Further Math with Ali Hashir

Vectors

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 Paper 1 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/11, 9231/12 • May/June 2015 • Question 11 (OR)

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Mark scheme.

$\mathbf{p} = \begin{pmatrix} 8+\lambda \\ 2-2\lambda \\ 3 \end{pmatrix}, \mathbf{q} = \begin{pmatrix} 5 \\ 3+2\mu \\ -14-3\mu \end{pmatrix} \Rightarrow \overrightarrow{QP} = \begin{pmatrix} 3+\lambda \\ -1-2\lambda-2\mu \\ 17+3\mu \end{pmatrix}$	**M1A1**
$\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3+\lambda \\ -1-2\lambda-2\mu \\ 17+3\mu \end{pmatrix} = 0 \Rightarrow 5\lambda + 4\mu = -5$	**M1A1**
$\begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3+\lambda \\ -1-2\lambda-2\mu \\ 17+3\mu \end{pmatrix} = 0 \Rightarrow -4\lambda - 13\mu = 53$	**A1**
Solving : $\lambda = 3, \ \mu = -5$	**M1A1**
Whence: $\mathbf{p} = \begin{pmatrix} 11 \\ -4 \\ 3 \end{pmatrix}, \mathbf{q} = \begin{pmatrix} 5 \\ -7 \\ 1 \end{pmatrix}$.	**A1**
	(8)
(ii)(a) $\overrightarrow{AP} = \begin{pmatrix} 3 \\ -6 \\ 0 \end{pmatrix}, \overrightarrow{AQ} = \begin{pmatrix} -3 \\ -9 \\ -2 \end{pmatrix}$	**B1**
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -6 & 0 \\ -3 & -9 & -2 \end{vmatrix} = \begin{pmatrix} 12 \\ 6 \\ -45 \end{pmatrix}$ (CAO) ; Area $= \frac{1}{2}\sqrt{2205}$ $(= 23.5)$	**M1A1**

(b) $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 1 \\ -17 \end{pmatrix} \Rightarrow \text{Volume} = \frac{1}{3} \times \frac{1}{2}\sqrt{2205}\frac{\left|\begin{pmatrix} -3 \\ 1 \\ -17 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 6 \\ -45 \end{pmatrix}\right|}{\sqrt{2205}} = \frac{735}{6}$ (= 122.5) **M1A1**

(6)

Total: 14

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Q2

9231/13 • May/June 2015 • Question 8

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Mark scheme.

$\begin{pmatrix} 3+2\lambda \\ \lambda \\ 2-2\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 3$	**M1**
$3 + 2\lambda + 2\lambda + 2 - 2\lambda = 3 \Rightarrow \lambda = -1$	**A1**
P is $(1, -1, 4)$ (Accept position vector.)	**A1**
	(3)
$\mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$	**B1**
$\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 1 \\ 2 & 1 & -2 \end{vmatrix} = \begin{pmatrix} -5 \\ 4 \\ -3 \end{pmatrix}$	**M1A1**
	(3)
Direction of PQ is $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 4 & -3 \\ 1 & 2 & 1 \end{vmatrix} = \begin{pmatrix} 10 \\ 2 \\ -14 \end{pmatrix} \sim \begin{pmatrix} 5 \\ 1 \\ -7 \end{pmatrix}$ (OE)	**M1A1**
Equation of PQ is $\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 1 \\ -7 \end{pmatrix}$	**dM1A1**
	(4)
[N.B. For methods not using **w** at most three B1 marks.]	
	Total: 10

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Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 11 (EITHER)

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Mark scheme.

E.g. $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 0 \\ -1 & 0 & 4 \end{vmatrix} = \begin{pmatrix} 8 \\ 4 \\ 2 \end{pmatrix} \sim \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$	**M1A1**
$\dfrac{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}}{\sqrt{4^2+2^2+1^2}} = \dfrac{4}{\sqrt{21}}$ (AG)	**M1A1**
	(4)
$\mathbf{p} = \dfrac{3}{\sqrt{21}}\left(\dfrac{4\mathbf{i}+2\mathbf{j}+\mathbf{k}}{\sqrt{21}}\right) = \dfrac{1}{7}(4\mathbf{i}+2\mathbf{j}+\mathbf{k})$	**B1**
Line AP: $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$	**M1A1**
For Q: $1-3t=0 \Rightarrow t=\dfrac{1}{3} \Rightarrow \mathbf{q} = \dfrac{1}{3}\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$	**M1A1**
	(5)
E.g. $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$, $\overrightarrow{BQ} = \dfrac{1}{3}\begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}$	**B1**
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 0 \\ 0 & -4 & 1 \end{vmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$	**M1A1**

$\cos^{-1}\left\|\frac{\begin{pmatrix}4\\2\\1\end{pmatrix}\cdot\begin{pmatrix}2\\1\\4\end{pmatrix}}{\sqrt{21}.\sqrt{21}}\right\| = \cos^{-1}\frac{8+2+4}{21} = \cos^{-1}\frac{14}{21} = \cos^{-1}\frac{2}{3}$ (AG)	**M1A1**
	(5)
	Total: 14

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Q4

9231/11, 9231/12 • May/June 2016 • Question 8

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Mark scheme.

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & -8 \\ -3 & 4 & -5 \end{vmatrix} = \begin{pmatrix} 17 \\ 34 \\ 17 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$	**M1 A1**
So $x + 2y + z = \text{const} \Rightarrow \text{const} = 2 - 2 + 3 = 3$ (using a point) $\Rightarrow$	**M1**
$x + 2y + z = 3$	**A1**
	(4)
$\sqrt{9+1+4}\sqrt{1+4+1}\cos\theta = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \Rightarrow \cos\theta = \frac{3}{\sqrt{14}\sqrt{6}} = \frac{3}{\sqrt{84}}$	**M1 M1**
$\Rightarrow \theta = 70.9$ or 1.24 radians	**A1**
	(3)
Direction of line of intersection is $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 1 \\ 3 & -1 & 2 \end{vmatrix} = \begin{pmatrix} 5 \\ 1 \\ -7 \end{pmatrix}$	**M1A1**
Finds point common to both planes is $(-1, 0, 4)$ or $\left(\frac{13}{7}, \frac{4}{7}, 0\right)$ or $\left(0, \frac{1}{5}, \frac{13}{5}\right)$	**M1**
Equation of line of intersection is eg $\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} + t\begin{pmatrix} 5 \\ 1 \\ -7 \end{pmatrix}$.	**A1✓**
	(4)
	Total: 11

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Q5

9231/13 • May/June 2016 • Question 11 (OR)

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Mark scheme.

Direction perpendicular to AB and CD: $\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 1 \\ 4 & 3-\lambda & 1 \end{vmatrix} = \begin{pmatrix} \lambda-2 \\ 4 \\ -4 \end{pmatrix}$	**M1A1**
$\overrightarrow{DB} = \begin{pmatrix} 5 \\ -4 \\ -6 \end{pmatrix}$. Hence $\dfrac{\left\lvert \begin{pmatrix} 5 \\ -4 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} \lambda-2 \\ 4 \\ -4 \end{pmatrix} \right\rvert}{\sqrt{(\lambda-2)^2+16+16}} = 3$ or equivalent	**M1A1**
$\Rightarrow (5\lambda-2)^2 = 9(\lambda^2-4\lambda+36)$	**M1M1**
$\Rightarrow \cdots \Rightarrow \lambda^2+\lambda-20=0$ (AG)	**A1**
	(7)
$(\lambda+5)(\lambda-4)=0 \Rightarrow \lambda=-5, 4$	**B1**
$\lambda=4 \Rightarrow \mathbf{a} = \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix} \Rightarrow$ Normal to $ABD = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 1 \\ -1 & 3 & 7 \end{vmatrix} = \begin{pmatrix} -10 \\ -29 \\ 11 \end{pmatrix}$	**M1A1**
$\lambda=-5 \Rightarrow \mathbf{a} = \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} \Rightarrow$ Normal to $ABD = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 8 & 1 \\ -1 & 12 & 7 \end{vmatrix} = \begin{pmatrix} 44 \\ -29 \\ 56 \end{pmatrix}$	**A1**
$\cos\theta = \dfrac{-440+841+616}{\sqrt{10^2+29^2+11^2}\sqrt{44^2+29^2+56^2}} = \dfrac{1017}{\sqrt{1062}\sqrt{5913}}$	**M1A1**
$\Rightarrow \theta = 66.1$	**A1**
	(7)
	Total: 14

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Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 11 (EITHER)

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Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & -1 \\ 3 & -4 & -2 \end{vmatrix} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$	**M1A1**
Obtains three equations. E.g. from $\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$	
$t - 3s + 2k = 4$	
$-3t + 4s - k = -2$	**M1A1**
$-t + 2s + 5k = 4$	
Solves: $s = -1$, $t = -1$, $k = 1$	**M1A1**
$\mathbf{p} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{q} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ (Both required)	**A1**
	(7)
ALT METHOD: Find PQ in terms of s,t **M1** Calculate scalar products direction vectors for l_1 and l_2 **M1,A1,A1** Solve simultaneous equations **M1 A1** p and q **A1**	
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -4 & -2 \\ 2 & -1 & 5 \end{vmatrix} = \begin{pmatrix} -22 \\ -19 \\ 5 \end{pmatrix}$	**B1**
$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -22 \\ -19 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$	**B1B1**
	(3)
Direction of l_3 is $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -4 & -2 \\ 0 & 0 & 1 \end{vmatrix} = \begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}$	**B1**

Normal to Π is $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 22 & 19 & -5 \\ 2 & -1 & 5 \end{vmatrix} = \begin{pmatrix} 90 \\ -120 \\ -60 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix}$	**M1**
Cartesian equation of Π is $3x - 4y - 2z = 1$ (Since P lies in Π)	**A1✓**
Vector equation of l_3 is $\mathbf{r} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}$ (OE) (Since $z = 0$.)	**B1**
	(4)
	Total: 14

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Q7

9231/11, 9231/12 • May/June 2017 • Question 12 (OR)

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Mark scheme.

$\overrightarrow{AC} = \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$ $\overrightarrow{CD} = \begin{pmatrix} 2 \\ -4 \\ \alpha - 1 \end{pmatrix}$ (May find $\overrightarrow{AD}, \overrightarrow{BC}$ or $\overrightarrow{BD}$ instead of AC)	**B1**
$\overrightarrow{AB} \times \overrightarrow{CD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 2 & -4 & \alpha - 1 \end{vmatrix} = \begin{pmatrix} 5-\alpha \\ 3-\alpha \\ -2 \end{pmatrix} \sim \begin{pmatrix} \alpha-5 \\ \alpha-3 \\ 2 \end{pmatrix}$	**M1A1**
$\dfrac{\begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix} \begin{pmatrix} \alpha-5 \\ \alpha-3 \\ 2 \end{pmatrix}}{\sqrt{(\alpha-5)^2 + (\alpha-3)^2 + 4}} = 2\sqrt{2} \Rightarrow 2\sqrt{2}\sqrt{2\alpha^2 - 16\alpha + 38} = 8$ (Substitutes their vectors into correct formula)	**M1A1**
$\Rightarrow 2\alpha^2 - 16\alpha + 38 = 8 \Rightarrow \alpha^2 - 8\alpha + 15 = 0$	**A1**
$\Rightarrow (\alpha - 3)(\alpha - 5) = 0 \Rightarrow \alpha = 3$ or 5 (AG)	**A1**
	(7)
$\overrightarrow{AD} = \begin{pmatrix} 4 \\ -6 \\ 0 \end{pmatrix}$ (or $\overrightarrow{CD} = \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}$) (Alt method: Let P be point on AC with parameter λ **M1**)	**B1**
Distance of D from $AC = \dfrac{1}{\sqrt{1+1+1}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -6 & 0 \\ 1 & -1 & -1 \end{vmatrix} = \sqrt{\dfrac{56}{3}} = 4.32$ (or with CD) (Use DP.AC = 0 to find $\lambda(= -5/3)$ **M1**; Find length **A1**)	**M1A1**
	(3)

ABC: $\mathbf{n}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ ABD: $\mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 2 & -3 & 0 \end{vmatrix} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$	**B1B1**
$\cos\theta = \left\lvert \dfrac{-3-2+0}{\sqrt{2}\sqrt{14}} \right\rvert \Rightarrow \theta = 19.1^\circ$ or 0.333 rads	**M1A1**
	(4)
	Total: 14

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Q8

9231/13 • May/June 2017 • Question 9

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Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = \begin{pmatrix} -7 \\ -1 \\ 3 \end{pmatrix}$	**M1A1**
Cartesian equation of Π_1 is $7x + y - 3z = \text{const}$	**M1**
$7 \times 1 + 2 - 3 \times 1 = 6 \Rightarrow 7x + y - 3z = 6$	**A1**
	(4)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & -1 \\ 2 & 3 & -1 \end{vmatrix} = \begin{pmatrix} 5 \\ -1 \\ 7 \end{pmatrix}$	**M1A1**
$5 \times 2 - (-3) + 7 \times 1 = 20 \Rightarrow 5x - y + 7z = 20$	**M1A1**
	(4)
$\cos\theta = \left\lvert \dfrac{\begin{pmatrix} 7 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -1 \\ 7 \end{pmatrix}}{\sqrt{49+1+9}\sqrt{25+1+49}} \right\rvert$	**M1M1**
$\Rightarrow \cos\theta = \dfrac{13}{\sqrt{59}\sqrt{75}} \Rightarrow \theta = 78.7^\circ$ or 1.37 rad.	**A1**
	(3)
	Total: 11

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Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 6

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Mark scheme.

$\overrightarrow{AB} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ $\quad \overrightarrow{BC} = -4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ $\quad \overrightarrow{AC} = -3\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ (2 correct required)	**B1**
$\overrightarrow{AB} \times \overrightarrow{BC} = 21\mathbf{i} + 3\mathbf{j} + 18\mathbf{k}$ $(*)$ (OE)	**M1A1**
Area of triangle $ABC = \frac{1}{2}\sqrt{21^2 + 3^2 + 18^2} = 13.9$ $\left(\frac{3}{2}\sqrt{86}\right)$	**A1**
Alt method: Use scalar product to find angle	**(M1A1**
Find area using Area $= \frac{1}{2}ab\sin C$ or equivalent	**M1A1)**
	(4)
$\mathrm{d} = \dfrac{\lvert\overrightarrow{AB} \times \overrightarrow{BC}\rvert}{\lvert\overrightarrow{BC}\rvert} = \dfrac{\sqrt{21^2 + 3^2 + 18^2}}{\sqrt{4^2 + 2^2 + 5^2}}$	**M1A1**
$= 4.15$ $\left(\frac{1}{5}\sqrt{430}\right)$ (Alt method: Area triangle $= \sin C \times \lvert AC\rvert$)	**A1**
Alt method: Use equation of BC to find D (foot of perpendicular) in terms of parameter **and** scalar product to find parameter, $\lambda = 8/15$. Find length	**(M1A1)**
	(3)
From (*) Cartesian equation is $7x + y + 6z = \text{const.}$	**M1**
Through $(2, -1, 1)$ Hence $7x + y + 6z = 19$	**A1**
	(2)
	Total: 9

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Q10

9231/11, 9231/12 • May/June 2018 • Question 10

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Mark scheme.

Point on l_1 is $\begin{pmatrix} \lambda a \\ 9-\lambda \\ 2+\lambda \end{pmatrix}$ and on l_2 is $\begin{pmatrix} -6-\mu a \\ -5+2\mu \\ 10+4\mu \end{pmatrix}$	**B1**
Point of intersection: $\begin{pmatrix} \lambda a \\ 9-\lambda \\ 2+\lambda \end{pmatrix} = \begin{pmatrix} -6-\mu a \\ -5+2\mu \\ 10+4\mu \end{pmatrix}$. Equates coordinates of points	**M1**
$\Rightarrow \mu = 1,\ \lambda = 12,\ a = -\dfrac{6}{13}$. AG	**A1**
	(3)
Normal to the plane: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\frac{6}{13} & -1 & 1 \\ \frac{6}{13} & 2 & 4 \end{vmatrix} = -6\mathbf{i} + \dfrac{30}{13}\mathbf{j} - \dfrac{6}{13}\mathbf{k} \sim -13\mathbf{i} + 5\mathbf{j} - \mathbf{k}$. Uses cross product to find normal to the plane. AEF	**M1A1**
Using point on plane: e.g. $5(9) - 2 = 43$. Substitutes a point	**M1**
Equation of plane: $-13x + 5y - z = 43$. AEF	**A1**
	(4)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & -1 & 1 \\ -a & 2 & 4 \end{vmatrix} = -6i - 5aj + ak$, $\left\lvert \begin{pmatrix} -6 \\ -5a \\ a \end{pmatrix} \right\rvert = \sqrt{36 + 26a^2}$. Finds the magnitude of the cross product of the direction vectors of the lines	**B1**
$\begin{pmatrix} -6 \\ -5-9 \\ 10-2 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ -5a \\ a \end{pmatrix} = 36 + 78a$. Takes dot product of the correct vectors	**M1A1**
$3\sqrt{30}\sqrt{36+26a^2} = \lvert 36 + 78a \rvert$. Puts distance equal to $3\sqrt{30}$	**M1**

$\Rightarrow 15(18+13a^2)=(6+13a)^2$	**A1**
$\Rightarrow 26(a-3)^2=0 \Rightarrow a=3$	
	(5)
	Total: 12

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Q11

9231/13 • May/June 2018 • Question 7

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Mark scheme.

Solve two equations $9+2\lambda = 7+2\mu$, $13+3\lambda = -2-3\mu$	**M1**
to obtain $\lambda = -3$ and $\mu = -2$.	**A1**
Use third equation to obtain $a = 2$.	**A1**
	(3)
Normal to the plane is $\mathbf{n} = -12\mathbf{i} + 4\mathbf{k}$	**M1A1**
Perpendicular distance $= \dfrac{1}{\sqrt{160}} \begin{pmatrix} -6 \\ 6 \\ -8 \end{pmatrix} \cdot \begin{pmatrix} -12 \\ 0 \\ 4 \end{pmatrix} = \sqrt{10} = 3.16$	**M1A1**
Alt method: Finds equation of plane M1, A1: Finds foot of perpendicular from P to plane M1 Hence length A1	
Alt method: Find equation of plane $3x - z + 7 = 0$ M1, A1	
Use formula $\dfrac{\lvert 3\times 3 + 0\times 1 - 6 + 7\rvert}{\sqrt{9+1}} = \sqrt{10}$ M1, A1	
	(4)
Cross product of direction of P to l_2 with direction of l_2 $\begin{pmatrix} -6 \\ 6 \\ -8 \end{pmatrix} \times \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ -10 \\ -6 \end{pmatrix}$. Alt method: Find N (foot of perpendicular) in terms of parameter and uses scalar product with n to find parameter M1, A1 so $PN = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$	**M1A1**

Perpendicular distance from P to l_2 is $\dfrac{\lvert 2\mathbf{i}+10\mathbf{j}+6\mathbf{k}\rvert}{\lvert -\mathbf{i}+2\mathbf{j}-3\mathbf{k}\rvert} = \sqrt{10} = 3.16$. Find length PN M1, A1	**M1A1**
	(4)
	Total: 11

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Q12

9231/11, 9231/13 • October/November 2018 • Question 8

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Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 1 & 3 \\ 0 & 1 & 2 \end{vmatrix} = -\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$ (Finds normal to Π_1.)	**M1**
$-x + 8y - 4z = 3$ (Uses point on plane to find cartesian equation, AEF.)	**M1A1**
	(3)
$\begin{pmatrix} -1 \\ 8 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = 9\sqrt{11}\cos\theta$ (Uses scalar product.)	**M1**
$\Rightarrow \theta = 72.5°$ (Accept 1.26 rad.)	**A1**
	(2)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & -1 \\ -1 & 8 & -4 \end{vmatrix} = 4\mathbf{i} + 13\mathbf{j} + 25\mathbf{k}$ (Finds direction of line of intersection.)	**M1A1**
Point on both planes is, e.g. $(1, 1, 1)$. (Finds point common to both planes.)	**M1A1**
$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(4\mathbf{i} + 13\mathbf{j} + 25\mathbf{k})$ (States vector equation of line.)	**A1**
	(5)
	Total: 10

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Q13

9231/12 • October/November 2018 • Question 10

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Mark scheme.

$\overrightarrow{AB} = \begin{pmatrix}2\\3\\2\end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix}1\\1.5\\1\end{pmatrix} = \frac{1}{2}\overrightarrow{AB}$ (Or shows if parallel, then $m = 3/2$)	**B1**
	(1)
$\overrightarrow{AB} = \begin{pmatrix}2\\3\\2\end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix}1\\m\\1\end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix}-2\\-1\\0\end{pmatrix}$ or $\overrightarrow{AD} = \begin{pmatrix}-1\\m-1\\1\end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix}-4\\-4\\-2\end{pmatrix}$	**B1**
$\mathbf{n} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\2 & 3 & 2\\1 & m & 1\end{vmatrix} = \begin{pmatrix}3-2m\\0\\2m-3\end{pmatrix}$ (so parallel to $\begin{pmatrix}1\\0\\-1\end{pmatrix}$) (Finds common perpendicular using cross product.)	**M1A1**
$\frac{\lvert\overrightarrow{AC}\cdot\mathbf{n}\rvert}{\lvert\mathbf{n}\rvert} = \frac{\lvert -2(3-2m)+0+0\rvert}{\sqrt{(3-2m)^2+(2m-3)^2}}$ o.e. (Uses formula for shortest distance.)	**M1**
$= \frac{2}{\sqrt{2}} = \sqrt{2}$	**A1**
	(5)
$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\2 & 3 & 2\\-2 & -1 & 0\end{vmatrix} = \begin{pmatrix}2\\-4\\4\end{pmatrix} \sim \begin{pmatrix}1\\-2\\2\end{pmatrix}$ o.e. (Finds normal to plane ABC (AEF).)	**M1A1**
$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\2 & 3 & 2\\-1 & 1 & 1\end{vmatrix} = \begin{pmatrix}1\\-4\\5\end{pmatrix}$ o.e. (Finds normal to plane ABD (AEF).)	**A1**

$\cos\theta = \dfrac{1+8+10}{\sqrt{1^2+2^2+2^2}\sqrt{1^2+4^2+5^2}} \left(= \dfrac{19}{\sqrt{378}}\right)$ (Uses formula for angle between two lines.)	**M1A1FT**
$\Rightarrow \theta = 12.2°$ (CAO.)	**A1**
	(6)
	Total: 12

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Q14

9231/11, 9231/12 • May/June 2019 • Question 3

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Mark scheme.

$\overrightarrow{OP} = \begin{pmatrix} 6+\lambda \\ 2+\lambda \\ 7 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 4 \\ 4-6\mu \\ \mu \end{pmatrix} \Rightarrow \overrightarrow{PQ} = \begin{pmatrix} -2-\lambda \\ 2-\lambda-6\mu \\ -7+\mu \end{pmatrix}$	**M1A1**
$\begin{pmatrix} -2-\lambda \\ 2-\lambda-6\mu \\ -7+\mu \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0$ or $\begin{pmatrix} -2-\lambda \\ 2-\lambda-6\mu \\ -7+\mu \end{pmatrix} = k \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & -6 & 1 \end{vmatrix} = k \begin{pmatrix} 1 \\ -1 \\ -6 \end{pmatrix}$	**M1**
$-2\lambda - 6\mu = 0$	**A1**
$\begin{pmatrix} -2-\lambda \\ 2-\lambda-6\mu \\ -7+\mu \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -6 \\ 1 \end{pmatrix} = 0 \Rightarrow 6\lambda + 37\mu = 19$	**A1**
$\lambda = -3, \mu = 1$	**M1A1**
$\overrightarrow{OP} = \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$	**A1**

Total: 8

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Q15

9231/13 • May/June 2019 • Question 7

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Mark scheme.

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}, \overrightarrow{CD} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	**B1**
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & 5 \\ 1 & 1 & 0 \end{vmatrix} = \begin{pmatrix} -5 \\ 5 \\ -2 \end{pmatrix}$	**M1A1**
$\dfrac{\begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 5 \\ -2 \end{pmatrix}}{\sqrt{5^2+5^2+2^2}} = \dfrac{18}{\sqrt{54}} = \sqrt{6} = 2.45$	**M1A1**
	(5)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & 5 \\ 2 & 6 & 1 \end{vmatrix} = \begin{pmatrix} -26 \\ 8 \\ 4 \end{pmatrix} \sim \begin{pmatrix} -13 \\ 4 \\ 2 \end{pmatrix}$	**M1A1**
$\dfrac{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -13 \\ 4 \\ 2 \end{pmatrix}}{\sqrt{1^2+1^2+0^2}\sqrt{13^2+4^2+2^2}} = -\dfrac{9}{\sqrt{378}} = -\dfrac{\sqrt{42}}{14}$	**M1A1**
$\cos^{-1}\left(-\dfrac{\sqrt{42}}{14}\right) - 90 = 27.6^\circ$ (or 0.481 rad)	**A1**
	(5)
	Total: 10

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Q16

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 6

↑ Back to contents

Mark scheme.

$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}$	**B1**
$\overrightarrow{OC} \times \overrightarrow{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 1 & 2 & 7 \end{vmatrix} = \begin{pmatrix} -9 \\ -6 \\ 3 \end{pmatrix} = t\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$	**M1A1**
Finds direction of common perpendicular.	
$\dfrac{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{3^2+2^2+1^2}} = \dfrac{1}{\sqrt{14}} = 0.267$	**M1A1**
Uses formula for shortest distance.	
	(5)
$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 3 & 2 & -1 \end{vmatrix} = t\begin{pmatrix} -1 \\ 4 \\ 5 \end{pmatrix}$	**M1A1**
Finds normal to plane.	
$-(0)+4(0)+5(0)=0$	**M1**
Uses point on plane.	
$-x+4y+5z=0$	**A1**
AEF	
	(4)
	Total: 9

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Q17

9231/11, 9231/12 • May/June 2020 • Question 5

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Mark scheme.

$\begin{pmatrix}3\\-5\\-6\end{pmatrix}-\begin{pmatrix}3\\0\\3\end{pmatrix}=\begin{pmatrix}0\\-5\\-9\end{pmatrix}$	**B1**
$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&4&4\\0&5&6\end{vmatrix}=\begin{pmatrix}4\\-6\\5\end{pmatrix}$	**M1A1**
$\frac{1}{\sqrt{77}}\left\vert\begin{pmatrix}0\\-5\\-9\end{pmatrix}\cdot\begin{pmatrix}4\\-6\\5\end{pmatrix}\right\vert=\frac{15}{\sqrt{77}}=1.71$	**M1A1**
	(5)
$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&4&4\\1&0&1\end{vmatrix}=\begin{pmatrix}3\\4\\-4\end{pmatrix}$	**M1A1**
$4(3)+3(0)-4(3)=0\Rightarrow 4x+3y-4z=0$	**M1A1**
	(4)
$\begin{pmatrix}0\\5\\6\end{pmatrix}\cdot\begin{pmatrix}4\\3\\-4\end{pmatrix}=\sqrt{61}\sqrt{41}\cos\alpha\Rightarrow\cos\alpha=\frac{-9}{\sqrt{61}\sqrt{41}}$	**M1A1 FT**
(A1 FT *their* normal)	
Acute angle between l_2 and Π is $\alpha-90=10.4°$	**A1**
	(3)
	Total: 12

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Q18

9231/13 • May/June 2020 • Question 7

↑ Back to contents

Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = \begin{pmatrix} -2 \\ -5 \\ 5 \end{pmatrix}$	**M1A1**
$-5(-5) = 25$	**M1**
$-2x - 5y + 5z = 25$	**A1**
	(4)
$\begin{pmatrix} 0 \\ -5 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -7 \\ 2 \end{pmatrix}$	**M1**
$\frac{1}{\sqrt{54}} \left\lvert \begin{pmatrix} -4 \\ -7 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -5 \\ 5 \end{pmatrix} \right\rvert = \frac{53}{\sqrt{54}} = 7.21$	**M1A1**
	(3)
$\overrightarrow{OP} = \begin{pmatrix} 5\lambda \\ -5 \\ 2\lambda \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 4 \\ 2+\mu \\ -2+\mu \end{pmatrix} \Rightarrow \overrightarrow{PQ} = \begin{pmatrix} 4-5\lambda \\ 7+\mu \\ -2+\mu-2\lambda \end{pmatrix}$	**M1A1**
$\begin{pmatrix} 4-5\lambda \\ 7+\mu \\ -2+\mu-2\lambda \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 0$ or $\begin{pmatrix} 4-5\lambda \\ 7+\mu \\ -2+\mu-2\lambda \end{pmatrix} = k \begin{pmatrix} -2 \\ -5 \\ 5 \end{pmatrix}$	**M1**
$-29\lambda + 2\mu = -16$	**A1**
$\begin{pmatrix} 4-5\lambda \\ 7+\mu \\ -2+\mu-2\lambda \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 0 \Rightarrow -2\lambda + 2\mu = -5$	**A1**

$\lambda = \dfrac{11}{27} \Rightarrow \overrightarrow{OP} = \dfrac{1}{27}\begin{pmatrix} 55 \\ -135 \\ 22 \end{pmatrix} = \dfrac{55}{27}\mathbf{i} - 5\mathbf{j} + \dfrac{22}{27}\mathbf{k}$	**M1A1**
$\mathbf{r} = \dfrac{1}{27}\begin{pmatrix} 55 \\ -135 \\ 22 \end{pmatrix} + k\begin{pmatrix} -2 \\ -5 \\ 5 \end{pmatrix}$	**B1 FT**
(B1 FT *their* common perpendicular)	
	(8)
	Total: 15

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Q19

9231/11, 9231/13 • October/November 2020 • Question 4

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Mark scheme.

$\overrightarrow{AB} = -\mathbf{i} - 2\mathbf{j} - 2\mathbf{k} \quad \overrightarrow{AC} = 3\mathbf{i} - \mathbf{j}$	**B1**
Finds direction vectors of two lines in the plane. $\overrightarrow{BC} = 4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 2 \\ 3 & -1 & 0 \end{vmatrix} = \begin{pmatrix} -2 \\ -6 \\ 7 \end{pmatrix}$	**M1A1**
Finds normal to the plane ABC.	
$-2(-1) - 6(1) + 7(2) = 10 \Rightarrow -2x - 6y + 7z = 10$	**M1A1**
Substitutes point.	
Alternative method for question 4(a)	
Setting up 3 equations using points given.	**M1**
$-2x - 6y + 7z = 10$	**A1 A1 A1 A1**
OE	
	(5)
$\dfrac{10}{\sqrt{2^2 + 6^2 + 7^2}} = \dfrac{10}{\sqrt{89}}$ OE	**M1A1**
Divides by magnitude of normal vector. 1.06...	
	(2)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 2 \\ -2 & -1 & 0 \end{vmatrix} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$	**M1A1**
Finds normal to the plane OAB.	

$\begin{pmatrix} -2 \\ -6 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} = \sqrt{89}\sqrt{29}\cos\theta$	**M1**
Uses dot product correctly.	
36.2°	**A1**
	(4)
	Total: 11

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Q20

9231/12 • October/November 2020 • Question 7

↑ Back to contents

Mark scheme.

$\overrightarrow{AB} = -\mathbf{j} + 3\mathbf{k} \quad \overrightarrow{AC} = 2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$	**B1**
Finds direction vectors of two lines in the plane. $\overrightarrow{BC} = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$	
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 3 \\ 1 & -2 & 1 \end{vmatrix} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$	**M1A1**
Finds normal to the plane ABC. OE	
$5(0) + 3(-2) + 1(1) = -5 \Rightarrow 5x + 3y + z = -5$	**M1A1**
Substitutes point. OE	
	(5)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 2 \\ 0 & -2 & 1 \end{vmatrix} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}$	**M1A1**
Finds normal to the plane OBC.	
$\begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix} = \sqrt{35}\sqrt{45}\cos\theta$	**M1**
Uses dot product of normal vectors.	
$28.1°$	**A1**
$0.49(0)$ rad	
	(4)

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 3 \\ t & 1 & -1 \end{vmatrix} = \begin{pmatrix} -2 \\ 3t \\ t \end{pmatrix}$	**M1A1**
Finds common perpendicular.	
$\left\lvert \frac{1}{\sqrt{4+10t^2}} \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3t \\ t \end{pmatrix} \right\rvert = \left\lvert \frac{-4-10t}{\sqrt{4+10t^2}} \right\rvert$	**M1A1**
Uses formula for perpendicular distance.	
$\left\lvert \frac{-4-10t}{\sqrt{4+10t^2}} \right\rvert = \sqrt{10} \Rightarrow (4+10t)^2 = 10(4+10t^2)$	**M1**
Sets equal to $\sqrt{10}$ and solves for t.	
$t = \frac{3}{10}$	**A1**
	(6)
	Total: 15

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Q21

9231/11, 9231/12 • May/June 2021 • Question 6

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Mark scheme.

$\begin{pmatrix}0\\1\\t\end{pmatrix} - \begin{pmatrix}t\\1\\0\end{pmatrix} = t\begin{pmatrix}-1\\0\\1\end{pmatrix}$	**B1**
$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -2 & -1 & 0\\ 0 & -2 & 1\end{vmatrix} = \begin{pmatrix}-1\\2\\4\end{pmatrix}$	**M1 A1**
$\frac{t}{\sqrt{21}}\begin{pmatrix}-1\\0\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\2\\4\end{pmatrix} = \sqrt{21} \Rightarrow t = \frac{21}{5} = 4.2$	**M1 A1**
	(5)
$\mathbf{r} = \frac{21}{5}\mathbf{i} + \mathbf{j} + \lambda(2\mathbf{i}+\mathbf{j}) + \mu(-2\mathbf{j}+\mathbf{k})$ (Using their value of t)	**B1 FT**
	(1)
$\begin{pmatrix}0\\-2\\1\end{pmatrix}\cdot\begin{pmatrix}5\\-6\\7\end{pmatrix} = \sqrt{5}\sqrt{110}\cos\alpha$ leading to $\cos\alpha = \frac{19}{\sqrt{5}\sqrt{110}}$	**M1 A1**
Acute angle between l_2 and Π_2 is $90 - \alpha = 54.1$	**A1**
	(3)
$\begin{pmatrix}-1\\2\\4\end{pmatrix}\cdot\begin{pmatrix}5\\-6\\7\end{pmatrix} = \sqrt{21}\sqrt{110}\cos\alpha$ leading to $\cos\alpha = \frac{11}{\sqrt{21}\sqrt{110}}$ (Dot product using their normal to Π_1)	**M1 A1 FT**
76.8	**A1**
	(3)

Total: 12

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Q22

9231/13 • May/June 2021 • Question 6

↑ Back to contents

Mark scheme.

$-(-2\mathbf{i}-2\mathbf{j}+4\mathbf{k})+(-\mathbf{i}-2\mathbf{j}+\mathbf{k})=\mathbf{i}-3\mathbf{k}$ (OE. Finds direction vector from P to a point of l_1.)	**B1**
$\mathbf{r}=-2\mathbf{i}-2\mathbf{j}+4\mathbf{k}+\lambda(2\mathbf{i}-3\mathbf{j})+\mu(\mathbf{i}-3\mathbf{k})$ or $\mathbf{r}=-\mathbf{i}-2\mathbf{j}+\mathbf{k}+\lambda(2\mathbf{i}-3\mathbf{j})+\mu(\mathbf{i}-3\mathbf{k})$ (OE. FT their $\mathbf{i}-3\mathbf{k}$)	**B1 FT**
	(2)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 0 \\ 3 & -1 & 3 \end{vmatrix} = \begin{pmatrix} -9 \\ -6 \\ 7 \end{pmatrix}$ (OE. Finds vector perpendicular to Π_2.)	**M1 A1**
$-9(3)-6(0)+7(-2)=-41$ (Uses point on Π_2.)	**M1**
$9x+6y-7z=41$	**A1**
	(4)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 0 \\ 1 & 0 & -3 \end{vmatrix} = \begin{pmatrix} 9 \\ 6 \\ 3 \end{pmatrix} \sim \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ (Finds vector perpendicular to Π_1.)	**M1 A1**
$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ 6 \\ -7 \end{pmatrix} = \sqrt{14}\sqrt{166}\cos\alpha$ leading to $\cos\alpha = \dfrac{32}{\sqrt{14}\sqrt{166}}$ (Dot product using their normal vectors.)	**DM1 A1 FT**
48.4 (0.845 rad)	**A1**
	(5)
$\overrightarrow{OF}=\overrightarrow{OQ}+\overrightarrow{QF}=-5\begin{pmatrix} -2 \\ -2 \\ 4 \end{pmatrix}+t\begin{pmatrix} 9 \\ 6 \\ -7 \end{pmatrix}=\begin{pmatrix} 10+9t \\ 10+6t \\ -20-7t \end{pmatrix}$ (Scales $\overrightarrow{OP}$ by a factor of ± 5 and uses multiple of their normal to Π_2.)	**M1 A1 FT**

$9(10+9t)+6(10+6t)-7(-20-7t)=41$ leading to $290+166t=41$ (Substitutes into the equation of Π_2.)	**DM1**
$t=-\dfrac{3}{2}$ leading to $\overrightarrow{OF}=\begin{pmatrix}-\frac{7}{2}\\ 1\\ -\frac{19}{2}\end{pmatrix}$	**A1**
	(4)
	Total: 15

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Q23

9231/11, 9231/13 • October/November 2021 • Question 5

↑ Back to contents

Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 2 & 3 & 0 \end{vmatrix} = \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix}$	**M1A1**
Finds vector perpendicular to Π.	
$-3(-2) + 2(3) + 3(3) = 21$	**M1**
Substitutes point on Π.	
$-3x + 2y + 3z = 21$	**A1**
	(4)
$\begin{pmatrix} 2 \\ -3 \\ 5+t \end{pmatrix}$	**B1**
Forms general point on line (given as a single vector).	
$-3(2) + 2(-3) + 3(5+t) = 21$ leading to $t = 6$	**M1**
Substitutes into the equation for Π.	
$\begin{pmatrix} 2 \\ -3 \\ 11 \end{pmatrix}$	**A1**
	(3)
$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix} = \sqrt{1}\sqrt{22}\cos\alpha$ leading to $\cos\alpha = \dfrac{3}{\sqrt{22}}$	**M1** **A1FT**
Uses dot product of $\mathbf{k}$ and their normal.	

Acute angle between l and Π is $90 - \alpha = 39.8°$	**A1**
	(3)
$\begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$	**B1**
Finds direction vector from P to plane.	
$\frac{1}{\sqrt{22}} \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix} = \frac{18}{\sqrt{22}} = 3.84$	**M1A1**
Uses dot product of their direction and normal vectors.	
	(3)
	Total: 13

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Q24

9231/12 • October/November 2021 • Question 7

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Mark scheme.

$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 0 \\ 0 & -1 & 1 \end{vmatrix} = \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix} \sim \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$	**M1A1**
Finds common perpendicular.	
$\mathbf{r}.\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 0$	**A1**
	(3)
$\dfrac{1}{\sqrt{1^2+3^2+2^2}} = \dfrac{1}{\sqrt{14}}$	**B1**
Divides by magnitude of the normal to Π. 0.267	
	(1)
$\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} = \sqrt{3}\sqrt{14}\cos\alpha$ leading to $\cos\alpha = \dfrac{6}{\sqrt{3}\sqrt{14}}$	**M1** **A1FT**
Takes dot product of normal vectors.	
$22.2°$	**A1**
Accept 0.388 radians. Mark final answer.	
	(3)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -7 \\ -2 & -3 & 1 \end{vmatrix} = \begin{pmatrix} -20 \\ 12 \\ -4 \end{pmatrix} \sim \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$	**M1A1**
Finds direction of common perpendicular.	

$\overrightarrow{OP} = \lambda \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 2-2\mu \\ 2-3\mu \\ \mu \end{pmatrix}$ leading to $\overrightarrow{PQ} = \begin{pmatrix} 2-2\mu-2\lambda \\ 2-3\mu-\lambda \\ \mu+7\lambda \end{pmatrix}$	**M1A1**
Finds $\overrightarrow{PQ}$.	
$\begin{pmatrix} 2-2\mu-2\lambda \\ 2-3\mu-\lambda \\ \mu+7\lambda \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = 0$ or $\begin{pmatrix} 2-2\mu-2\lambda \\ 2-3\mu-\lambda \\ \mu+7\lambda \end{pmatrix} = k \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$	**M1**
Uses that dot product of $\overrightarrow{PQ}$ with line direction is zero, or, alternatively, $\overrightarrow{PQ}$ is a multiple of the common perpendicular (parameter k not 1).	
$14\mu + 14\lambda = 10$	**A1**
Deduces one equation.	
$\begin{pmatrix} 2-2\mu-2\lambda \\ 2-3\mu-\lambda \\ \mu+7\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix} = 0 \Rightarrow 14\mu + 54\lambda = 6$	**A1**
Deduces second equation.	
$\lambda = -\frac{1}{10}$ leading to $\overrightarrow{OP} = -\frac{1}{10} \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix}$	**M1A1**
Solves for λ and substitutes into $\overrightarrow{OP}$.	
$\mathbf{r} = \begin{pmatrix} -0.2 \\ -0.1 \\ 0.7 \end{pmatrix} + k \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$	**B1FT**
FT using their common perpendicular.	
	(10)
	Total: 17

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Q25

9231/11, 9231/12 • May/June 2022 • Question 2

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Mark scheme.

$\overrightarrow{AB} = -8\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} \quad \overrightarrow{AC} = 3\mathbf{j} - 3\mathbf{k}$	**B1**
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{vmatrix} = \begin{pmatrix} -6 \\ -24 \\ -24 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix}$	**M1 A1**
$(4) + 4(-4) + 4(1) = -8 \Rightarrow x + 4y + 4z = -8$	**M1 A1**
	(5)
$\dfrac{8}{\sqrt{1^2 + 4^2 + 4^2}} = 1.39$	**M1 A1**
	(2)
$\mathbf{r} = t\begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$	**B1**
$2t + 12t - 12t = -8$	**M1**
$(-8, -12, 12)$	**A1**
	(3)
	Total: 10

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Q26

9231/13 • May/June 2022 • Question 7

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Mark scheme.

$\overrightarrow{AB} = 4\mathbf{i} - \mathbf{j} + \mathbf{k} \quad \overrightarrow{CD} = \mathbf{j} + (\lambda - 3)\mathbf{k}$	**B1**
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 1 \\ 0 & 1 & \lambda - 3 \end{vmatrix} = \begin{pmatrix} 2-\lambda \\ 12-4\lambda \\ 4 \end{pmatrix}$	**M1 A1**
$\left\lvert \dfrac{1}{\sqrt{(\lambda-2)^2 + (4\lambda-12)^2 + 16}} \begin{pmatrix} -5 \\ 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2-\lambda \\ 12-4\lambda \\ 4 \end{pmatrix} \right\rvert$	**M1 A1**
$\left\lvert \dfrac{30-3\lambda}{\sqrt{17\lambda^2 - 100\lambda + 164}} \right\rvert = 3 \Rightarrow 9(\lambda - 10)^2 = 9(17\lambda^2 - 100\lambda + 164)$	**M1**
$16\lambda^2 - 80\lambda + 64 = 0$ leading to $\lambda^2 - 5\lambda + 4 = 0$	**A1**
	(7)
$\mathbf{r} = 7\mathbf{i} + 4\mathbf{j} - \mathbf{k} + s(4\mathbf{i} - \mathbf{j} + \mathbf{k}) + t(-5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$	**M1 A1**
	(2)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 1 \\ -5 & 3 & 5 \end{vmatrix} = \begin{pmatrix} -8 \\ -25 \\ 7 \end{pmatrix}$	**M1 A1**
$-8(7) - 25(4) + 7(-1)$ leading to $-8x - 25y + 7z = -163$	**M1 A1**
	(4)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 1 \\ -5 & 3 & 2 \end{vmatrix} = \begin{pmatrix} -5 \\ -13 \\ 7 \end{pmatrix}$	**M1 A1**
$\begin{pmatrix} -5 \\ -13 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} -8 \\ -25 \\ 7 \end{pmatrix} = \sqrt{243}\sqrt{738}\cos\theta$ leading to $\cos\theta = \dfrac{414}{\sqrt{243}\sqrt{738}}$	**M1 A1**

12.1	**A1**
	(5)
	Total: 18

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Q27

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Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = \begin{pmatrix} -4 \\ 2 \\ -8 \end{pmatrix} \sim \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$	**M1A1**
$2(3) - (-2) + 4(1) = 12$	**M1**
$2x - y + 4z = 12$	**A1**
	(4)
$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = \sqrt{2}\sqrt{21}\cos\alpha$ leading to $\cos\alpha = \frac{3}{\sqrt{42}}$	**M1A1FT**
Acute angle between l and Π is $90 - \alpha = 27.6°$	**A1**
	(3)
$\overrightarrow{OF} = \overrightarrow{OP} + t\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2+2t \\ 3-t \\ 1+4t \end{pmatrix}$	**M1A1**
$21t + 5 = 12$	**M1**
$\overrightarrow{OF} = \frac{1}{3}\begin{pmatrix} 8 \\ 8 \\ 7 \end{pmatrix}$	**A1**
	(4)
	Total: 11

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Q28

9231/12 • October/November 2022 • Question 6

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Mark scheme.

$\begin{pmatrix}0\\2\\6\end{pmatrix} - \begin{pmatrix}2\\0\\1\end{pmatrix} = \begin{pmatrix}-2\\2\\5\end{pmatrix}$	**B1**
$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & 2 & -2\\1 & -1 & 2\end{vmatrix} = \begin{pmatrix}2\\-4\\-3\end{pmatrix}$	**M1A1**
$\frac{1}{\sqrt{29}}\begin{pmatrix}-2\\2\\5\end{pmatrix} \cdot \begin{pmatrix}2\\-4\\-3\end{pmatrix} = \frac{27}{\sqrt{29}} = 5.01$	**M1A1**
	(5)
$\mathbf{r} = 2\mathbf{i} + \mathbf{k} + s(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t(2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k})$	**B1**
	(1)
$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & 2 & -2\\2 & -4 & -3\end{vmatrix} = \begin{pmatrix}-14\\-1\\-8\end{pmatrix}$	**M1A1FT**
$14(0) + (2) + 8(6) = 50 \Rightarrow 14x + y + 8z = 50$	**M1A1**
	(4)
$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & -1 & 2\\2 & -4 & -3\end{vmatrix} = \begin{pmatrix}11\\7\\-2\end{pmatrix}$	**M1A1FT**
$\begin{pmatrix}11\\7\\-2\end{pmatrix} \cdot \begin{pmatrix}14\\1\\8\end{pmatrix} = \sqrt{174}\sqrt{261}\cos\theta$ leading to $\cos\theta = \frac{145}{\sqrt{174}\sqrt{261}}$	**M1A1**

$47.1°$	**A1**
	(5)
	Total: 15

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Q29

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Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, finds common perpendicular.	**M1A1**
$(-4) + (-3) = -7 \Rightarrow y + z = -7$, substitutes point.	**M1A1**
	(4)
States point common to both planes e.g. $\begin{pmatrix} -7 \\ -2 \\ -5 \end{pmatrix}$.	**B1FT**
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 1 \\ -5 & 3 & 5 \end{vmatrix} = \begin{pmatrix} 2 \\ -5 \\ 5 \end{pmatrix}$, finds direction of line.	**M1A1FT**
$\mathbf{r} = \begin{pmatrix} -7 \\ -2 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -5 \\ 5 \end{pmatrix}$	**A1**
	(4)
$\left\lvert \dfrac{a + a - 7 + 7}{\sqrt{2}} \right\rvert = \sqrt{2}$, uses correct formula for distance from A to Π_1.	**M1**
$a = 1$	**A1**
	(2)
$\left\lvert \dfrac{b + b}{\sqrt{2}\sqrt{(1-b)^2 + 2b^2}} \right\rvert = \frac{1}{2}\sqrt{2}$, uses correct formula.	**M1A1**
$2b = \sqrt{(1-b)^2 + 2b^2} \Rightarrow b^2 + 2b - 1 = 0$, solves for b.	**M1**
$b = -1 + \sqrt{2}$	**A1**
	(4)

Total: 14

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Q30

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Mark scheme.

$\overrightarrow{AB} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $\overrightarrow{AC} = -3\mathbf{i} + 3\mathbf{k}$, $\overrightarrow{BC} = -\mathbf{i} - \mathbf{j} - \mathbf{k}$, finds direction vectors of two lines in the plane.	**B1**
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 4 \\ -3 & 0 & 3 \end{vmatrix} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix} \sim \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, finds normal to the plane ABC.	**M1A1FT**
$1(1) - 2(1) + 1(0) = -1 \Rightarrow x - 2y + z = -1$, substitutes point. CAO	**M1A1**
	(5)
$\frac{1}{\sqrt{1^2 + 2^2 + 1^2}} = \frac{1}{\sqrt{6}} = 0.408$, divides by magnitude of normal vector. FT their (a).	**M1A1FT**
	(2)
$\overrightarrow{OP} = \begin{pmatrix} -2\lambda \\ \lambda \\ 3\lambda \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 1-2\mu \\ 1+\mu \\ 4\mu \end{pmatrix} \Rightarrow \overrightarrow{PQ} = \begin{pmatrix} 1-2\mu+2\lambda \\ 1+\mu-\lambda \\ 4\mu-3\lambda \end{pmatrix}$, finds $\overrightarrow{PQ}$, where P is a point on OC and Q is a point on AB.	**M1A1**
$\begin{pmatrix} 1-2\mu+2\lambda \\ 1+\mu-\lambda \\ 4\mu-3\lambda \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} = 0$, uses that dot product of $\overrightarrow{PQ}$ with line direction is zero.	**M1***
$17\mu - 14\lambda = 1$, deduces one equation.	**dM1**
$\begin{pmatrix} 1-2\mu+2\lambda \\ 1+\mu-\lambda \\ 4\mu-3\lambda \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} = 0 \Rightarrow 21\mu - 17\lambda = 1$, deduces second equation.	**dM1**

$\lambda = -\frac{4}{5} \Rightarrow \overrightarrow{OP} = -\frac{4}{5}\begin{pmatrix}-2\\1\\3\end{pmatrix}$, solves for λ or μ and substitutes into $\overrightarrow{OP}$.	**dM1A1**
$\mathbf{r} = -\frac{4}{5}\begin{pmatrix}-2\\1\\3\end{pmatrix} + k\begin{pmatrix}1\\2\\0\end{pmatrix}$	**A1**
	(8)
	Total: 15

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Q31

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Mark scheme.

$\begin{pmatrix}2\\-2\\3\end{pmatrix}-\begin{pmatrix}-2\\-3\\-5\end{pmatrix}=\begin{pmatrix}4\\1\\8\end{pmatrix}$	**B1**
$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-4&3&5\\2&-3&1\end{vmatrix}=\begin{pmatrix}18\\14\\6\end{pmatrix}\sim\begin{pmatrix}9\\7\\3\end{pmatrix}$	**M1A1**
$\frac{1}{\sqrt{139}}\begin{pmatrix}4\\1\\8\end{pmatrix}\cdot\begin{pmatrix}9\\7\\3\end{pmatrix}=\frac{67}{\sqrt{139}}(=5.68)$ ((a) Total: 5)	**M1A1**
$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&0&1\\-4&3&5\end{vmatrix}=\begin{pmatrix}9\\-9\\3\end{pmatrix}\sim\begin{pmatrix}3\\-1\\-1\end{pmatrix}$	**M1A1**
$1(-1)+3(-3)-1(-4)=-6\Rightarrow x+3y-z=-6$ ((b) Total: 4)	**M1A1**
	Total: 9

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Q32

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Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & -3 \\ 3 & 0 & -1 \end{vmatrix} = \begin{pmatrix} 2 \\ -8 \\ 6 \end{pmatrix} \sim \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$	**M1A1**
$1(1) - 4(-1) + 3(-2) = -1$	**M1**
$x - 4y + 3z = -1$ ((a) Total: 4)	**A1**
$\begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 1 - 4 + 3 = 0$ ((b) Total: 2)	**M1A1**
$\frac{1}{\sqrt{1^2+4^2+3^2}} \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$ or $\frac{1}{\sqrt{1^2+4^2+3^2}} \left(\begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix} + 1 \right)$	**M1A1**
$\frac{1}{\sqrt{26}} (= 0.196)$ ((c) Total: 3)	**A1**
States point common to both planes e.g. $\begin{pmatrix} \frac{1}{15} \\ \frac{4}{15} \\ 0 \end{pmatrix}$.	**B1**
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 3 \\ 3 & 3 & 2 \end{vmatrix} = \begin{pmatrix} -17 \\ 7 \\ 15 \end{pmatrix}$	**M1A1**
$\mathbf{r} = \begin{pmatrix} \frac{5}{7} \\ 0 \\ -\frac{4}{7} \end{pmatrix} + \lambda \begin{pmatrix} -17 \\ 7 \\ 15 \end{pmatrix}$ ((d) Total: 4)	**A1**

Total: 13

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Q33

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Mark scheme.

$\overrightarrow{AB} = 2\mathbf{j} - 5\mathbf{k} \quad \overrightarrow{AC} = -5\mathbf{i} - 5\mathbf{j} \quad \overrightarrow{BC} = -5\mathbf{i} - 7\mathbf{j} + 5\mathbf{k}$	**B1**
Finds direction vectors of two lines in the plane.	
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & -5 \\ -5 & -5 & 0 \end{vmatrix} = \begin{pmatrix} -25 \\ 25 \\ 10 \end{pmatrix} \sim \begin{pmatrix} -5 \\ 5 \\ 2 \end{pmatrix}$	**M1 A1**
Finds normal to the plane ABC.	
$-5(2) + 5(2) + 2(4) = 8 \Rightarrow -5x + 5y + 2z = 8$	**M1 A1**
Substitutes point. AEF for final answer.	
	(5)
$\frac{8 - (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \cdot (-5\mathbf{i} + 5\mathbf{j} + 2\mathbf{k})}{\sqrt{5^2 + 5^2 + 2^2}} = \frac{7}{\sqrt{54}} = 0.953$	**M1 A1 Correct formula for distance from point to plane.**
	(2)
$\overrightarrow{CD} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ -1 \end{pmatrix}$	**B1**
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & -5 \\ 5 & 4 & -1 \end{vmatrix} = \begin{pmatrix} 18 \\ -25 \\ -10 \end{pmatrix}$	**M1 A1**
Find common perpendicular.	

$\frac{1}{\sqrt{1049}}\begin{pmatrix}-5\\-5\\0\end{pmatrix}\cdot\begin{pmatrix}18\\-25\\-10\end{pmatrix}=\frac{35}{\sqrt{1049}}=1.08$	**M1 A1** **Uses formula for shortest distance.**
	(5)
	Total: 12

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Q34

9231/13 • May/June 2024 • Question 5

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Mark scheme.

$\begin{pmatrix}-3\\4\\0\end{pmatrix}-\begin{pmatrix}1\\4\\-1\end{pmatrix}=\begin{pmatrix}-4\\0\\1\end{pmatrix}$	**B1**
OE. Finds direction of one line to another.	
$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\0&1&-2\\1&2&1\end{vmatrix}=\begin{pmatrix}5\\-2\\-1\end{pmatrix}$	**M1 A1**
Find common perpendicular.	
$\frac{1}{\sqrt{30}}\begin{pmatrix}-4\\0\\1\end{pmatrix}\cdot\begin{pmatrix}5\\-2\\-1\end{pmatrix}=\frac{21}{\sqrt{30}}(=3.83)$	**M1 A1 Uses formula for shortest distance.**
	(5)
$5(1)-2(4)-1(-1)=-2\Rightarrow 5x-2y-z=-2$	**M1**
Uses point in the plane.	
A1 FT	
Follow through their normal in part (a)	
	(2)
$-\mathbf{i}-\mathbf{j}+\mathbf{k}$	**B1**
Finding direction between the two given points.	

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 1 \\ 1 & 4 & -3 \end{vmatrix} = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$	**M1**
Find common perpendicular. Allow use of their normal from part (b).	
$x + 2y + 3z = 6$	**A1**
OE	
	(3)
	Total: 10

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Q35

9231/12 • October/November 2024 • Question 2

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Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & -4 \\ 2 & 5 & -4 \end{vmatrix} = \begin{pmatrix} 24 \\ -4 \\ 7 \end{pmatrix}$	**M1A1**
$24(1) - 4(3) + 7(-1) = d \Rightarrow 24x - 4y + 7z = 5$ ((a) Total: 4)	**M1A1**
$\begin{pmatrix} 24 \\ -4 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -5 \\ -2 \end{pmatrix} = \sqrt{641}\sqrt{54}\cos\alpha \Rightarrow \cos\alpha = \dfrac{126}{\sqrt{641}\sqrt{54}}$	**M1A1FT**
Acute angle between l_2 and Π is $90 - \alpha = 42.6°$ ((b) Total: 3)	**A1**
	Total: 7

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Q36

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Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 1 \\ 1 & -4 & 2 \end{vmatrix} = \begin{pmatrix} 6 \\ -3 \\ -9 \end{pmatrix} \sim \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$	**M1A1**
$-2(1) + (3) + 3(-2) = -5$	**M1**
$2x - y - 3z = 5$ ((a) Total: 4)	**A1**
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 2 \\ 1 & 1 & -2 \end{vmatrix} = \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix}$	**M1A1**
$\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix} = \sqrt{14}\sqrt{77}\cos\theta \Rightarrow \cos\theta = \frac{-7}{\sqrt{14}\sqrt{77}}$	**M1**
$77.7°$ ((b) Total: 4)	**A1**
$\overrightarrow{OP} = \begin{pmatrix} 1+2\lambda \\ 3+\lambda \\ -2+\lambda \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 1+\mu \\ -2-4\mu \\ 9+2\mu \end{pmatrix} \Rightarrow \overrightarrow{PQ} = \begin{pmatrix} \mu-2\lambda \\ -5-4\mu-\lambda \\ 11+2\mu-\lambda \end{pmatrix}$	**M1A1**
$\begin{pmatrix} \mu-2\lambda \\ -5-4\mu-\lambda \\ 11+2\mu-\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0$ or $\begin{pmatrix} \mu-2\lambda \\ -5-4\mu-\lambda \\ 11+2\mu-\lambda \end{pmatrix} = k\begin{pmatrix} 6 \\ -3 \\ -9 \end{pmatrix}$	**M1**
$-6\lambda + 6 = 0$	**A1**
$\begin{pmatrix} \mu-2\lambda \\ -5-4\mu-\lambda \\ 11+2\mu-\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} = 0 \Rightarrow 21\mu + 42 = 0$	**A1**

$\lambda = 1 \Rightarrow \overrightarrow{OP} = \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}$ or $\mu = -2 \Rightarrow \overrightarrow{OQ} = \begin{pmatrix} -1 \\ 6 \\ 5 \end{pmatrix}$	**M1**
$\mathbf{r} = \begin{pmatrix} -1 \\ 6 \\ 5 \end{pmatrix} + t \begin{pmatrix} 6 \\ -3 \\ -9 \end{pmatrix}$ ((c) Total: 7)	**A1FT**
	Total: 15

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Q37

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Mark scheme.

$\overrightarrow{AB} = 2\mathbf{j} + 4\mathbf{k} \quad \overrightarrow{AC} = \mathbf{i} - \mathbf{j} + \mathbf{k}$	**B1**
Finds direction vectors of two lines in the plane.	
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ 1 & -1 & 1 \end{vmatrix} = \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix} \sim \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$	**M1A1**
Finds normal to the plane ABC.	
$3(1) + 2(0) - (-2) = 5 \Rightarrow 3x + 2y - z = 5$	**M1A1**
Substitutes point.	
	(5)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ 0 & 0 & t+2 \end{vmatrix} = \begin{pmatrix} 2t+4 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	**M1A1**
Finds normal to the plane ABD.	
$\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \sqrt{14}\cos\theta$	**M1**
Uses dot product of normal vectors in correct formula.	
$(t \neq -2) \Rightarrow \theta = 36.7°$	**A1**
0.641 rad.	
	(4)

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ -1 & 1 & t+1 \end{vmatrix} = \begin{pmatrix} 2t-2 \\ -4 \\ 2 \end{pmatrix} \sim \begin{pmatrix} t-1 \\ -2 \\ 1 \end{pmatrix}$	**M1A1**

Finds common perpendicular.

$\left\lvert \dfrac{1}{\sqrt{t^2-2t+6}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} t-1 \\ -2 \\ 1 \end{pmatrix} \right\rvert = \left\lvert \dfrac{t+2}{\sqrt{t^2-2t+6}} \right\rvert$	**M1A1**

Uses formula for perpendicular distance.

$\left\lvert \dfrac{t+2}{\sqrt{t^2-2t+6}} \right\rvert = \sqrt{2} \Rightarrow (t+2)^2 = 2(t^2-2t+6)$	**M1**

Sets equal to $\sqrt{2}$ and solves for t.

$t^2 - 8t + 8 = 0 \Rightarrow t = 2(2+\sqrt{2}),\ t = 2(2-\sqrt{2})$	**A1A1**
$t = 6.83,\ t = 1.17.$	
	(7)
	Total: 16

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Q38

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Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & -1 \\ 3 & 2 & -2 \end{vmatrix} = \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix}$	**M1A1**
Finds vector perpendicular to Π.	
$6(2) - (3) + 8(-2) = -7$	**M1**
Substitutes point on Π.	
$6x - y + 8z = -7$	**A1**
	(4)
$\overrightarrow{OF} = \overrightarrow{OP} + \overrightarrow{PF} = \begin{pmatrix} 4 \\ 2 \\ 9 \end{pmatrix} + t\begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix} = \begin{pmatrix} 4+6t \\ 2-t \\ 9+8t \end{pmatrix}$	**M1A1FT**
Uses $\overrightarrow{OP}$ and multiple of their normal to Π.	
$6(4+6t) - (2-t) + 8(9+8t) = -7 \Rightarrow 101t + 94 = -7$	**M1**
Substitutes into the equation for Π.	
$t = -1 \Rightarrow \overrightarrow{OF} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$	**A1**
	(4)
$\begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix} = \sqrt{35}\sqrt{101}\cos\alpha \Rightarrow \cos\alpha = \dfrac{5}{\sqrt{35}\sqrt{101}}$	**M1A1FT**
Uses dot product of $3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ and their normal in a correct formula.	

Acute angle between l and Π is $90 - \alpha = 4.8°$	**A1**
Mark final answer. No ISW	
	(3)
	Total: 11

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Q39

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Mark scheme.

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 2 \\ 1 & 3 & 1 \end{vmatrix} = \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix} \sim \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$	**M1A1**
Finds perpendicular to Π_1.	
$5(-3) - 2(-1) + (-1) = -14$	**M1**
Uses point on Π_1.	
$5x - 2y + z = -14$	**A1**
	(4)
$\frac{1}{\sqrt{5^2+2^2+1^2}} \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix}$	**M1A1**
Uses correct formula for perpendicular distance from point to Π_1.	
$\frac{7}{\sqrt{30}}$	**A1**
Accept 1.28.	
	(3)
States point common to both planes e.g. $\begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix}$.	**B1**
Or $\begin{pmatrix} 0 \\ 0 \\ -14 \end{pmatrix}$.	

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = \begin{pmatrix} 0 \\ 8 \\ 16 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$	**M1A1**
Finds direction of line.	
$\mathbf{r} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$	**A1**
OE	
	(4)
	Total: 11

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Q40

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Mark scheme.

$\dfrac{1}{\sqrt{1^2+3^2+2^2}} = \dfrac{1}{\sqrt{14}}$	**M1A1**
Allow 0.267 or better	
	(2)
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$	**M1A1**
Finds normal to OAB. Allow 1 error for the M mark to be awarded.	
$\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \sqrt{21}\sqrt{14}\cos\alpha \Rightarrow \cos\alpha = \dfrac{17}{\sqrt{21}\sqrt{14}}$	**M1**
Takes dot product of normal vectors.	
$7.5°$	**A1**
Accept 0.131 radians. Mark final answer of $7.5°$ or better.	
	(4)
$\overrightarrow{OP} = \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 2\mu \\ -1+\mu \\ 2-3\mu \end{pmatrix} \Rightarrow \overrightarrow{PQ} = \begin{pmatrix} 2\mu-2\lambda \\ -1+\mu+\lambda \\ 2-3\mu+\lambda \end{pmatrix}$	**M1A1**
Finds $\overrightarrow{PQ}$ or direction of common perpendicular.	
$\begin{pmatrix} 2\mu-2\lambda \\ -1+\mu+\lambda \\ 2-3\mu+\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 0$	**M1**

Uses that dot product of $\overrightarrow{PQ}$ with line direction is zero, or, alternatively, $\overrightarrow{PQ}$ is a multiple of the common perpendicular $\begin{pmatrix}1\\1\\1\end{pmatrix}$.	
$6\mu - 6\lambda = 1$	**A1**
Deduces one equation.	
$\begin{pmatrix}2\mu-2\lambda\\-1+\mu+\lambda\\2-3\mu+\lambda\end{pmatrix}\cdot\begin{pmatrix}2\\1\\-3\end{pmatrix} = 0 \Rightarrow 14\mu - 6\lambda = 7$	**A1**
Deduces second equation.	
$\mu = \frac{3}{4} \Rightarrow \overrightarrow{OQ} = \begin{pmatrix}\frac{3}{2}\\-\frac{1}{4}\\-\frac{1}{4}\end{pmatrix} = \frac{1}{4}\begin{pmatrix}6\\-1\\-1\end{pmatrix}$.	**M1A1**
Solves for μ and substitutes into $\overrightarrow{OQ}$ and $\overrightarrow{PQ}$ or $\lambda = \frac{7}{12} \Rightarrow \overrightarrow{OP} = \frac{7}{12}\begin{pmatrix}2\\-1\\-1\end{pmatrix}$.	
$\mathbf{r} = \frac{1}{4}\begin{pmatrix}6\\-1\\-1\end{pmatrix} + k\begin{pmatrix}1\\1\\1\end{pmatrix}$	**B1FT**
OE. FT using their common perpendicular. Must have $\mathbf{r} =$	
	(8)
	Total: 14

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Q41

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Mark scheme.

$\overrightarrow{AB} = -\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}, \quad \overrightarrow{AC} = -\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$	**B1**
Finds direction vectors of two lines in the plane.	
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & -3 \\ -1 & -4 & -6 \end{vmatrix} = \begin{pmatrix} 6 \\ -3 \\ 1 \end{pmatrix}$	**M1A1**
Finds normal to the plane ABC.	
$6(3) - 3(5) + (5) = 8 \Rightarrow 6x - 3y + z = 8$	**M1A1**
Substitutes point. CAO	
	(5)
$\overrightarrow{CD} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix}$	**B1**
$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & -3 \\ -1 & -2 & 4 \end{vmatrix} = \begin{pmatrix} -18 \\ 7 \\ -1 \end{pmatrix}$	**M1A1**
Find common perpendicular.	
$\frac{1}{\sqrt{374}} \left\| \begin{pmatrix} -1 \\ -4 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} -18 \\ 7 \\ -1 \end{pmatrix} \right\| = \frac{4}{\sqrt{374}} = 0.207$	**M1A1**
Uses formula for shortest distance.	
	(5)
	Total: 10

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