parts (a) and (b), find the exact value of I_9. [5]

Q37

9231/22 • October/November 2024 • Question 4

(a) Use de Moivre's theorem to show that

$$\cot 6\theta = \frac{\cot^6\theta - 15\cot^4\theta + 15\cot^2\theta - 1}{6\cot^5\theta - 20\cot^3\theta + 6\cot\theta}.$$

[6]

(b) Hence obtain the roots of the equation

$$x^6 - 6x^5 - 15x^4 + 20x^3 + 15x^2 - 6x - 1 = 0$$

in the form $\cot(q\pi)$, where q is a rational number. [4]

Q38

9231/21, 9231/22 • May/June 2025 • Question 1

Find the roots of the equation $z^3 = 27 - 27\mathrm{i}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi \leqslant \theta < \pi$. [5]

Q39

9231/21, 9231/22 • May/June 2025 • Question 3

By considering the binomial expansion of $\left(z - \frac{1}{z}\right)^5$, where $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\operatorname{cosec}^5\theta = \frac{a}{\sin 5\theta + b\sin 3\theta + c\sin\theta},$$

where a, b and c are integers to be determined. [6]

Q40

9231/23 • May/June 2025 • Question 5

(a) Use de Moivre's theorem to show that

$$\sec 5\theta = \frac{\sec^5\theta}{5\sec^4\theta - 20\sec^2\theta + 16}.$$

[6]

(b) Hence, obtain the roots of the equation

$$\sqrt{3}x^5 - 10x^4 + 40x^2 - 32 = 0$$

in the form $\sec(q\pi)$, where q is rational. [4]

Q41

9231/24 • May/June 2025 • Question 5

(a) Use de Moivre's theorem to show that

$$\sin 7\theta = -64\sin^7\theta + 112\sin^5\theta - 56\sin^3\theta + 7\sin\theta.$$

[5]

(b) Hence find all roots of the equation

$$64x^6 - 112x^4 + 56x^2 - 7 = 0$$

in the form $\sin q\pi$, where q is a rational number. [3]

Q42

9231/21, 9231/23 • October/November 2025 • Question 8

(a) State the sum of the series $z + z^2 + \cdots + z^n$, for $z \neq 1$. [1]

(b) By letting $z = \frac{1}{2}(\cos\theta + \mathrm{i}\sin\theta)$ use de Moivre's theorem to deduce that

$$\sum_{m=1}^{n} \left(\tfrac{1}{2}\right)^m \sin m\theta = \frac{\left(\frac{1}{2}\right)^{n+2}\sin n\theta - \left(\frac{1}{2}\right)^{n+1}\sin(n+1)\theta + \frac{1}{2}\sin\theta}{\frac{5}{4} - \cos\theta}.$$

[6]

(c) Use the result in (b) to find $\sum_{m=1}^{n} \left(\tfrac{1}{2}\right)^m m\cos m\theta$ in terms of n and θ. [You do not need to simplify your answer.] [3]

(d) Hence find $\sum_{m=1}^{\infty} \left(\tfrac{1}{2}\right)^m m\cos m\theta$ in terms of $\cos\theta$. [You may assume that $\frac{n}{2^n} \to 0$ as $n \to \infty$.] [2]

Q43

9231/22 • October/November 2025 • Question 2

Find the roots of the equation $z^4 = 8 - 8\mathrm{i}\sqrt{3}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$. [5]

Q44

9231/22 • October/November 2025 • Question 4(a)

By considering the binomial expansion of $\left(z + \frac{1}{z}\right)^6$, where $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\cos^6\theta = a\cos 6\theta + b\cos 4\theta + c\cos 2\theta + d,$$

where a, b, c and d are constants to be determined. [5]

Q45

9231/24 • October/November 2025 • Question 3

(a) Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta.$$

[4]

(b) Hence obtain the roots of the equation

$$32x^5 - 40x^3 + 10x - \sqrt{3} = 0$$

in the form $\sin q\pi$, where q is a rational number. [4]

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Written by Ali Hashir.

Answers available in the companion mark scheme booklet.

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