Further Math with Ali Hashir

Complex Numbers

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/11, 9231/12 • May/June 2015 • Question 8

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Mark scheme.

$\sum_{r=1}^{n} z^{2r-1} = \frac{z(1-[z^2]^n)}{1-z^2} = \frac{z-z^{2n+1}}{1-z^2}$	**M1A1**
$= \frac{1-z^{2n}}{z^{-1}-z} = \frac{1-(\cos 2n\theta + \mathrm{i}\sin 2n\theta)}{\cos\theta - \mathrm{i}\sin\theta - (\cos\theta + \mathrm{i}\sin\theta)} = \frac{1-\cos 2n\theta - \mathrm{i}\sin 2n\theta}{-2\mathrm{i}\sin\theta}$	**M1A1** **A1**
Equating imaginary parts: $\sum_{r=1}^{n} \sin(2r-1)\theta = \frac{2\sin^2 n\theta}{2\sin\theta} = \frac{\sin^2 n\theta}{\sin\theta}$ (AG)	**M1A1**
Differentiating: $\sum_{r=1}^{n} \sin(2r-1)\cos(2r-1)\theta = 2n\sin n\theta\cos n\theta\,\mathrm{cosec}\,\theta - \sin^2 n\theta\,\mathrm{cosec}\,\theta\cot\theta$	**M1A1**
Putting $\theta = \frac{\pi}{2n}$: $\sum_{r=1}^{n}(2r-1)\cos\left[\frac{(2r-1)\pi}{2n}\right] = n\sin\frac{\pi}{2}\cos\frac{\pi}{2}\,\mathrm{cosec}\left(\frac{\pi}{2n}\right) - \sin^2\frac{\pi}{2}\,\mathrm{cosec}\left(\frac{\pi}{2n}\right)\cot\left(\frac{\pi}{2n}\right)$	**dM1**
$\Rightarrow \sum_{r=1}^{n}(2r-1)\cos\left[\frac{(2r-1)\pi}{2n}\right] = -\,\mathrm{cosec}\left(\frac{\pi}{2n}\right)\cot\left(\frac{\pi}{2n}\right)$ (AG)	**A1**
	Total: 11

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Q2

9231/13 • May/June 2015 • Question 6

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Mark scheme.

$(1+z)^n = 1 + \binom{n}{1} z + \binom{n}{2} z^2 + \cdots + \binom{n}{n} z^n$	**B1**
$\mathrm{Re}\{(1+z)^n\} = 1 + \binom{n}{1} \cos\theta + \binom{n}{2} \cos 2\theta + \cdots + \binom{n}{n} \cos n\theta$	**M1A1**
$\mathrm{Re}\left([1+\cos\theta] + \mathrm{i}\sin\theta\right)^n = \mathrm{Re}\left(2\cos^2\left[\tfrac{1}{2}\theta\right] + 2\mathrm{i}\sin\left[\tfrac{1}{2}\theta\right]\cos\left[\tfrac{1}{2}\theta\right]\right)^n$	**M1A1**
$= 2^n \cos^n\left(\tfrac{1}{2}\theta\right) \mathrm{Re}\left(\cos\left[\tfrac{1}{2}\theta\right] + \mathrm{i}\sin\left[\tfrac{1}{2}\theta\right]\right)^n$	**A1**
$\Rightarrow \binom{n}{1} \cos\theta + \binom{n}{2} \cos 2\theta + \cdots + \binom{n}{n} \cos n\theta = 2^n \cos^n\left(\tfrac{1}{2}\theta\right) \cos\left(\tfrac{1}{2}n\theta\right) - 1 \quad \text{(AG)}$	**A1**
$\mathrm{Im}\{(1+z)^n\} = \binom{n}{1} \sin\theta + \binom{n}{2} \sin 2\theta + \cdots + \binom{n}{n} \sin n\theta$	**M1**
$\Rightarrow \binom{n}{1} \sin\theta + \binom{n}{2} \sin 2\theta + \cdots + \binom{n}{n} \sin n\theta = 2^n \cos^n\left(\tfrac{1}{2}\theta\right) \sin\left(\tfrac{1}{2}n\theta\right)$	**A1**
	Total: 9

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Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 10

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Mark scheme.

$(\cos\theta + \mathrm{i}\sin\theta)^5 = \cos 5\theta + \mathrm{i}\sin 5\theta$	**B1**
$(c+\mathrm{i}s)^5 = c^5 + 5c^4 s\mathrm{i} - 10c^3s^2 - 10\mathrm{i}c^2s^3 + 5cs^4 + s^5\mathrm{i}$	**M1A1**
$\tan 5\theta = \dfrac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4}$	**M1**
Divide numerator and denominator by c^5 (stated or shown) $\Rightarrow \tan 5\theta = \dfrac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$ (AG)	**A1**
$\tan 5\theta = 0 \Rightarrow \theta = \frac{1}{5}\pi, \frac{2}{5}\pi, \frac{3}{5}\pi, \frac{4}{5}\pi, \pi$	**B1**
$t^5 - 10t^3 + 5t = 0$ has roots $\tan\left(\frac{1}{5}\pi\right), \tan\left(\frac{2}{5}\pi\right), \tan\left(\frac{3}{5}\pi\right), \tan\left(\frac{4}{5}\pi\right), \tan\pi$	
$\Rightarrow t^4 - 10t^2 + 5 = 0$ has roots $\tan\left(\frac{1}{5}\pi\right), \tan\left(\frac{2}{5}\pi\right), \tan\left(\frac{3}{5}\pi\right), \tan\left(\frac{4}{5}\pi\right)$.	**B1**
$\Rightarrow \left(t^2 - \tan^2\left(\frac{1}{5}\pi\right)\right)\left(t^2 - \tan^2\left(\frac{2}{5}\pi\right)\right) = 0$, since $\tan\left(\frac{1}{5}\pi\right) = -\tan\left(\frac{4}{5}\pi\right)$ and $\tan\left(\frac{2}{5}\pi\right) = -\tan\left(\frac{3}{5}\pi\right)$	**M1**
$\Rightarrow x^2 - 10x + 5 = 0$ has roots $\tan^2\left(\frac{1}{5}\pi\right)$ and $\tan^2\left(\frac{2}{5}\pi\right)$ (AG)	**A1**
$\sec^2\alpha = 1 + \tan^2\alpha$	**M1**
$y = 1 + x \Rightarrow x = y - 1 \Rightarrow (y-1)^2 - 10(y-1) + 5 = 0$	**M1**
$\Rightarrow y^2 - 12y + 16 = 0$	**A1**
	Total: 12

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Q4

9231/11, 9231/12 • May/June 2016 • Question 6

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Mark scheme.

$(c+is)^7 = c^7 + 7c^6(is) + \cdots + (is)^7$	**M1**
$\dfrac{\cos 7\theta}{\sin 7\theta} = \dfrac{c^7 - 21c^5s^2 + 35c^3s^4 - 7cs^6}{7c^6s - 35c^4s^3 + 21c^2s^5 - s^7}$	**A1**
$\cot 7\theta = \dfrac{\cot^7\theta - 21\cot^5\theta + 35\cot^3\theta - 7\cot\theta}{7\cot^6\theta - 35\cot^4\theta + 21\cot^2\theta - 1}$	**M1 A1**
$\cot 7\theta = 0$ and $\cot\theta \neq 0 \Rightarrow x^6 - 21x^4 + 35x^2 - 7 = 0$, where $x = \cot\theta$	
and $\theta = k\dfrac{\pi}{14}$ where $k = 1, 3, 5, 9, 11, 13$	**M1**
Product of roots $\Rightarrow \cot\dfrac{\pi}{14}\cot\dfrac{3\pi}{14}\cot\dfrac{5\pi}{14}\cot\dfrac{9\pi}{14}\cot\dfrac{11\pi}{14}\cot\dfrac{13\pi}{14} = -7$	**M1 A1**
But $\cot\dfrac{\pi}{14} = -\cot\dfrac{13\pi}{14}$, $\cot\dfrac{3\pi}{14} = -\cot\dfrac{11\pi}{14}$, $\cot\dfrac{5\pi}{14} = -\cot\dfrac{9\pi}{14}$	**M1**
Hence $\cot^2\dfrac{1}{14}\pi\cot^2\dfrac{3}{14}\pi\cot^2\dfrac{5}{14}\pi = 7$. (AG) (Penultimate line must be seen.)	**A1**
SC Award B1 if product of roots mentioned, without proper pairing seen.	

Total: 9

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Q5

9231/13 • May/June 2016 • Question 9

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Mark scheme.

$\left(z+\frac{1}{z}\right)^4 = \left(z^4+\frac{1}{z^4}\right)+4\left(z^2+\frac{1}{z^2}\right)+6$	**M1**
Taking $z=\cos\theta+i\sin\theta$, $(2\cos\theta)^4 = 2\cos 4\theta+8\cos 2\theta+6$	**M1A1**
$\Rightarrow \cos^4\theta = \frac{1}{16}(2\cos 4\theta+8\cos 2\theta+6) = \frac{1}{8}(\cos 4\theta+4\cos 2\theta+3)$ (AG)	**A1**
$\left(z-\frac{1}{z}\right)^4 = \left(z^4+\frac{1}{z^4}\right)-4\left(z^2+\frac{1}{z^2}\right)+6$	**M1**
$\Rightarrow (2i\sin\theta)^4 = 2\cos 4\theta-8\cos 2\theta+6$	**M1A1**
$\Rightarrow \sin^4\theta = \frac{1}{16}(2\cos 4\theta-8\cos 2\theta+6) = \frac{1}{8}(\cos 4\theta-4\cos 2\theta+3)$	**A1**
$\int_0^{\frac{1}{8}\pi}(\cos^4\theta+\sin^4\theta)\,d\theta = \frac{1}{4}\int_0^{\frac{1}{8}\pi}(\cos 4\theta+3)\,d\theta$	**M1**
$=\frac{1}{4}\left[\frac{\sin 4\theta}{4}+3\theta\right]_0^{\frac{1}{8}\pi} = \frac{1}{16}+\frac{3}{32}\pi$ (oe)	**M1A1**
	Total: 11

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Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 10

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Mark scheme.

$z^n = \cos n\theta + \mathrm{i}\sin n\theta$ and $z^{-n} = \cos(-n\theta) + \mathrm{i}\sin(-n\theta) = \cos n\theta - \mathrm{i}\sin n\theta$	
$\Rightarrow z^n + \dfrac{1}{z^n} = 2\cos n\theta$ and $z^n - \dfrac{1}{z^n} = 2\mathrm{i}\sin n\theta$. (AG)	**M1A1**
$\left(z - \dfrac{1}{z}\right)^4 \left(z + \dfrac{1}{z}\right)^2 = \left(z - \dfrac{1}{z}\right)^2 \left(z^2 - \dfrac{1}{z^2}\right)^2$ (Or 1st bracket expanded)	**M1**
$= \left(z^2 - 2 + \dfrac{1}{z^2}\right)\left(z^4 - 2 + \dfrac{1}{z^4}\right)$ (and 2nd bracket expanded)	**M1**
$= \left(z^6 + \dfrac{1}{z^6}\right) - 2\left(z^4 + \dfrac{1}{z^4}\right) - \left(z^2 + \dfrac{1}{z^2}\right) + 4$ (Expanded and grouped)	**M1A1**
$\Rightarrow 64\sin^4\theta\cos^2\theta = 2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4$	**M1A1**
$\Rightarrow \sin^4\theta\cos^2\theta = \dfrac{1}{32}(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2)$ (AG) (Uses initial result)	**A1**
$\displaystyle\int_0^{\frac{1}{4}\pi} \sin^4\theta\cos^2\theta\,\mathrm{d}\theta = \frac{1}{32}\left[\frac{1}{6}\sin 6\theta - \frac{1}{2}\sin 4\theta - \frac{1}{2}\sin 2\theta + 2\theta\right]_0^{\frac{1}{4}\pi}$	**M1A1**
$= \dfrac{1}{32}\left(-\dfrac{1}{6} - 0 - \dfrac{1}{2} + \dfrac{\pi}{2}\right) = \left(\dfrac{3\pi - 4}{192}\right)$ or 0.0283 (3sf)	**A1**

ALT METHOD: Expand and group each bracket separately and sub trig: **M1, A1, A1** Multiply together two brackets and manipulate trig expressions **M1,M1** LHS = RHS **M1 A1**

Total: 12

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Q7

9231/11, 9231/12 • May/June 2017 • Question 8

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Mark scheme.

$z - z^{-1} = \cos\theta + \mathrm{i}\sin\theta - \cos(-\theta) - \mathrm{i}\sin(-\theta) = 2\mathrm{i}\sin\theta$	**B1**
$\left(z - \frac{1}{z}\right)^5 = \left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$	**M1A1**
$\Rightarrow 32\sin^5\theta\,\mathrm{i} = 2\mathrm{i}\sin 5\theta - 10\mathrm{i}\sin 3\theta + 20\mathrm{i}\sin\theta$ (Grouping not required at this stage.)	**M1A1**
$\Rightarrow 16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta$ (**M1** for grouping and applying initial result)	**A1**
$\int_0^{\frac{1}{3}\pi} 16\sin^5\theta\,\mathrm{d}\theta = \left[-\frac{\cos 5\theta}{5} + \frac{5\cos 3\theta}{3} - 10\cos\theta\right]_0^{\frac{1}{3}\pi}$	**M1A1FT**
$= \left[-\frac{1}{10} - \frac{5}{3} - 5\right] - \left[-\frac{1}{5} + \frac{5}{3} - 10\right] = \frac{53}{30}$	**A1**
	Total: 9

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Q8

9231/13 • May/June 2017 • Question 7

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Mark scheme.

$(c+\mathrm{i}s)^4 = c^4 + 4c^3(\mathrm{i}s) + 6c^2(\mathrm{i}s)^2 + 4c(\mathrm{i}s)^3 + (\mathrm{i}s)^4$ (SOI)	**B1**
Equate real and imaginary parts	**M1**
$\cos 4\theta = c^4 - 6c^2s^2 + s^4$ (SOI)	**A1**
$\sin 4\theta = 4c^3s - 4cs^3$ (SOI)	**A1**
$\tan 4\theta = \dfrac{(4c^3s - 4cs^3) \div c^4}{(c^4 - 6c^2s^2 + s^4) \div c^4} \left(= \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta} \right)$ (AG)	**A1**
$\tan 4\theta = -1 \Rightarrow t^4 - 4t^3 - 6t^2 + 4t + 1 = 0$	**M1M1**
so $4\theta = \dfrac{3\pi}{4} \left(, \dfrac{7\pi}{4}, \dfrac{11\pi}{4}, \dfrac{15\pi}{4} \right)$	**M1**
$t = \tan\dfrac{3\pi}{16}, \tan\dfrac{7\pi}{16}, \tan\dfrac{11\pi}{16}, \tan\dfrac{15\pi}{16}$. Allow $\left(\dfrac{k}{4} - \dfrac{1}{16}\right)\pi$, $k = 0, 1, 2, 3$ oe	**A1A1**
	Total: 10

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Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 10

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Mark scheme.

$\sin 5\theta = \mathrm{Im}(c+\mathrm{i}s)^5 = \mathrm{Im}\left(c^5 + 5c^4\mathrm{i}s + 10c^3(\mathrm{i}s)^2 + 10c^2(\mathrm{i}s)^3 + 5c(\mathrm{i}s)^4 + (\mathrm{i}s)^5\right)$ (SOI)	**B1**
$\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$	**M1A1**
$= s\left(5[1-s^2]^2 - 10s^2[1-s^2] + s^4\right)$	**M1**
$= \ldots = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$	**A1**
If $\theta = 0, \pm\frac{1}{5}\pi, \pm\frac{2}{5}\pi$ then $\sin 5\theta = 0$	**B1**
$\Rightarrow 16s^5 - 20s^3 + 5s = 0$, where $s = \sin\theta$, $\Rightarrow s(16s^4 - 20s^2 + 5) = 0$	**B1**
$s = 0 \Rightarrow \theta = 0$ Hence roots of $16s^4 - 20s^2 + 5 = 0$ are $\pm\sin\frac{1}{5}\pi, \pm\sin\frac{2}{5}\pi$ (AG)	**B1**
Since $\sin\frac{4}{5}\pi = -\sin\left(-\frac{1}{5}\pi\right)$ and $\sin\frac{3}{5}\pi = -\sin\left(-\frac{2}{5}\pi\right)$	**B1**
$\sin\left(\frac{4}{5}\pi\right)\sin\left(\frac{3}{5}\pi\right)\sin\left(\frac{2}{5}\pi\right)\sin\left(\frac{1}{5}\pi\right) = \sin\left(-\frac{1}{5}\pi\right)\sin\left(-\frac{2}{5}\pi\right)\sin\left(\frac{1}{5}\pi\right)\sin\left(\frac{2}{5}\pi\right) = \frac{5}{16}$	**M1A1**
$\sin^2\frac{1}{5}\pi + \sin^2\frac{2}{5}\pi = -\frac{(-20)}{16} = \frac{5}{4}$	**A1**
	Total: 12

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Q10

9231/11, 9231/12 • May/June 2018 • Question 11

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Mark scheme.

If $z \neq -1$ then $\dfrac{z-1}{z+1} = \dfrac{z^{1/2}-z^{-1/2}}{z^{1/2}+z^{-1/2}} \left(\text{or} = \dfrac{2i\sin\theta}{2(\cos\theta+1)}\right)$. Using de Moivre's theorem or multiplying numerator and denominator by $\cos\theta + 1 - i\sin\theta$	**M1A1**
$= \dfrac{2i\sin\frac{1}{2}\theta}{2\cos\frac{1}{2}\theta} = i\tan\frac{1}{2}\theta$	**A1**
If $z = 1$ then $z^r - 1 = 0$ so the given sum is zero.	**B1**
If $z \neq 1$ then $z = e^{i\frac{2\pi}{3}}$ or $z = e^{-i\frac{2\pi}{3}}$, so. Must consider all three roots of unity	**B1**
$\dfrac{z^3-1}{z^3+1} + \dfrac{z^2-1}{z^2+1} + \dfrac{z-1}{z+1} = 0 + \dfrac{e^{\pm i\frac{4\pi}{3}}-1}{e^{\pm i\frac{4\pi}{3}}+1} + \dfrac{e^{\pm i\frac{2\pi}{3}}-1}{e^{\pm i\frac{2\pi}{3}}+1}$. AEF two of three terms correct for M1	**M1A1**
Using (ii), or by direct calculation, $\dfrac{e^{\pm i\frac{4\pi}{3}}-1}{e^{\pm i\frac{4\pi}{3}}+1} + \dfrac{e^{\pm i\frac{2\pi}{3}}-1}{e^{\pm i\frac{2\pi}{3}}+1} = i\tan\frac{1}{2}\left(\frac{4\pi}{3}\right) + i\tan\frac{1}{2}\left(\frac{2\pi}{3}\right) = 0$. AG	**A1**
$z = 1,\ e^{\pm i\frac{2\pi}{3}}$, $(z^3-1)(z^2+1)(z+1) + (z^2-1)(z^3+1)(z+1) + (z-1)(z^3+1)(z^2+1) = 0$. Writes down the cube roots of unity. AEF, must be exact	**B1**
$\Rightarrow 3z^6 + z^5 + z^4 - z^2 - z - 3 = 0$. Expands	**M1**
$3z^6 + z^5 + z^4 - z^2 - z - 3 = (z+1)(z^3-1)(3z^2-2z+3)$. Factorises	**M1A1**
$\Rightarrow z = -1,\ \dfrac{2 \pm i\sqrt{32}}{6} = \dfrac{1}{3} \pm i\dfrac{2}{3}\sqrt{2}$. Finds other three roots. AEF, must be exact	**M1A1**
	Total: 14

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Q11

9231/13 • May/June 2018 • Question 3

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Mark scheme.

$(c+is)^4 = c^4 + 4c^3(is) + 6c^2(-s^2) + 4c(is)^3 + (is)^4$. Uses binomial theorem to expand $(c+is)^4$.	**M1**
$\Rightarrow \cos 4\theta = c^4 - 6c^2s^2 + s^4$. Takes real part, AG.	**M1A1**
$\dfrac{\cos 4\theta}{\cos^4\theta} = \tan^4\theta - 6\tan^2\theta + 1$. Divides through by $\cos^4\theta$.	**M1A1**
Let $x = \tan\theta$, then $x^4 - 6x^2 + 1 = 0 \Rightarrow \cos 4\theta = 0$. Following on from finding correct quartic. Solves $\cos 4\theta = 0$.	**dM1**
$\Rightarrow 4\theta = \pm\dfrac{\pi}{2} + 2m\pi,\ m \in \mathbb{Z}$	
Roots are $\tan q\pi$ where $q = \dfrac{1}{8}, \dfrac{3}{8}, \dfrac{5}{8}, \dfrac{7}{8}$ $(q > 0)$	**A1A1**
	Total: 8

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Q12

9231/11, 9231/13 • October/November 2018 • Question 7

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Mark scheme.

Write $c = \cos\theta$, $s = \sin\theta$. $\cos 8\theta + \mathrm{i}\sin 8\theta = (c + \mathrm{i}s)^8$ (Uses binomial theorem.)	**M1**
$\Rightarrow \sin 8\theta = 8c^7s - 56c^5s^3 + 56c^3s^5 - 8cs^7$	**A1**
$= 8cs(c^6 - 7c^4s^2 + 7c^2s^4 - s^6)$ (Factorises.)	**M1**
$c^6 - 7c^4s^2 + 7c^2s^4 - s^6 = (1-s^2)^3 - 7(1-s^2)^2s^2 + 7(1-s^2)s^4 - s^6$ (Uses $c^2 = 1 - s^2$.)	**M1**
$= (1 - 3s^2 + 3s^4 - s^6) - 7(s^2 - 2s^4 + s^6) + 7(s^4 - s^6) - s^6$	**A1**
$= 1 - 10s^2 + 24s^4 - 16s^6$	
$\Rightarrow \sin 8\theta = 8cs(1 - 10s^2 + 24s^4 - 16s^6)$ (AG)	**A1**
$\sin 2\theta = 2cs \Rightarrow \dfrac{\sin 8\theta}{\sin 2\theta} = 4(1 - 10s^2 + 24s^4 - 16s^6)$ (Uses $\sin 2\theta = 2cs$ to relate with equation in part (i))	**M1**
$\sin 8\theta = 0 \Rightarrow \theta = \dfrac{n\pi}{8}$ (Solves $\sin 8\theta = 0$)	**M1**
$x = \sin\dfrac{\pi}{8}$ (Gives one correct solution)	**A1**
$\sin\dfrac{n\pi}{8}$, $n = \pm1, \pm2 \pm 3$ *or* $1, 2, 3, 9, 10, 11$ (Gives five other solutions. Allow different values of k as long as six distinct solutions are found. $\sin 0$ and $\sin\frac{\pi}{2}$ must be excluded.)	**A1**
	Total: 10

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Q13

9231/12 • October/November 2018 • Question 8

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Mark scheme.

$z + z^{-1} = 2\cos\theta$ (Use of $z + z^{-1} = 2\cos\theta$.)	**B1**
$(z + z^{-1})^6 = (z^6 + z^{-6}) + 6(z^4 + z^{-4}) + 15(z^2 + z^{-2}) + 20$ (Expands and groups.)	**M1A1**
$64\cos^6\theta = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$ (Substitutes $z^n + z^{-n} = 2\cos n\theta$.)	**M1A1**
$\Rightarrow \cos^6\theta = \frac{1}{32}(10 + 15\cos 2\theta + 6\cos 4\theta + \cos 6\theta)$ ((Allow $p = 10,\ q = 15,\ r = 6,\ s = 1$.))	**A1**
$\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \cos^6 \frac{x}{2}\, dx = \frac{1}{32}\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} 10 + 15\cos x + 6\cos 2x + \cos 3x\, dx$ (Applies part (i))	**M1**
Integrates correctly (3/4 terms correct).	**M1**
$\frac{1}{32}\left[10x + 15\sin x + 3\sin 2x + \frac{1}{3}\sin 3x\right]_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi}$ (Inserts limits and evaluates.)	**M1**
$= \frac{1}{32}\left\{\left(5\pi + 15 + 0 - \frac{1}{3}\right) - \left(-5\pi - 15 + 0 + \frac{1}{3}\right)\right\} = \frac{1}{16}\left(5\pi + \frac{44}{3}\right)$	**A1**
	Total: 10

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Q14

9231/13 • May/June 2019 • Question 3

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Mark scheme.

$\exp\left(\mathrm{i}\dfrac{2\pi k}{5}\right), k = 0, \pm1, \pm2$	**B2**
$z^5 = -\dfrac{1}{2} \pm \mathrm{i}\dfrac{\sqrt{3}}{2}$	**M1**
$z^5 = \exp\left(\mathrm{i}2\pi\left(\dfrac{1}{3} + k\right)\right)$ or $\exp\left(\mathrm{i}2\pi\left(-\dfrac{1}{3} + k\right)\right)$	**M1A1**
$\exp\left(\pm\mathrm{i}\dfrac{2\pi}{15}\right), \exp\left(\pm\mathrm{i}\dfrac{4\pi}{15}\right), \exp\left(\pm\mathrm{i}\dfrac{8\pi}{15}\right), \exp\left(\pm\mathrm{i}\dfrac{2\pi}{3}\right), \exp\left(\pm\mathrm{i}\dfrac{14\pi}{15}\right)$	**A2**
Alternative method for 3(ii)	
$z^5 + z^{-5} = -1$	**M1**
$2\cos 5\theta = -1$	**M1A1**
$\exp\left(\pm\mathrm{i}\dfrac{2\pi}{15}\right), \exp\left(\pm\mathrm{i}\dfrac{4\pi}{15}\right), \exp\left(\pm\mathrm{i}\dfrac{8\pi}{15}\right), \exp\left(\pm\mathrm{i}\dfrac{2\pi}{3}\right), \exp\left(\pm\mathrm{i}\dfrac{14\pi}{15}\right)$	**A2**
	Total: 7

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Q15

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 9

↑ Back to contents

Mark scheme.

Write $c = \cos\theta,\ s = \sin\theta.\ \cos 6\theta + i\sin 6\theta = (c + is)^6$	**M1**
Uses binomial theorem.	
$\Rightarrow \cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	**A1**
$c^6 - 15c^4s^2 + 15c^2s^4 - s^6 = c^6 - 15c^4(1-c^2) + 15c^2(1-c^2)^2 - (1-c^2)^3$	**M1**
Uses $c^2 = 1 - s^2$.	
$= c^6 - 15c^4(1-c^2) + 15c^2(1 - 2c^2 + c^4) - (1 - 3c^2 + 3c^4 - c^6)$	**A1**
$= 32c^6 - 48c^4 + 18c^2 - 1$	**M1**
Divides numerator and denominator by c^6.	
$\Rightarrow \sec 6\theta = \dfrac{1}{32c^6 - 48c^4 + 18c^2 - 1} = \dfrac{\sec^6\theta}{32 - 48\sec^2\theta + 18\sec^4\theta - \sec^6\theta}$	**A1**
AG	
$x^6 = 2(32 - 48x^2 + 18x^4 - x^6) \Rightarrow \dfrac{x^6}{32 - 48x^2 + 18x^4 - x^6} = 2$	**M1A1**
Relates with equation in part (i).	
$\sec 6\theta = 2 \Rightarrow \cos 6\theta = \dfrac{1}{2}$	**M1**
Solves $\cos 6\theta = \dfrac{1}{2}$.	
$x = \sec\dfrac{\pi}{18}$	**A1**
Gives one correct solution.	
$x = \sec q\pi,\ q = \dfrac{5}{18}, \dfrac{7}{18}, \dfrac{11}{18}, \dfrac{13}{18}, \dfrac{17}{18}$	**A1**
Gives five other solutions. Allow different values of q as long as all six solutions are found.	

Total: 11

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Q16

9231/21, 9231/22 • May/June 2020 • Question 3

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Mark scheme.

(a)	
$z^3 = -1 - \mathrm{i} = 2^{\frac{1}{2}} \mathrm{e}^{\mathrm{i}\frac{5}{4}\pi}$	**B1**
$z_1 = 2^{\frac{1}{6}} \mathrm{e}^{\mathrm{i}\frac{5}{12}\pi}$	**M1 A1**
$z_2 = 2^{\frac{1}{6}} \mathrm{e}^{\mathrm{i}\frac{13}{12}\pi}$, $z_3 = 2^{\frac{1}{6}} \mathrm{e}^{\mathrm{i}\frac{21}{12}\pi}$	**A1 A1**
(b)	
$z_1^{3k} + z_2^{3k} + z_3^{3k} = 3\left(2^{\frac{1}{2}k} \mathrm{e}^{\mathrm{i}\frac{5}{4}k\pi}\right)$	**M1**
$R = \lvert w \rvert = 3\left(2^{\frac{1}{2}k}\right)$	**A1**
$\alpha = \frac{5}{4}k\pi$	**A1**
	Total: 8

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Q17

9231/23 • May/June 2020 • Question 8

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Mark scheme.

(a)	
$z - z^{-1} = 2\mathrm{i}\sin\theta$	**B1**
$(z - z^{-1})^6 = (z^6 + z^{-6}) - 6(z^4 + z^{-4}) + 15(z^2 + z^{-2}) - 20$	**M1 A1**
$(2\mathrm{i}\sin\theta)^6 = 2\cos 6\theta - 6(2\cos 4\theta) + 15(2\cos 2\theta) - 20$	**M1 A1**
$\sin^6\theta = \frac{1}{32}(-\cos 6\theta + 6\cos 4\theta - 15\cos 2\theta + 10)$	**A1**
(b)	
$\int_0^{\frac{1}{3}\pi} \cos^6(\frac{1}{4}x) + \sin^6(\frac{1}{4}x)\,\mathrm{d}x = \frac{1}{8}\int_0^{\frac{1}{3}\pi} 3\cos x + 5\,\mathrm{d}x$	**M1 A1**
$\frac{1}{8}\left[3\sin x + 5x\right]_0^{\frac{1}{3}\pi} = \frac{1}{8}\left(\frac{5}{3}\pi + \frac{3}{2}\sqrt{3}\right)$	**M1 A1**
(c)	
$c = \cos\theta \Rightarrow 1 - c^2 = \sin^2\theta$	**B1**
$16c^6 + 16(1 - c^2)^3 - 13 = 0 \Rightarrow 6\cos 4\theta - 3 = 0$	**M1 A1**
$4\theta = \frac{1}{3}\pi, \frac{5}{3}\pi, \frac{7}{3}\pi, \frac{11}{3}\pi$	**M1**
$c = \cos\left(\frac{1}{12}\pi\right), \cos\left(\frac{5}{12}\pi\right), \cos\left(\frac{7}{12}\pi\right), \cos\left(\frac{11}{12}\pi\right)$	**A1**
	Total: 15

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Q18

9231/21, 9231/23 • October/November 2020 • Question 6(a)

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Mark scheme.

Use of $z - z^{-1} = 2\mathrm{i}\sin\theta$.	**B1**
Expands and groups: $(z - z^{-1})^4 = (z^4 + z^{-4}) - 4(z^2 + z^{-2}) + 6$.	**M1 A1**
Substitutes $z^n + z^{-n} = 2\cos n\theta$: $(2\mathrm{i}\sin\theta)^4 = (2\cos 4\theta) - 4(2\cos 2\theta) + 6$.	**M1**
AG: $\sin^4\theta = \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3)$.	**A1**
	Total: 5

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Q19

9231/22 • October/November 2020 • Question 3

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Mark scheme.

Uses the 6th roots of unity: $w+1=\cos\frac{2n}{6}\pi+\mathrm{i}\sin\frac{2n}{6}\pi$, $n=0,\ldots,5$.	**M1 A1**
$w=0$, $-\frac{1}{2}\pm\frac{1}{2}\mathrm{i}\sqrt{3}$, $-\frac{3}{2}\pm\frac{1}{2}\mathrm{i}\sqrt{3}$, -2. (A1 for 3 correct roots; A1 for all six roots in exact form.)	**A1 A1**
	Total: 4

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Q20

9231/22 • October/November 2020 • Question 7

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Mark scheme.

(a) Uses sum of geometric series: $z^2 + z^2(z^2) + \cdots + z^2(z^2)^{n-1} = \dfrac{z^2(z^{2n}-1)}{z^2-1}$.	**M1**
Divides numerator and denominator by z, must see at least $\dfrac{z^{2n+2}-z^2}{z^2-1}$, AG: $\dfrac{z^2(z^{2n}-1)}{z^2-1} \times \dfrac{z^{-1}}{z^{-1}} = \dfrac{z^{2n+1}-z}{z-z^{-1}}$.	**A1**
(b) Simplifies denominator: $z - z^{-1} = 2\mathrm{i}\sin\theta$.	**B1**
Applies de Moivre's theorem to numerator: $\dfrac{z^{2n+1}-z}{z-z^{-1}} = \dfrac{\cos(2n+1)\theta + \mathrm{i}\sin(2n+1)\theta - \cos\theta - \mathrm{i}\sin\theta}{2\mathrm{i}\sin\theta}$.	**M1 A1**
Equates real parts: $\sum_{r=1}^{n} \cos(2r\theta) = \dfrac{\sin(2n+1)\theta - \sin\theta}{2\sin\theta} = \dfrac{\sin(2n+1)\theta}{2\sin\theta} - \dfrac{1}{2}$.	**M1**
AG: $1 + 2\sum_{r=1}^{n} \cos(2r\theta) = \dfrac{\sin(2n+1)\theta}{\sin\theta}$.	**A1**
	Total: 7

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Q21

9231/21, 9231/22 • May/June 2021 • Question 5

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Mark scheme.

(a)

$\dfrac{z - z^{n+1}}{1 - z}$ or $\dfrac{z^{n+1} - z}{z - 1}$.	**B1**

(b)

Must see $z^n = 1$: $z^n = 1$ and $z \neq 1$ leading to $z + z^2 + z^3 + \cdots + z^n = 0$ leading to $1 + z + z^2 + \cdots + z^{n-1} = 0$.	**M1 A1**

(c)

Applies sum to infinity and substitutes for z: $\sum_{m=1}^{\infty} z^m = \dfrac{z}{1-z} = \dfrac{\cos\theta + \mathrm{i}\sin\theta}{3 - \cos\theta - \mathrm{i}\sin\theta}$.	**M1 A1**
Rationalises denominator: $\dfrac{(\cos\theta + \mathrm{i}\sin\theta)(3 - \cos\theta + \mathrm{i}\sin\theta)}{(3 - \cos\theta)^2 + \sin^2\theta}$.	**M1 A1**
Applies $\sin^2\theta + \cos^2\theta = 1$ or $\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta} = 2\cos\theta$.	**M1**
Takes the real part (AG): $\mathrm{Re}\left(\sum_{m=1}^{\infty} z^m\right) = \dfrac{3\cos\theta - 1}{10 - 6\cos\theta}$.	**M1 A1**

Total: 10

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Q22

9231/23 • May/June 2021 • Question 1

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Mark scheme.

(a)	
$a = 1$, $b = \mathrm{i}$.	**B1**
(b)	
Finds one fifth root of unity: $z = \mathrm{e}^{\mathrm{i}\frac{2}{5}\pi}$.	**M1**
Gives all fifth roots of unity: $z = \mathrm{e}^{\mathrm{i}\frac{2k}{5}\pi}$, $k = 0, 1, 2, 3, 4$ (accept 1 not in exponential form; A0 if $r = 1$ not seen or implied).	**A1**
Argument of i is $\frac{1}{2}\pi$.	**B1**
Finds one root of $z^3 = \mathrm{i}$: $z = \mathrm{e}^{\mathrm{i}\frac{1}{6}\pi}$ (A0 if first root not given in exponential form).	**M1 A1**
Finds other two roots of $z^3 = \mathrm{i}$: $z = \mathrm{e}^{\mathrm{i}\frac{5}{6}\pi}$, $\mathrm{e}^{\mathrm{i}\frac{3}{2}\pi}$ (A0 if $r = 1$ not seen or implied).	**A1**
	Total: 7

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Q23

9231/23 • May/June 2021 • Question 4

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Mark scheme.

Use of $z - z^{-1} = 2\mathrm{i}\sin\theta$ and $z + z^{-1} = 2\cos\theta$.	**B1**
Expands and groups (A0 if grouping not shown): $\left(z - z^{-1}\right)^5 = \left(z^5 - z^{-5}\right) - 5\left(z^3 - z^{-3}\right) + 10\left(z - z^{-1}\right)$.	**M1 A1**
Expands and groups (A0 if grouping not shown): $\left(z + z^{-1}\right)^5 = \left(z^5 + z^{-5}\right) + 5\left(z^3 + z^{-3}\right) + 10\left(z + z^{-1}\right)$.	**M1 A1**
Substitutes $z^n + z^{-n} = 2\cos n\theta$ and $z^n - z^{-n} = 2\mathrm{i}\sin n\theta$: $\dfrac{2^5\mathrm{i}\sin^5\theta}{2^5\cos^5\theta} = \dfrac{2\mathrm{i}\sin 5\theta - 5(2\mathrm{i}\sin 3\theta) + 10(2\mathrm{i}\sin\theta)}{2\cos 5\theta + 5(2\cos 3\theta) + 10(2\cos\theta)}$.	**M1**
Justifies cancellation of constants: $\tan^5\theta = \dfrac{\sin 5\theta - 5\sin 3\theta + 10\sin\theta}{\cos 5\theta + 5\cos 3\theta + 10\cos\theta}$.	**A1**
	Total: 7

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Q24

9231/21, 9231/23 • October/November 2021 • Question 6

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Mark scheme.

Uses binomial theorem: $(\cos 5\theta + \mathrm{i}\sin 5\theta) = (c + \mathrm{i}s)^5$.	**M1**
$\sin 5\theta = s^5 - 10c^2s^3 + 5c^4s$.	**A1**
Uses $c^2 = 1 - s^2$: $s^5 - 10(1-s^2)s^3 + 5(1-s^2)^2s$.	**M1**
$16s^5 - 20s^3 + 5s$.	**A1**
Divides simplified numerator and denominator by s^5: $\operatorname{cosec} 5\theta = \frac{1}{16s^5 - 20s^3 + 5s} \times \frac{\operatorname{cosec}^5\theta}{\operatorname{cosec}^5\theta}$.	**M1**
$\frac{\operatorname{cosec}^5\theta}{5\operatorname{cosec}^4\theta - 20\operatorname{cosec}^2\theta + 16}$ (AG).	**A1**
Relates with equation in part (a): $x^5 = 2(5x^4 - 20x^2 + 16)$ leading to $\frac{x^5}{5x^4 - 20x^2 + 16} = 2$.	**M1**
$\operatorname{cosec} 5\theta = 2$ leading to $\sin 5\theta = \frac{1}{2}$.	**M1**
$x = \operatorname{cosec}\left(\frac{1}{30}\pi\right)$.	**A1**
$\operatorname{cosec}\left(\frac{5}{30}\pi\right), \operatorname{cosec}\left(-\frac{7}{30}\pi\right), \operatorname{cosec}\left(-\frac{11}{30}\pi\right), \operatorname{cosec}\left(\frac{13}{30}\pi\right)$ (allow different q as long as all five solutions found: $9.56, 2, -1.49, -1.09, 1.02$).	**A1**
	Total: 10

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Q25

9231/22 • October/November 2021 • Question 4

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Mark scheme.

$e^{i\frac{2}{5}k\pi}$, $k = 0, 1, 2, 3, 4$ (OE; B1 for 1 correct fifth root of unity, B2 for exactly 5 distinct correct roots).	**B2**
Uses binomial expansion: $(c + is)^4 = c^4 + 4c^3(is) + 6c^2(is)^2 + 4c(is)^3 + (is)^4$.	**M1 A1**
Applies $s^2 = 1 - c^2$: $\cos 4\theta = c^4 - 6c^2s^2 + s^4 = c^4 - 6c^2(1 - c^2) + (1 - c^2)^2$.	**M1**
$8\cos^4\theta - 8\cos^2\theta + 1$ (AG).	**A1**
$8x^9 - 8x^7 + x^5 - 8x^4 + 8x^2 - 1 = (x^5 - 1)(8x^4 - 8x^2 + 1)$.	**B1**
Solves $\cos 4\theta = 0$ leading to $4\theta = \frac{1}{2}(2k + 1)\pi$.	**M1 A1**
$\cos 0, \cos\frac{1}{8}\pi, \cos\frac{3}{8}\pi, \cos\frac{5}{8}\pi, \cos\frac{7}{8}\pi$ (gives exactly 5 distinct real roots; accept $1, \pm\cos\frac{1}{8}\pi, \pm\cos\frac{3}{8}\pi$).	**A1**
	Total: 10

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Q26

9231/21, 9231/22 • May/June 2022 • Question 7

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Mark scheme.

Uses binomial theorem: $\sin 7\theta = \mathrm{Im}(c+is)^7$	**(a) M1**
$\sin 7\theta = -s^7 + 21c^2s^5 - 35c^4s^3 + 7c^6 s$	**A1**
Uses $c^2 = 1 - s^2$: $\sin 7\theta = -s^7 + 21(1-s^2)s^5 - 35(1-s^2)^2 s^3 + 7(1-s^2)^3 s$	**M1**
$\sin 7\theta = -64s^7 + 112s^5 - 56s^3 + 7s$	**A1**
Divides numerator and denominator by s^7: $\operatorname{cosec} 7\theta = \dfrac{1}{-64s^7 + 112s^5 - 56s^3 + 7s}$	**M1**
AG, CWO: $\operatorname{cosec} 7\theta = \dfrac{\operatorname{cosec}^7\theta}{7\operatorname{cosec}^6\theta - 56\operatorname{cosec}^4\theta + 112\operatorname{cosec}^2\theta - 64}$	**A1**
Relates with equation in part (a): $x^7 = 2(7x^6 - 56x^4 + 112x^2 - 64)$ leading to $\dfrac{x^7}{7x^6 - 56x^4 + 112x^2 - 64} = 2$	**(b) M1** **A1**
Solves $\sin 7\theta = \frac{1}{2}$: $\operatorname{cosec} 7\theta = 2$ leading to $\sin 7\theta = \frac{1}{2}$	**M1**
Gives one correct solution: $x = \operatorname{cosec}\left(\frac{1}{42}\pi\right)$	**A1**
Gives six other solutions (allow different values of q as long as all seven solutions are found): $x = \operatorname{cosec}(q\pi)$, $q = -\frac{19}{42}, -\frac{11}{42}, -\frac{7}{42}, \frac{5}{42}, \frac{13}{42}, \frac{17}{42}$	**A1**
	Total: 11

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Q27

9231/23 • May/June 2022 • Question 1

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Mark scheme.

Converts $7\sqrt{3} - 7\mathrm{i}$ to exponential form: $z^3 = 7\sqrt{3} - 7\mathrm{i} = 14e^{-\mathrm{i}\frac{1}{6}\pi}$	**B1**
Finds one root: $z_1 = 14^{\frac{1}{3}}e^{-\mathrm{i}\frac{1}{18}\pi}$	**M1 A1**
Finds other two roots (FT on modulus; others given score A1 A0): $z_2 = 14^{\frac{1}{3}}e^{\mathrm{i}\frac{11}{18}\pi}$, $z_3 = 14^{\frac{1}{3}}e^{-\mathrm{i}\frac{13}{18}\pi}$	**A1FT** **A1FT**
	Total: 5

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Q28

9231/21, 9231/23 • October/November 2022 • Question 7

↑ Back to contents

Mark scheme.

(a)	
$\dfrac{w^n - 1}{w - 1}$.	**B1**
(b)	
$(1 + \mathrm{i}\tan\theta)^k = \sec^k\theta(\cos\theta + \mathrm{i}\sin\theta)^k = \sec^k\theta(\cos k\theta + \mathrm{i}\sin k\theta)$. Applies de Moivre's theorem, AG.	**M1 A1**
(c)	
$\displaystyle\sum_{k=0}^{n-1}(1 + \mathrm{i}\tan\theta)^k = \frac{(1 + \mathrm{i}\tan\theta)^n - 1}{\mathrm{i}\tan\theta}$. Applies part (a).	**M1 A1**
$= -\cot\theta\sec^n\theta(\mathrm{i}\cos n\theta - \sin n\theta) + \mathrm{i}\cot\theta$.	**A1**
$\displaystyle\sum_{k=0}^{n-1}\sec^k\theta\sin k\theta = \cot\theta(1 - \sec^n\theta\cos n\theta)$. Takes imaginary part, AG.	**M1 A1**
(d)	
Sets $\theta = \frac{1}{3}\pi$: $\displaystyle\sum_{k=0}^{6m-1} 2^k\sin(\tfrac{1}{3}k\pi) = \tfrac{1}{3}\sqrt{3}\left(1 - 2^{6m}\cos(2m\pi)\right) = \tfrac{1}{3}\sqrt{3}\left(1 - 2^{6m}\right)$.	**M1 A1**
	Total: 10

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Q29

9231/22 • October/November 2022 • Question 5

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Mark scheme.

(a)	
$\pm 1, \pm \mathrm{i}$ (four values, exponential or trig form accepted).	**B1**
(b)	
$\cos 4\theta = \mathrm{Re}(\cos\theta + \mathrm{i}\sin\theta)^4 = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$. Expands and takes real part.	**M1 A1**
Applies $\sin^2\theta = 1 - \cos^2\theta$: $\cos^4\theta - 6\cos^2\theta(1-\cos^2\theta) + (1 - 2\cos^2\theta + \cos^4\theta)$.	**M1**
$8\cos^4\theta - 8\cos^2\theta + 1$. AG.	**A1**
(c)	
$(8x^4 - 8x^2 + 1)^4 = \frac{9}{16}$, $x = \cos\theta$. Applies identity from (b).	**M1**
$\cos 4\theta = \pm\frac{1}{2}\sqrt{3}$ (need $\pm$ and exact value).	**A1**
Solves $\cos 4\theta = \frac{1}{2}\sqrt{3}$ or $\cos 4\theta = -\frac{1}{2}\sqrt{3}$: $4\theta = \pm\frac{1}{6}\pi + 2k\pi$, $4\theta = \pm\frac{5}{6}\pi + 2k\pi$.	**M1**
$\cos(\frac{1}{24}\pi)$ (one correct solution).	**A1**
Other solutions: $\cos(\frac{11}{24}\pi), \cos(\frac{13}{24}\pi), \cos(\frac{23}{24}\pi), \cos(\frac{5}{24}\pi), \cos(\frac{7}{24}\pi), \cos(\frac{17}{24}\pi), \cos(\frac{19}{24}\pi)$.	**A1**
	Total: 10

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Q30

9231/21, 9231/22 • May/June 2023 • Question 3

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Mark scheme.

Use of $z + z^{-1} = 2\cos\theta$.	**B1**
Expands and groups: $(z + z^{-1})^4 = (z^4 + z^{-4}) + 4(z^2 + z^{-2}) + 6$ (M1 A0 for no clear grouping; correct substitution of $z^n = \cos n\theta + \mathrm{i}\sin n\theta$ for each term scores M1 A1).	**M1 A1**
Substitutes $z^n + z^{-n} = 2\cos n\theta$: $(2\cos\theta)^4 = 2\cos 4\theta + 4(2\cos 2\theta) + 6$ (if $z^n = \cos n\theta + \mathrm{i}\sin n\theta$ used, must cancel sin terms).	**M1**
$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$ (AG). SC B1 expands $(\cos\theta + \mathrm{i}\sin\theta)^4$ and uses trigonometric identities. **(b)**	**A1**
Applies substitution (M1), gets to $\int \cos^4\theta\,\mathrm{d}\theta$, changes limits, integration correct (A1): $\int_0^{1/2}(1-x^2)^{3/2}\,\mathrm{d}x = \int_0^{\pi/6}\cos^4\theta\,\mathrm{d}\theta = \frac{1}{8}\left[\frac{1}{4}\sin 4\theta + 2\sin 2\theta + 3\theta\right]_0^{\pi/6}$.	**M1 A1**
$\frac{1}{16}\left(\frac{9}{4}\sqrt{3} + \pi\right)$.	**A1**
	Total: 8

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Q31

9231/23 • May/June 2023 • Question 3

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Mark scheme.

Use of $z - z^{-1} = 2\mathrm{i}\sin\theta$ and $z + z^{-1} = 2\cos\theta$.	**B1**
Expands and groups: $(z + z^{-1})^4 = (z^4 + z^{-4}) + 4(z^2 + z^{-2}) + 6$.	**M1 A1**
Expands and groups (only withhold one A1 mark for no clear grouping of either): $(z - z^{-1})^4 = (z^4 + z^{-4}) - 4(z^2 + z^{-2}) + 6$.	**M1 A1**
Substitutes $z^n + z^{-n} = 2\cos n\theta$ once in LHS: $\dfrac{2^4\cos^4\theta}{2^4\sin^4\theta} = \dfrac{2\cos 4\theta + 4(2\cos 2\theta) + 6}{2\cos 4\theta - 4(2\cos 2\theta) + 6}$.	**M1**
$\cot^4\theta = \dfrac{\cos 4\theta + 4\cos 2\theta + 3}{\cos 4\theta - 4\cos 2\theta + 3}$.	**A1**
	Total: 7

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Q32

9231/21, 9231/23 • October/November 2023 • Question 2

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Mark scheme.

Finds modulus and argument of $4+4\sqrt{3}\,\mathrm{i}$: $(z+5\mathrm{i})^3 = 4+4\mathrm{i}\sqrt{3} = 8\mathrm{e}^{\frac{1}{3}\mathrm{i}\pi}$.	**B1**
Finds one root: $z_1 = 2\left(\cos\frac{1}{9}\pi + \mathrm{i}\sin\frac{1}{9}\pi\right) - 5\mathrm{i} = 2\cos\frac{1}{9}\pi + \mathrm{i}\left(2\sin\frac{1}{9}\pi - 5\right)$.	**M1 A1**
Finds other two roots (FT on their modulus): $z_2 = 2\cos\frac{7}{9}\pi + \mathrm{i}\left(2\sin\frac{7}{9}\pi - 5\right)$.	**A1 FT**
$z_3 = 2\cos\frac{13}{9}\pi + \mathrm{i}\left(2\sin\frac{13}{9}\pi - 5\right)$.	**A1 FT**
	Total: 5

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Q33

9231/22 • October/November 2023 • Question 3

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Mark scheme.

(a)	
Expands and takes real part. Accept RHS to LHS using $2\cos\theta = z + \frac{1}{z}$: $\cos 5\theta = \mathrm{Re}(\cos\theta + \mathrm{i}\sin\theta)^5 = \cos^5\theta - 10\sin^2\theta\cos^3\theta + 5\sin^4\theta\cos\theta$.	**M1 A1**
Applies $\sin^2\theta = 1 - \cos^2\theta$: $= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta)$.	**M1**
$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (AG).	**A1**
(b)	
Applies identity given in (a): $x = \cos\theta$, $\cos 5\theta = \frac{1}{2}\sqrt{2}$.	**M1**
Solves $\cos 5\theta = \frac{1}{2}\sqrt{2}$: $5\theta = \pm\frac{1}{4}\pi + 2k\pi$.	**M1**
Gives one correct solution (accept $q = \frac{1}{20}$): $\cos\left(\frac{1}{20}\pi\right)$.	**A1**
Gives other solutions (OE, A0 for repeated roots): $\cos\left(\frac{9}{20}\pi\right), \cos\left(\frac{17}{20}\pi\right), \cos\left(\frac{25}{20}\pi\right), \cos\left(\frac{33}{20}\pi\right)$.	**A1**
	Total: 8

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Q34

9231/21, 9231/22 • May/June 2024 • Question 1

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Mark scheme.

Finds modulus and argument of $-108\sqrt{3}+108\mathrm{i}$: $z^3 = 216\cos\frac{5\pi}{6} + \mathrm{i}\,216\sin\frac{5\pi}{6}$.	**B1**
Finds one root. FT on their modulus. For M1 must have divided their argument of $-108\sqrt{3}+108\mathrm{i}$ by 3.	**M1**
$z_1 = 6\cos\frac{5\pi}{18} + \mathrm{i}\,6\sin\frac{5\pi}{18}$.	**A1FT**
Finds other two roots.	**M1**
$z_2 = 6\cos\frac{17\pi}{18} + \mathrm{i}\,6\sin\frac{17\pi}{18}$, $\quad z_3 = 6\cos\frac{29\pi}{18} + \mathrm{i}\,6\sin\frac{29\pi}{18}$.	**A1**
	Total: 5

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Q35

9231/23 • May/June 2024 • Question 6

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Mark scheme.

(a)

Uses sum of geometric series.	**M1**
Divides numerator and denominator by z^2. AG.	**A1**

(b)

Simplifies denominator: $z^2 - z^{-2} = 2\mathrm{i}\sin 2\theta$.	**B1**
Applies de Moivre's theorem to numerator.	**M1**
$\dfrac{z^{4n+2} - z^2}{z^2 - z^{-2}} = \dfrac{\cos(4n+2)\theta + \mathrm{i}\sin(4n+2)\theta - \cos 2\theta - \mathrm{i}\sin 2\theta}{2\mathrm{i}\sin 2\theta}$.	**A1**
Equates imaginary parts.	**M1**
AG: $\sum_{r=1}^{n} \sin(4r\theta) = \dfrac{\cos 2\theta - \cos(4n+2)\theta}{2\sin 2\theta}$. Must see evidence of eliminating i from the denominator.	**A1**

Total: 7

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Q36

9231/21, 9231/23 • October/November 2024 • Question 8

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Mark scheme.

(a)	
$z + z^{-1} = 2\cos\theta$.	**B1**
Expands and takes real part: $\cos 6\theta + \ldots$ type expansion: $\left(z+\frac{1}{z}\right)^7 = (z^7 + z^{-7}) + 7(z^5+z^{-5}) + 21(z^3+z^{-3}) + 35(z+z^{-1})$.	**M1A1**
Substitutes $z^n + z^{-n} = 2\cos n\theta$: $2^7\cos^7\theta = 2\cos 7\theta + 7(2\cos 5\theta) + 21(2\cos 3\theta) + 35(2\cos\theta)$.	**M1**
$\cos^7\theta = \frac{1}{64}\cos 7\theta + \frac{7}{64}\cos 5\theta + \frac{21}{64}\cos 3\theta + \frac{35}{64}\cos\theta$.	**A1**
(b)	
Applies integration by parts: $I_n = \left[\cos^{n-1}\theta\sin\theta\right]_0^{\pi/4} + (n-1)\int_0^{\pi/4}\cos^{n-2}\theta\sin^2\theta\,d\theta$.	**M1A1**
Applies $\sin^2\theta = 1 - \cos^2\theta$.	**M1**
$I_n = 2^{-n/2} + (n-1)I_{n-2} - (n-1)I_n \Rightarrow nI_n = 2^{-n/2} + (n-1)I_{n-2}$ (AG).	**A1**
(c)	
Applies reduction formula from (b): $9I_9 = 2^{-9/2} + 8I_7$.	**M1A1**
Applies identity from (a): $I_7 = \int_0^{\pi/4}\frac{1}{64}\cos 7\theta + \frac{7}{64}\cos 5\theta + \frac{21}{64}\cos 3\theta + \frac{35}{64}\cos\theta\,d\theta$.	**M1**
$I_7 = \frac{1}{64}\left[\frac{1}{7}\sin 7\theta + \frac{7}{5}\sin 5\theta + 7\sin 3\theta + 35\sin\theta\right]_0^{\pi/4}$ (with appropriate coefficients).	**A1**
$I_9 = \dfrac{2867}{5040}\left(2^{-1/2}\right) = \dfrac{2867}{10080}\sqrt{2}$.	**A1**
	Total: 14

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Q37

9231/22 • October/November 2024 • Question 4

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Mark scheme.

(a)	
Expands and takes real part: $\cos 6\theta = \operatorname{Re}(\cos\theta + i\sin\theta)^6 = \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta$.	**M1A1**
Takes imaginary part: $\sin 6\theta = \operatorname{Im}(\cos\theta + i\sin\theta)^6 = 6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta$.	**M1A1**
Divides numerator and denominator by $\sin^6\theta$.	**M1**
$\cot 6\theta = \dfrac{\cot^6\theta - 15\cot^4\theta + 15\cot^2\theta - 1}{6\cot^5\theta - 20\cot^3\theta + 6\cot\theta}$ (AG).	**A1**
(b)	
Applies identity from (a): $x = \cot\theta$, $\cot 6\theta = 1$.	**B1**
Solves $\cot 6\theta = 1 \Rightarrow 6\theta = \frac{1}{4}\pi + k\pi$.	**M1**
Gives one correct solution: $\cot\left(\frac{1}{24}\pi\right)$.	**A1**
Gives other five solutions: $\cot\left(\frac{5}{24}\pi\right), \cot\left(\frac{3}{8}\pi\right), \cot\left(\frac{13}{24}\pi\right), \cot\left(\frac{17}{24}\pi\right), \cot\left(\frac{7}{8}\pi\right)$ (withhold for repeated roots).	**A1**
	Total: 10

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Q38

9231/21, 9231/22 • May/June 2025 • Question 1

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Mark scheme.

$z^3 = 27 - 27\mathrm{i} = 27\sqrt{2}\,\mathrm{e}^{-\mathrm{i}\frac{1}{4}\pi}$. Modulus and argument of z^3 (does not need to be in exponential form).	**B1**
Finds one root. Cube root must be taken for M1.	**M1**
$z_1 = 3 \times 2^{\frac{1}{6}}\mathrm{e}^{-\mathrm{i}\frac{1}{12}\pi}$.	**A1**
$z_2 = 3 \times 2^{\frac{1}{6}}\mathrm{e}^{\mathrm{i}\frac{7}{12}\pi}$. FT on modulus. Withhold one answer mark for omission of i.	**A1FT**
$z_3 = 3 \times 2^{\frac{1}{6}}\mathrm{e}^{-\mathrm{i}\frac{9}{12}\pi}$. FT on modulus. Withhold one answer mark for omission of i.	**A1FT**
	Total: 5

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Q39

9231/21, 9231/22 • May/June 2025 • Question 3

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Mark scheme.

Use of $z - z^{-1} = 2\mathrm{i}\sin\theta$.	**B1**
Expands $\left(z - z^{-1}\right)^5 = \left(z^5 - z^{-5}\right) - 5\left(z^3 - z^{-3}\right) + 10\left(z - z^{-1}\right)$ and groups. No clear grouping is M1 A0.	**M1**
Correct expansion as above.	**A1**
Substitutes $z^n - z^{-n} = 2\mathrm{i}\sin n\theta$ on RHS: $2^5\mathrm{i}\sin^5\theta = 2\mathrm{i}\sin 5\theta - 5(2\mathrm{i}\sin 3\theta) + 10(2\mathrm{i}\sin\theta)$.	**M1**
FT on signs of coefficients only. Substituting $z^n - z^{-n} = \mathrm{i}\sin n\theta$ on RHS is M1 A0. Omission of i is M0.	**A1FT**
$\operatorname{cosec}^5\theta = \dfrac{16}{\sin 5\theta - 5\sin 3\theta + 10\sin\theta}$. Cancellation of i justified.	**A1**
	Total: 6

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Q40

9231/23 • May/June 2025 • Question 5

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Mark scheme.

(a)	
Uses binomial theorem: $(\cos 5\theta + \mathrm{i}\sin 5\theta) = (c + \mathrm{i}s)^5$.	**M1**
$\cos 5\theta = c^5 - 10s^2c^3 + 5s^4c$.	**A1**
Uses $s^2 = 1 - c^2$: $= c^5 - 10(1-c^2)c^3 + 5(1-c^2)^2c$.	**M1**
$= 16c^5 - 20c^3 + 5c$.	**A1**
Divides simplified numerator and denominator by c^5: $\sec 5\theta = \dfrac{1}{16c^5 - 20c^3 + 5c} \times \dfrac{\sec^5\theta}{\sec^5\theta}$.	**M1**
$= \dfrac{\sec^5\theta}{5\sec^4\theta - 20\sec^2\theta + 16}$, AG.	**A1**
(b)	
Relates with equation in part (a): $x^5 = \frac{2}{\sqrt{3}}(5x^4 - 20x^2 + 16) \Rightarrow \dfrac{x^5}{5x^4 - 20x^2 + 16} = \frac{2}{\sqrt{3}}$. Can be implied by M1 mark.	**B1**
Solves $\cos 5\theta = \frac{\sqrt{3}}{2}$: $\sec 5\theta = \frac{2}{\sqrt{3}} \Rightarrow \cos 5\theta = \frac{\sqrt{3}}{2}$.	**M1**
Gives one correct solution: $x = \sec\left(\frac{1}{30}\pi\right)$.	**A1**
Gives four other solutions: $\sec\left(\frac{11}{30}\pi\right), \sec\left(\frac{13}{30}\pi\right), \sec\left(\frac{23}{30}\pi\right), \sec\left(\frac{25}{30}\pi\right)$. Allow different values of q as long as all five solutions are found. Numerically: $-1.35, -1.15, 1.01, 2.46, 4.81$.	**A1**
	Total: 10

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Q41

9231/24 • May/June 2025 • Question 5

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Mark scheme.

(a)

Uses binomial expansion and takes imaginary part: $\sin 7\theta = \operatorname{Im}\left((c+\mathrm{i}s)^7\right)$.	**M1**
$= -s^7 + 21c^2s^5 - 35c^4s^3 + 7c^6s$.	**A1**
Applies $c^2 = 1 - s^2$: $= -s^7 + 21(1-s^2)s^5 - 35(1-s^2)^2s^3 + 7(1-s^2)^3s$.	**M1**
$(1-s^2)^3 = -s^6 + 3s^4 - 3s^2 + 1$. No sight of $(1-s^2)^3$ fully expanded is B0.	**B1**
$\sin 7\theta = -64s^7 + 112s^5 - 56s^3 + 7s$, AG.	**A1**

(b)

Solves $\sin 7\theta = 0$ and excludes $\sin\theta = 0$, where $x = \sin\theta$. For M1 the roots from $\sin\theta = 0$ must be excluded at some stage of their working (this could be implied by their final answers).	**M1**
One correct root: $x = \sin\left(\frac{1}{7}\pi\right)$.	**A1**
Exactly five other correct roots: $\sin\left(-\frac{1}{7}\pi\right)$, $\sin\left(\pm\frac{2}{7}\pi\right)$, $\sin\left(\pm\frac{3}{7}\pi\right)$. Allow different values of q giving these roots.	**A1**

Total: 8

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Q42

9231/21, 9231/23 • October/November 2025 • Question 8

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Mark scheme.

$\dfrac{z^{n+1}-z}{z-1}$.	**(a) B1**
$\dfrac{z^{n+1}-z}{z-1} = \dfrac{\left(\frac{1}{2}\right)^{n+1}\cos(n+1)\theta - \frac{1}{2}\cos\theta + \mathrm{i}\left(\left(\frac{1}{2}\right)^{n+1}\sin(n+1)\theta - \frac{1}{2}\sin\theta\right)}{\frac{1}{2}\cos\theta - 1 + \mathrm{i}\frac{1}{2}\sin\theta}$.	**(b) B1**
Multiplies numerator and denominator by complex conjugate of $z-1$.	**M1***
Takes imaginary part.	**M1 (dep)**
Correct imaginary part expression.	**A1**
Uses $\sin(A-B) = \sin A\cos B - \cos A\sin B$ in numerator.	**M1**
AG, result shown.	**A1**
Differentiates the result from (b) with respect to θ.	**(c) M1**
Correct differentiation applying quotient rule.	**M1**
Correct final expression (unsimplified acceptable).	**A1**
Uses $n\left(\frac{1}{2}\right)^n \to 0$ as $n \to \infty$.	**(d) M1**
$\sum_{m=1}^{\infty}\left(\frac{1}{2}\right)^m m\cos m\theta = \dfrac{\frac{5}{8}\cos\theta - \frac{1}{2}}{\left(\frac{5}{4}-\cos\theta\right)^2}$.	**A1**
	Total: 12

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Q43

9231/22 • October/November 2025 • Question 2

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Mark scheme.

Finds the modulus and argument of z^4: $z^4 = 8 - 8\mathrm{i}\sqrt{3} = 16\mathrm{e}^{-\mathrm{i}\frac{1}{3}\pi}$. Does not have to be in exponential form.	**B1**
Finds one root, using their r and $\theta/4$.	**M1**
$z_1 = 2\mathrm{e}^{-\mathrm{i}\frac{1}{12}\pi}$.	**A1**
$z_2 = 2\mathrm{e}^{\mathrm{i}\frac{5}{12}\pi}$. Follow through on modulus.	**A1 (FT)**
$z_3 = 2\mathrm{e}^{-\mathrm{i}\frac{7}{12}\pi}$, $z_4 = 2\mathrm{e}^{\mathrm{i}\frac{11}{12}\pi}$. Follow through on modulus; withhold 1 mark for omission of i.	**A1 (FT)**
	Total: 5

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Q44

9231/22 • October/November 2025 • Question 4(a)

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Mark scheme.

$z + z^{-1} = 2\cos\theta$ (SOI).	**B1**
Expands and groups: $(z+z^{-1})^6 = (z^6+z^{-6}) + 6(z^4+z^{-4}) + 15(z^2+z^{-2}) + 20$.	**M1 A1**
Substitutes $z^n + z^{-n} = 2\cos n\theta$: $2^6\cos^6\theta = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$.	**M1**
$\cos^6\theta = \frac{1}{32}\cos 6\theta + \frac{3}{16}\cos 4\theta + \frac{15}{32}\cos 2\theta + \frac{5}{16}$. OE, fractions need not be simplified.	**A1**
	Total: 5

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Q45

9231/24 • October/November 2025 • Question 3

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Mark scheme.

Expands and clearly takes imaginary part, enough terms for M1: $\sin 5\theta = \operatorname{Im}(\cos\theta + \mathrm{i}\sin\theta)^5 = \sin^5\theta - 10\sin^3\theta\cos^2\theta + 5\sin\theta\cos^4\theta$.	**(a) M1** **A1**
Applies $\cos^2\theta = 1 - \sin^2\theta$: $= \sin^5\theta - 10\sin^3\theta(1-\sin^2\theta) + 5\sin\theta(1-\sin^2\theta)^2$.	**M1**
$= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$. AG, intermediate expansion line needed.	**A1**
Applies identity from (a) with $x = \sin\theta$: $\sin 5\theta = \frac{1}{2}\sqrt{3}$.	**(b) M1**
Solves $\sin 5\theta = \frac{1}{2}\sqrt{3}$: $5\theta = \frac{1}{3}\pi + 2k\pi$ or $5\theta = \frac{2}{3}\pi + 2k\pi$.	**M1**
Gives one correct solution $\sin(\frac{1}{15}\pi)$.	**A1**
Gives other solutions: $\sin(\frac{2}{15}\pi), \sin(-\frac{5}{15}\pi), \sin(-\frac{4}{15}\pi), \sin(\frac{7}{15}\pi)$ (OE). A0 for repeated roots.	**A1**
	Total: 8

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Written by Ali Hashir.

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