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Further Math with Ali Hashir

Vectors

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics Paper 1 past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/11, 9231/12 • May/June 2015 • Question 11 (OR)

Answer only **one** of the following two alternatives.

OR

The lines l_1 and l_2 have equations $\mathbf{r} = 8\mathbf{i}+2\mathbf{j}+3\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j})$ and $\mathbf{r} = 5\mathbf{i}+3\mathbf{j}-14\mathbf{k}+\mu(2\mathbf{j}-3\mathbf{k})$ respectively. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. Find the position vector of the point P and the position vector of the point Q. [8]

The points with position vectors $8\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $5\mathbf{i} + 3\mathbf{j} - 14\mathbf{k}$ are denoted by A and B respectively. Find

(i) $\overrightarrow{AP} \times \overrightarrow{AQ}$ and hence the area of the triangle APQ,

(ii) the volume of the tetrahedron $APQB$. (You are given that the volume of a tetrahedron is $\frac{1}{3}\times$ area of base $\times$ perpendicular height.)

[6]

Q2

9231/13 • May/June 2015 • Question 8

A line, passing through the point $A(3, 0, 2)$, has vector equation $\mathbf{r} = 3\mathbf{i} + 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. It meets the plane Π, which has equation $\mathbf{r} \cdot (\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = 3$, at the point P. Find the coordinates of P. [3]

Write down a vector $\mathbf{n}$ which is perpendicular to Π, and calculate the vector $\mathbf{w}$, where

$$\mathbf{w} = \mathbf{n} \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}).$$

[3]

The point Q lies in Π and is the foot of the perpendicular from A to Π. Use the vector $\mathbf{w}$ to determine an equation of the line PQ in the form $\mathbf{r} = \mathbf{u} + \mu\mathbf{v}$. [4]

Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 11 (EITHER)

Answer only **one** of the following two alternatives.

The points A, B and C have position vectors $\mathbf{i}$, $2\mathbf{j}$ and $4\mathbf{k}$ respectively, relative to an origin O. The point N is the foot of the perpendicular from O to the plane ABC. The point P on the line-segment ON is such that $OP = \frac{3}{4}ON$. The line AP meets the plane OBC at Q. Find a vector perpendicular to the plane ABC and show that the length of ON is $\dfrac{4}{\sqrt{(21)}}$. [4]

Find the position vector of the point Q. [5]

Show that the acute angle between the planes ABC and ABQ is $\cos^{-1}\left(\dfrac{2}{3}\right)$. [5]

Q4

9231/11, 9231/12 • May/June 2016 • Question 8

Find a cartesian equation of the plane Π_1 passing through the points with coordinates $(2, -1, 3)$, $(4, 2, -5)$ and $(-1, 3, -2)$. [4]

The plane Π_2 has cartesian equation $3x - y + 2z = 5$. Find the acute angle between Π_1 and Π_2. [3]

Find a vector equation of the line of intersection of the planes Π_1 and Π_2. [4]

Q5

9231/13 • May/June 2016 • Question 11 (OR)

Answer only **one** of the following two alternatives.

OR

The position vectors of the points A, B, C, D are

$$\mathbf{a} = 2\mathbf{i} + \lambda\mathbf{j} - 3\mathbf{k}, \quad \mathbf{b} = 6\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}, \quad \mathbf{c} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}, \quad \mathbf{d} = \mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$$

respectively. It is given that the shortest distance between the lines AB and CD is 3.

(i) Show that $\lambda^2 + \lambda - 20 = 0$. [7]

(ii) The planes p_1 and p_2 are the planes through A, B and D corresponding to the two values of λ satisfying the equation in part (i). Find the acute angle between p_1 and p_2. [7]

Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 11 (EITHER)

Answer only **one** of the following two alternatives.

EITHER

The lines l_1 and l_2 have equations

$$\mathbf{r} = 6\mathbf{i} - 3\mathbf{j} + s(3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = 2\mathbf{i} - \mathbf{j} - 4\mathbf{k} + t(\mathbf{i} - 3\mathbf{j} - \mathbf{k})$$

respectively. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. Show that the position vector of P is $3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and find the position vector of Q. [7]

Find, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, an equation of the plane Π which passes through P and is perpendicular to l_1. [3]

The plane Π meets the plane $\mathbf{r} = p\mathbf{i} + q\mathbf{j}$ in the line l_3. Find a vector equation of l_3. [4]

Q7

9231/11, 9231/12 • May/June 2017 • Question 12 (OR)

Answer only **one** of the following two alternatives.

OR

The position vectors of the points A, B, C, D are

$$\mathbf{i}+\mathbf{j}+3\mathbf{k}, \quad 3\mathbf{i}-\mathbf{j}+5\mathbf{k}, \quad 3\mathbf{i}-\mathbf{j}+\mathbf{k}, \quad 5\mathbf{i}-5\mathbf{j}+\alpha\mathbf{k},$$

respectively, where α is a positive integer. It is given that the shortest distance between the line AB and the line CD is equal to $2\sqrt{2}$.

(i) Show that the possible values of α are 3 and 5. [7]

(ii) Using $\alpha = 3$, find the shortest distance of the point D from the line AC, giving your answer correct to 3 significant figures. [3]

(iii) Using $\alpha = 3$, find the acute angle between the planes ABC and ABD, giving your answer in degrees. [4]

Q8

9231/13 • May/June 2017 • Question 9

The plane Π_1 passes through the points $(1, 2, 1)$ and $(5, -2, 9)$ and is parallel to the vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

(i) Find the cartesian equation of Π_1. [4]

The plane Π_2 contains the lines

$$\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k} + \mu(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}).$$

(ii) Find the cartesian equation of Π_2. [4]

(iii) Find the acute angle between Π_1 and Π_2. [3]

Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 6

The points A, B and C have position vectors $2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $-\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ respectively.

(i) Find the area of the triangle ABC. [4]

(ii) Find the perpendicular distance of the point A from the line BC. [3]

(iii) Find the cartesian equation of the plane through A, B and C. [2]

Q10

9231/11, 9231/12 • May/June 2018 • Question 10

The line l_1 is parallel to the vector $a\mathbf{i} - \mathbf{j} + \mathbf{k}$, where a is a constant, and passes through the point whose position vector is $9\mathbf{j} + 2\mathbf{k}$. The line l_2 is parallel to the vector $-a\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ and passes through the point whose position vector is $-6\mathbf{i} - 5\mathbf{j} + 10\mathbf{k}$.

(i) It is given that l_1 and l_2 intersect.

(a) Show that $a = -\frac{6}{13}$. [3]

(b) Find a cartesian equation of the plane containing l_1 and l_2. [4]

(ii) Given instead that the perpendicular distance between l_1 and l_2 is $3\sqrt{30}$, find the value of a. [5]

Q11

9231/13 • May/June 2018 • Question 7

The lines l_1 and l_2 have vector equations

$$\mathbf{r} = a\mathbf{i} + 9\mathbf{j} + 13\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -3\mathbf{i} + 7\mathbf{j} - 2\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

respectively. It is given that l_1 and l_2 intersect.

(i) Find the value of the constant a. [3]

The point P has position vector $3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$.

(ii) Find the perpendicular distance from P to the plane containing l_1 and l_2. [4]

(iii) Find the perpendicular distance from P to l_2. [4]

Q12

9231/11, 9231/13 • October/November 2018 • Question 8

The plane Π_1 has equation

$$\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}.$$

(i) Find a cartesian equation of Π_1. [3]

The plane Π_2 has equation $3x + y - z = 3$.

(ii) Find the acute angle between Π_1 and Π_2, giving your answer in degrees. [2]

(iii) Find an equation of the line of intersection of Π_1 and Π_2, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$. [5]

Q13

9231/12 • October/November 2018 • Question 10

The position vectors of the points A, B, C, D are

$$\mathbf{i}+\mathbf{j}+3\mathbf{k}, \qquad 3\mathbf{i}+4\mathbf{j}+5\mathbf{k}, \qquad -\mathbf{i}+3\mathbf{k}, \qquad m\mathbf{j}+4\mathbf{k},$$

respectively, where m is a constant.

(i) Show that the lines AB and CD are parallel when $m=\frac{3}{2}$. [1]

(ii) Given that $m \neq \frac{3}{2}$, find the shortest distance between the lines AB and CD. [5]

(iii) When $m=2$, find the acute angle between the planes ABC and ABD, giving your answer in degrees. [6]

Q14

9231/11, 9231/12 • May/June 2019 • Question 3

The lines l_1 and l_2 have equations $\mathbf{r} = 6\mathbf{i} + 2\mathbf{j} + 7\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + \mu(-6\mathbf{j} + \mathbf{k})$ respectively. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. Find the position vectors of P and Q. [8]

Q15

9231/13 • May/June 2019 • Question 7

The line l_1 passes through the points $A(-3, 1, 4)$ and $B(-1, 5, 9)$. The line l_2 passes through the points $C(-2, 6, 5)$ and $D(-1, 7, 5)$.

(i) Find the shortest distance between the lines l_1 and l_2. [5]

(ii) Find the acute angle between the line l_2 and the plane containing A, B and D. [5]

Q16

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 6

With O as the origin, the points A, B, C have position vectors

$$\mathbf{i} - \mathbf{j}, \quad 2\mathbf{i} + \mathbf{j} + 7\mathbf{k}, \quad \mathbf{i} - \mathbf{j} + \mathbf{k}$$

respectively.

(i) Find the shortest distance between the lines OC and AB. [5]

(ii) Find the cartesian equation of the plane containing the line OC and the common perpendicular of the lines OC and AB. [4]

Q17

9231/11, 9231/12 • May/June 2020 • Question 5

The lines l_1 and l_2 have equations $\mathbf{r} = 3\mathbf{i}+3\mathbf{k}+\lambda(\mathbf{i}+4\mathbf{j}+4\mathbf{k})$ and $\mathbf{r} = 3\mathbf{i}-5\mathbf{j}-6\mathbf{k}+\mu(5\mathbf{j}+6\mathbf{k})$ respectively.

(a) Find the shortest distance between l_1 and l_2. [5]

The plane Π contains l_1 and is parallel to the vector $\mathbf{i}+\mathbf{k}$.

(b) Find the equation of Π, giving your answer in the form $ax+by+cz=d$. [4]

(c) Find the acute angle between l_2 and Π. [3]

Q18

9231/13 • May/June 2020 • Question 7

The lines l_1 and l_2 have equations $\mathbf{r} = -5\mathbf{j} + \lambda(5\mathbf{i} + 2\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{j} + \mathbf{k})$ respectively. The plane Π contains l_1 and is parallel to l_2.

(a) Find the equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

(b) Find the distance between l_2 and Π. [3]

The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2.

(c) Show that P has position vector $\frac{55}{27}\mathbf{i} - 5\mathbf{j} + \frac{22}{27}\mathbf{k}$ and state a vector equation for PQ. [8]

Q19

9231/11, 9231/13 • October/November 2020 • Question 4

The points A, B, C have position vectors

$$-\mathbf{i}+\mathbf{j}+2\mathbf{k}, \quad -2\mathbf{i}-\mathbf{j}, \quad 2\mathbf{i}+2\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

(b) Find the perpendicular distance from O to the plane ABC. [2]

(c) Find the acute angle between the planes OAB and ABC. [4]

Q20

9231/12 • October/November 2020 • Question 7

The points A, B, C have position vectors

$$-2\mathbf{i}+2\mathbf{j}-\mathbf{k}, \quad -2\mathbf{i}+\mathbf{j}+2\mathbf{k}, \quad -2\mathbf{j}+\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

(b) Find the acute angle between the planes OBC and ABC. [4]

The point D has position vector $t\mathbf{i}-\mathbf{j}$.

(c) Given that the shortest distance between the lines AB and CD is $\sqrt{10}$, find the value of t. [6]

Q21

9231/11, 9231/12 • May/June 2021 • Question 6

Let t be a positive constant.

The line l_1 passes through the point with position vector $t\mathbf{i} + \mathbf{j}$ and is parallel to the vector $-2\mathbf{i} - \mathbf{j}$. The line l_2 passes through the point with position vector $\mathbf{j} + t\mathbf{k}$ and is parallel to the vector $-2\mathbf{j} + \mathbf{k}$.

It is given that the shortest distance between the lines l_1 and l_2 is $\sqrt{21}$.

(a) Find the value of t. [5]

The plane Π_1 contains l_1 and is parallel to l_2.

(b) Write down an equation of Π_1, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. [1]

The plane Π_2 has Cartesian equation $5x - 6y + 7z = 0$.

(c) Find the acute angle between l_2 and Π_2. [3]

(d) Find the acute angle between Π_1 and Π_2. [3]

Q22

9231/13 • May/June 2021 • Question 6

The lines l_1 and l_2 have equations $\mathbf{r} = -\mathbf{i} - 2\mathbf{j} + \mathbf{k} + s(2\mathbf{i} - 3\mathbf{j})$ and $\mathbf{r} = 3\mathbf{i} - 2\mathbf{k} + t(3\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ respectively.

The plane Π_1 contains l_1 and the point P with position vector $-2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$.

(a) Find an equation of Π_1, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. [2]

The plane Π_2 contains l_2 and is parallel to l_1.

(b) Find an equation of Π_2, giving your answer in the form $ax + by + cz = d$. [4]

(c) Find the acute angle between Π_1 and Π_2. [5]

The point Q is such that $\overrightarrow{OQ} = -5\overrightarrow{OP}$.

(d) Find the position vector of the foot of the perpendicular from the point Q to Π_2. [4]

Q23

9231/11, 9231/13 • October/November 2021 • Question 5

The plane Π has equation $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} + \mathbf{k}) + \mu(2\mathbf{i} + 3\mathbf{j})$.

(a) Find a Cartesian equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

The line l passes through the point P with position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and is parallel to the vector $\mathbf{k}$.

(b) Find the position vector of the point where l meets Π. [3]

(c) Find the acute angle between l and Π. [3]

(d) Find the perpendicular distance from P to Π. [3]

Q24

9231/12 • October/November 2021 • Question 7

The points A, B, C have position vectors

$$2\mathbf{i} + 2\mathbf{j}, \quad -\mathbf{j} + \mathbf{k} \quad \text{and} \quad 2\mathbf{i} + \mathbf{j} - 7\mathbf{k}$$

respectively, relative to the origin O.

(a) Find an equation of the plane OAB, giving your answer in the form $\mathbf{r}.\mathbf{n} = p$. [3]

The plane Π has equation $x - 3y - 2z = 1$.

(b) Find the perpendicular distance of Π from the origin. [1]

(c) Find the acute angle between the planes OAB and Π. [3]

(d) Find an equation for the common perpendicular to the lines OC and AB. [10]

Q25

9231/11, 9231/12 • May/June 2022 • Question 2

The points A, B, C have position vectors

$$4\mathbf{i} - 4\mathbf{j} + \mathbf{k}, \quad -4\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}, \quad 4\mathbf{i} - \mathbf{j} - 2\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [5]

(b) Find the perpendicular distance from O to the plane ABC. [2]

(c) The point D has position vector $2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$.

Find the coordinates of the point of intersection of the line OD with the plane ABC. [3]

Q26

9231/13 • May/June 2022 • Question 7

The position vectors of the points A, B, C, D are

$$7\mathbf{i} + 4\mathbf{j} - \mathbf{k}, \qquad 11\mathbf{i} + 3\mathbf{j}, \qquad 2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}, \qquad 2\mathbf{i} + 7\mathbf{j} + \lambda\mathbf{k}$$

respectively.

(a) Given that the shortest distance between the line AB and the line CD is 3, show that $\lambda^2 - 5\lambda + 4 = 0$. [7]

Let Π_1 be the plane ABD when $\lambda = 1$.

Let Π_2 be the plane ABD when $\lambda = 4$.

(b) (i) Write down an equation of Π_1, giving your answer in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$. [2]

(ii) Find an equation of Π_2, giving your answer in the form $ax + by + cz = d$. [4]

(c) Find the acute angle between Π_1 and Π_2. [5]

Q27

9231/11, 9231/13 • October/November 2022 • Question 4

The plane Π contains the lines $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$.

(a) Find a Cartesian equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

The line l passes through the point P with position vector $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and is parallel to the vector $\mathbf{j} + \mathbf{k}$.

(b) Find the acute angle between l and Π. [3]

(c) Find the position vector of the foot of the perpendicular from P to Π. [4]

Q28

9231/12 • October/November 2022 • Question 6

The lines l_1 and l_2 have equations $\mathbf{r} = 2\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ respectively.

The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2.

(a) Find the length PQ. [5]

The plane Π_1 contains PQ and l_1.

The plane Π_2 contains PQ and l_2.

(b) (i) Write down an equation of Π_1, giving your answer in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$. [1]

(ii) Find an equation of Π_2, giving your answer in the form $ax + by + cz = d$. [4]

(b) Find the acute angle between Π_1 and Π_2. [5]

Q29

9231/11, 9231/12 • May/June 2023 • Question 7

The plane Π_1 has equation $r = -4\mathbf{j} - 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(a) Obtain an equation of Π_1 in the form $px + qy + rz = d$. [4]

(b) The plane Π_2 has equation $\mathbf{r}.(-5\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) = 4$.

Find a vector equation of the line of intersection of Π_1 and Π_2. [4]

The line l passes through the point A with position vector $a\mathbf{i} + a\mathbf{j} + (a-7)\mathbf{k}$ and is parallel to $(1-b)\mathbf{i} + b\mathbf{j} + b\mathbf{k}$, where a and b are positive constants.

(c) Given that the perpendicular distance from A to Π_1 is $\sqrt{2}$, find the value of a. [2]

(d) Given that the obtuse angle between l and Π_1 is $\frac{3}{4}\pi$, find the exact value of b. [4]

Q30

9231/13 • May/June 2023 • Question 6

The points A, B, C have position vectors

$$\mathbf{i} + \mathbf{j}, \quad -\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}, \quad -2\mathbf{i} + \mathbf{j} + 3\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [5]

(b) Find the perpendicular distance from O to the plane ABC. [2]

(c) Find a vector equation of the common perpendicular to the lines OC and AB. [8]

Q31

9231/11, 9231/13 • October/November 2023 • Question 4

The lines l_1 and l_2 have equations

$$\mathbf{r} = -2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k} + \lambda(-4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$$

respectively.

(a) Find the shortest distance between l_1 and l_2. [5]

The plane Π contains l_1 and the point with position vector $-\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$.

(b) Find an equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

Q32

9231/12 • October/November 2023 • Question 5

The plane Π_1 has equation $\mathbf{r} = \mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$. [4]

The line l, which does not lie in Π_1, has equation $\mathbf{r} = -3\mathbf{i} + \mathbf{k} + t(\mathbf{i} + \mathbf{j} + \mathbf{k})$.

(b) Show that l is parallel to Π_1. [2]

(c) Find the distance between l and Π_1. [3]

The plane Π_2 has equation $3x + 3y + 2z = 1$.

(d) Find a vector equation of the line of intersection of Π_1 and Π_2. [4]

Q33

9231/11, 9231/12 • May/June 2024 • Question 5

The points A, B, C have position vectors

$$2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}, \qquad 2\mathbf{i} + 4\mathbf{j} - \mathbf{k}, \qquad -3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [5]

The point D has position vector $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

(b) Find the perpendicular distance from D to the plane ABC. [2]

(c) Find the shortest distance between the lines AB and CD. [5]

Q34

9231/13 • May/June 2024 • Question 5

The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} - 2\mathbf{k})$ and $\mathbf{r} = -3\mathbf{i} + 4\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.

(a) Find the shortest distance between l_1 and l_2. [5]

The plane Π_1 contains l_1 and is parallel to l_2.

(b) Obtain an equation of Π_1 in the form $px + qy + rz = s$. [2]

The point $(1, 1, 1)$ lies on the plane Π_2.

It is given that the line of intersection of the planes Π_1 and Π_2 passes through the point $(0, 0, 2)$ and is parallel to the vector $\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$.

(c) Obtain an equation of Π_2 in the form $ax + by + cz = d$. [3]

Q35

9231/12 • October/November 2024 • Question 2

The line l_1 has equation $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} - 4\mathbf{k})$.

The plane Π contains l_1 and is parallel to the vector $2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$.

(a) Find the equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

The line l_2 is parallel to the vector $5\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$.

(b) Find the acute angle between l_2 and Π. [3]

Q36

9231/11, 9231/13 • October/November 2024 • Question 7

The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i}+3\mathbf{j}-2\mathbf{k}+\lambda(2\mathbf{i}+\mathbf{j}+\mathbf{k})$ and $\mathbf{r} = \mathbf{i}-2\mathbf{j}+9\mathbf{k}+\mu(\mathbf{i}-4\mathbf{j}+2\mathbf{k})$ respectively. The plane Π_1 contains l_1 and is parallel to l_2.

(a) Find the equation of Π_1, giving your answer in the form $ax + by + cz = d$. [4]

The plane Π_2 contains l_2 and the point with coordinates $(2, -1, 7)$.

(b) Find the acute angle between Π_1 and Π_2. [4]

The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2.

(c) Find a vector equation for PQ. [7]

Q37

9231/11, 9231/12 • May/June 2025 • Question 6

The points A, B, C have position vectors

$$\mathbf{i} - 2\mathbf{k}, \quad \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

(a) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [5]

A point D has position vector $\mathbf{i} + t\mathbf{k}$, where $t \neq -2$.

(b) Find the acute angle between the planes ABC and ABD. [4]

(c) Find the values of t such that the shortest distance between the lines AB and CD is $\sqrt{2}$. [7]

Q38

9231/13 • May/June 2025 • Question 5

The plane Π has equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

(a) Find a Cartesian equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

The point P has position vector $4\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$.

(b) Find the position vector of the foot of the perpendicular from P to Π. [4]

The line l is parallel to the vector $3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

(c) Find the acute angle between l and Π. [3]

Q39

9231/11, 9231/13 • October/November 2025 • Question 5

The plane Π_1 has equation $\mathbf{r} = -3\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$. [4]

(b) Find the perpendicular distance from the point with position vector $-\mathbf{i} - 2\mathbf{k}$ to Π_1. [3]

The plane Π_2 has equation $3x + 2y - z = 14$.

(c) Find a vector equation of the line of intersection of Π_1 and Π_2. [4]

Q40

9231/12 • October/November 2025 • Question 6

The plane Π has equation $x + 3y + 2z = 1$.

(a) Find the perpendicular distance from the origin O to the plane Π. [2]

Relative to O, the points A, B, C have position vectors

$$-\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

(b) Find the acute angle between the planes OAB and Π. [4]

(c) Find an equation for the common perpendicular to the lines OC and AB. [8]

Q41

9231/14 • October/November 2025 • Question 4

The points A, B, C have position vectors

$$3\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}, \qquad 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \qquad 2\mathbf{i} + \mathbf{j} - \mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [5]

(b) The point D has position vector $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.
Find the shortest distance between the lines AB and CD. [5]

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Written by Ali Hashir.

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