Further Math with Ali Hashir

Summation of Series

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics Paper 1 past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/11, 9231/12 • May/June 2015 • Question 1

Use the List of Formulae (MF10) to show that $\sum_{r=1}^{13}(3r^2 - 5r + 1)$ and $\sum_{r=0}^{9}(r^3 - 1)$ have the same numerical value. [4]

Q2

9231/13 • May/June 2015 • Question 4

Use the formula for $\tan(A - B)$ in the List of Formulae (MF10) to show that

$$\tan^{-1}(x+1) - \tan^{-1}(x-1) = \tan^{-1}\left(\frac{2}{x^2}\right).$$

[3]

Deduce the sum to n terms of the series

$$\tan^{-1}\left(\frac{2}{1^2}\right) + \tan^{-1}\left(\frac{2}{2^2}\right) + \tan^{-1}\left(\frac{2}{3^2}\right) + \dots .$$

[4]

Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 4

The sequence $a_1, a_2, a_3, \ldots$ is such that, for all positive integers n,

$$a_n = \frac{n+5}{\sqrt{(n^2-n+1)}} - \frac{n+6}{\sqrt{(n^2+n+1)}}.$$

The sum $\sum_{n=1}^{N} a_n$ is denoted by S_N. Find

(i) the value of S_{30} correct to 3 decimal places, [3]

(ii) the least value of N for which $S_N > 4.9$. [4]

Q4

9231/11, 9231/12 • May/June 2016 • Question 2

Express $\dfrac{4}{r(r+1)(r+2)}$ in partial fractions and hence find $\displaystyle\sum_{r=1}^{n} \frac{4}{r(r+1)(r+2)}$. [5]

Deduce the value of $\displaystyle\sum_{r=1}^{\infty} \frac{4}{r(r+1)(r+2)}$. [1]

Q5

9231/13 • May/June 2016 • Question 1

Verify that $\dfrac{1}{(3r+1)(3r+4)} = \dfrac{1}{3}\left(\dfrac{1}{3r+1} - \dfrac{1}{3r+4}\right)$. [1]

Let S_N denote $\displaystyle\sum_{r=1}^{N} \frac{1}{(3r+1)(3r+4)}$ and let S denote $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(3r+1)(3r+4)}$. Find the least value of N such that $S - S_N < \dfrac{1}{10000}$. [5]

Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 1

Use the method of differences to find $\sum_{r=1}^{n} \frac{1}{(2r)^2 - 1}$. [4]

Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(2r)^2 - 1}$. [1]

Q7

9231/13 • May/June 2017 • Question 2

(i) Verify that $\dfrac{2r+1}{r(r+1)(r+2)} = \dfrac{1}{2}\left\{\dfrac{(2r+1)(2r+3)}{(r+1)(r+2)} - \dfrac{(2r-1)(2r+1)}{r(r+1)}\right\}$. [2]

(ii) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{2r+1}{r(r+1)(r+2)} = \frac{1}{2}\left\{\frac{(2n+1)(2n+3)}{(n+1)(n+2)} - \frac{3}{2}\right\}$. [2]

(iii) Deduce the value of $\displaystyle\sum_{r=1}^{\infty} \frac{2r+1}{r(r+1)(r+2)}$. [2]

Q8

9231/11, 9231/12 • May/June 2017 • Question 1

It is given that $\sum_{r=1}^{n} u_r = n^2(2n+3)$, where n is a positive integer.

(i) Find $\sum_{r=n+1}^{2n} u_r$. [2]

(ii) Find u_r. [3]

Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 1

Find $\sum_{r=1}^{n}(4r-3)(4r+1)$, giving your answer in its simplest form. [4]

Q10

9231/11, 9231/12 • May/June 2018 • Question 5

Let $S_n = \sum_{r=1}^{n} (-1)^{r-1} r^2$.

(i) Use the standard result for $\sum_{r=1}^{n} r^2$ given in the List of Formulae (MF10) to show that

$$S_{2n} = -n(2n+1).$$

[4]

(ii) State the value of $\lim_{n\to\infty} \frac{S_{2n}}{n^2}$ and find $\lim_{n\to\infty} \frac{S_{2n+1}}{n^2}$. [4]

Q11

9231/13 • May/June 2018 • Question 2

(i) Verify that

$$\frac{n(\mathrm{e}-1)+\mathrm{e}}{n(n+1)\mathrm{e}^{n+1}} = \frac{1}{n\mathrm{e}^n} - \frac{1}{(n+1)\mathrm{e}^{n+1}}.$$

[1]

Let $S_N = \sum_{n=1}^{N} \frac{n(\mathrm{e}-1)+\mathrm{e}}{n(n+1)\mathrm{e}^{n+1}}$.

(ii) Express S_N in terms of N and e. [2]

Let $S = \lim_{N\to\infty} S_N$.

(iii) Find the least value of N such that $(N+1)(S-S_N) < 10^{-3}$. [3]

Q12

9231/11, 9231/13 • October/November 2018 • Question 11 (EITHER)

Answer only **one** of the following two alternatives.

EITHER

(i) By considering $(2r+1)^2 - (2r-1)^2$, use the method of differences to prove that

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1).$$

[3]

(ii) By considering $(2r+1)^4 - (2r-1)^4$, use the method of differences and the result given in part (i) to prove that

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2.$$

[5]

The sums S and T are defined as follows:

$$S = 1^3 + 2^3 + 3^3 + 4^3 + \ldots + (2N)^3 + (2N+1)^3,$$

$$T = 1^3 + 3^3 + 5^3 + 7^3 + \ldots + (2N-1)^3 + (2N+1)^3.$$

(iii) Use the result given in part (ii) to show that $S = (2N+1)^2(N+1)^2$. [1]

(iv) Hence, or otherwise, find an expression in terms of N for T, factorising your answer as far as possible. [2]

(v) Deduce the value of $\dfrac{S}{T}$ as $N \to \infty$. [2]

Q13

9231/12 • October/November 2018 • Question 7

Let

$$S_N = \sum_{r=1}^{N}(3r+1)(3r+4) \quad \text{and} \quad T_N = \sum_{r=1}^{N}\frac{1}{(3r+1)(3r+4)}.$$

(i) Use standard results from the List of Formulae (MF10) to show that

$$S_N = N(3N^2 + 12N + 13).$$

[3]

(ii) Use the method of differences to show that

$$T_N = \frac{1}{12} - \frac{1}{3(3N+4)}.$$

[3]

(iii) Deduce that $\frac{S_N}{T_N}$ is an integer. [2]

(iv) Find $\lim_{N\to\infty} \frac{S_N}{N^3 T_N}$. [2]

Q14

9231/13 • May/June 2019 • Question 4

(i) Use the method of differences to show that $\sum_{r=1}^{N} \frac{1}{(3r+1)(3r-2)} = \frac{1}{3} - \frac{1}{3(3N+1)}$. [4]

(ii) Find the limit, as $N \to \infty$, of $\sum_{r=N+1}^{N^2} \frac{N}{(3r+1)(3r-2)}$. [4]

Q15

9231/11, 9231/12 • May/June 2019 • Question 2

Let $u_n = \dfrac{4\sin\left(n-\frac{1}{2}\right)\sin\frac{1}{2}}{\cos(2n-1)+\cos 1}$.

(i) Using the formulae for $\cos P \pm \cos Q$ given in the List of Formulae MF10, show that

$$u_n = \frac{1}{\cos n} - \frac{1}{\cos(n-1)}.$$

[2]

(ii) Use the method of differences to find $\sum_{n=1}^{N} u_n$. [2]

(iii) Explain why the infinite series $u_1 + u_2 + u_3 + \ldots$ does not converge. [1]

Q16

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 5

Let $S_N = \sum_{r=1}^{N}(5r+1)(5r+6)$ and $T_N = \sum_{r=1}^{N}\frac{1}{(5r+1)(5r+6)}$.

(i) Use standard results from the List of Formulae (MF10) to show that

$$S_N = \tfrac{1}{3}N(25N^2 + 90N + 83).$$

[3]

(ii) Use the method of differences to express T_N in terms of N. [4]

(iii) Find $\lim_{N\to\infty}\left(N^{-3}S_N T_N\right)$. [2]

Q17

9231/11, 9231/12 • May/June 2020 • Question 4

(a) By first expressing $\dfrac{1}{r^2-1}$ in partial fractions, show that

$$\sum_{r=2}^{n} \frac{1}{r^2-1} = \frac{3}{4} - \frac{an+b}{2n(n+1)},$$

where a and b are integers to be found. [5]

(b) Deduce the value of $\displaystyle\sum_{r=2}^{\infty} \frac{1}{r^2-1}$. [1]

(c) Find the limit, as $n \to \infty$, of $\displaystyle\sum_{r=n+1}^{2n} \frac{n}{r^2-1}$. [4]

Q18

9231/13 • May/June 2020 • Question 3

Let $S_n = 2^2 + 6^2 + 10^2 + \ldots + (4n-2)^2$.

(a) Use standard results from the List of Formulae (MF19) to show that $S_n = \frac{4}{3}n(4n^2-1)$. [4]

(b) Express $\frac{n}{S_n}$ in partial fractions and find $\sum_{n=1}^{N} \frac{n}{S_n}$ in terms of N. [4]

(c) Deduce the value of $\sum_{n=1}^{\infty} \frac{n}{S_n}$. [1]

Q19

9231/11, 9231/13 • October/November 2020 • Question 2

(a) Use standard results from the List of Formulae (MF19) to show that

$$\sum_{r=1}^{n}(7r+1)(7r+8) = an^3 + bn^2 + cn,$$

where a, b and c are constants to be determined. [3]

(b) Use the method of differences to find $\displaystyle\sum_{r=1}^{n}\frac{1}{(7r+1)(7r+8)}$ in terms of n. [4]

(c) Deduce the value of $\displaystyle\sum_{r=1}^{\infty}\frac{1}{(7r+1)(7r+8)}$. [1]

Q20

9231/12 • October/November 2020 • Question 3

(a) By simplifying $\left(x^n - \sqrt{x^{2n}+1}\right)\left(x^n + \sqrt{x^{2n}+1}\right)$, show that $\dfrac{1}{x^n - \sqrt{x^{2n}+1}} = -x^n - \sqrt{x^{2n}+1}$. [1]

Let $u_n = x^{n+1} + \sqrt{x^{2n+2}+1} + \dfrac{1}{x^n - \sqrt{x^{2n}+1}}$.

(b) Use the method of differences to find $\displaystyle\sum_{n=1}^{N} u_n$ in terms of N and x. [3]

(c) Deduce the set of values of x for which the infinite series

$$u_1 + u_2 + u_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

Q21

9231/11, 9231/12 • May/June 2021 • Question 2

(a) Use standard results from the List of formulae (MF19) to find $\sum_{r=1}^{n}\left(1-r-r^2\right)$ in terms of n, simplifying your answer. [3]

(b) Show that

$$\frac{1-r-r^2}{(r^2+2r+2)(r^2+1)} = \frac{r+1}{(r+1)^2+1} - \frac{r}{r^2+1}$$

and hence use the method of differences to find $\sum_{r=1}^{n}\frac{1-r-r^2}{(r^2+2r+2)(r^2+1)}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty}\frac{1-r-r^2}{(r^2+2r+2)(r^2+1)}$. [1]

Q22

9231/13 • May/June 2021 • Question 1

(a) Show that
$$\tan(r+1) - \tan r = \frac{\sin 1}{\cos(r+1)\cos r}.$$
[2]

Let $u_r = \dfrac{1}{\cos(r+1)\cos r}$.

(b) Use the method of differences to find $\displaystyle\sum_{r=1}^{n} u_r$. [3]

(c) Explain why the infinite series $u_1 + u_2 + u_3 + \ldots$ does not converge. [1]

Q23

9231/11, 9231/13 • October/November 2021 • Question 2

(a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n} r(r+1)(r+2)$ in terms of n, fully factorising your answer. [3]

(b) Express $\frac{1}{r(r+1)(r+2)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}.$$

[5]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$. [1]

Q24

9231/12 • October/November 2021 • Question 3

Let $S_n = \sum_{r=1}^{n} \ln \frac{r(r+2)}{(r+1)^2}$.

(a) Using the method of differences, or otherwise, show that $S_n = \ln \frac{n+2}{2(n+1)}$. [4]

Let $S = \sum_{r=1}^{\infty} \ln \frac{r(r+2)}{(r+1)^2}$.

(b) Find the least value of n such that $S_n - S < 0.01$. [3]

Q25

9231/11, 9231/12 • May/June 2022 • Question 1

Let a be a positive constant.

(a) Use the method of differences to find $\displaystyle\sum_{r=1}^{n} \frac{1}{(ar+1)(ar+a+1)}$ in terms of n and a. [4]

(b) Find the value of a for which $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(ar+1)(ar+a+1)} = \frac{1}{6}$. [3]

Q26

9231/13 • May/June 2022 • Question 4

Let $u_r = e^{rx}(e^{2x} - 2e^{x} + 1)$.

(a) Using the method of differences, or otherwise, find $\sum_{r=1}^{n} u_r$ in terms of n and x. [3]

(b) Deduce the set of non-zero values of x for which the infinite series

$$u_1 + u_2 + u_3 + \ldots$$

is convergent and give the sum to infinity when this exists. [3]

(c) Using a standard result from the list of formulae (MF19), find $\sum_{r=1}^{n} \ln u_r$ in terms of n and x. [3]

Q27

9231/11, 9231/13 • October/November 2022 • Question 3

(a) By considering $(2r+1)^3 - (2r-1)^3$, use the method of differences to prove that

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1).$$

[5]

Let $S_n = 1^2 + 3 \times 2^2 + 3^2 + 3 \times 4^2 + 5^2 + 3 \times 6^2 + \ldots + \big(2 + (-1)^n\big)n^2$.

(b) Show that $S_{2n} = \frac{1}{3}n(2n+1)(an+b)$, where a and b are integers to be determined. [3]

(c) State the value of $\lim\limits_{n\to\infty} \dfrac{S_{2n}}{n^3}$. [1]

Q28

9231/12 • October/November 2022 • Question 1

(a) Use the list of formulae (MF19) to find $\sum_{r=1}^{n} r(r+2)$ in terms of n, simplifying your answer. [2]

(b) Express $\frac{1}{r(r+2)}$ in partial fractions and hence find $\sum_{r=1}^{n} \frac{1}{r(r+2)}$ in terms of n. [4]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+2)}$. [1]

Q29

9231/11, 9231/12 • May/June 2023 • Question 2(b)

The cubic equation $x^3 + 4x^2 + 6x + 1 = 0$ has roots α, β, γ.

Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n} \left((\alpha + r)^2 + (\beta + r)^2 + (\gamma + r)^2\right) = n(n^2 + an + b),$$

where a and b are constants to be determined. [6]

Q30

9231/11, 9231/12 • May/June 2023 • Question 3

(a) Use the method of differences to find $\displaystyle\sum_{r=1}^{n} \frac{1}{(kr+1)(kr-k+1)}$ in terms of n and k, where k is a positive constant. [4]

(b) Deduce the value of $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(kr+1)(kr-k+1)}$. [1]

(c) Find also $\displaystyle\sum_{r=n}^{n^2} \frac{1}{(kr+1)(kr-k+1)}$ in terms of n and k. [2]

Q31

9231/13 • May/June 2023 • Question 2

(a) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n}(6r^2+6r-5)=an^3+bn^2+cn,$$

where a, b and c are integers to be determined. [2]

(b) Use the method of differences to find $\displaystyle\sum_{r=1}^{n}\frac{6r^2+6r-5}{r^2+r}$ in terms of n. [4]

(c) Find also $\displaystyle\sum_{r=n+1}^{2n}\frac{6r^2+6r-5}{r^2+r}$ in terms of n. [2]

Q32

9231/11, 9231/13 • October/November 2023 • Question 1

(a) By considering $(r+1)^2 - r^2$, use the method of differences to prove that

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1).$$

[4]

(b) Given that $\sum_{r=1}^{n}(r+a) = n$, find a in terms of n. [3]

Q33

9231/12 • October/November 2023 • Question 1

(a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n}(3r^2+3r+1)$ in terms of n, simplifying your answer. [3]

(b) Show that

$$\frac{1}{r^3}-\frac{1}{(r+1)^3}=\frac{3r^2+3r+1}{r^3(r+1)^3}.$$

Hence find $\sum_{r=1}^{n}\frac{3r^2+3r+1}{r^3(r+1)^3}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty}\frac{3r^2+3r+1}{r^3(r+1)^3}$. [1]

Q34

9231/11, 9231/12 • May/June 2024 • Question 3

(a) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{N} r(r+1)(3r+4) = \tfrac{1}{12}N(N+1)(N+2)(9N+19).$$

[3]

(b) Express $\dfrac{3r+4}{r(r+1)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^{N} \frac{3r+4}{r(r+1)}\left(\frac{1}{4}\right)^{r+1}$$

in terms of N. [4]

(c) Deduce the value of $\displaystyle\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)}\left(\frac{1}{4}\right)^{r+1}$. [1]

Q35

9231/13 • May/June 2024 • Question 4

(a) Prove by mathematical induction that, for all positive integers n,

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1).$$

[5]

The sum S_n is defined by $S_n = \sum_{r=1}^{n} r^4$.

(b) Using the identity

$$(2r+1)^5 - (2r-1)^5 \equiv 160r^4 + 80r^2 + 2,$$

show that $S_n = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$. [6]

(c) Find the value of $\lim_{n\to\infty} \left(n^{-5}S_n\right)$. [2]

Q36

9231/11, 9231/13 • October/November 2024 • Question 4

(a) Use the method of differences to find $\displaystyle\sum_{r=1}^{n} \frac{5k}{(5r+k)(5r+5+k)}$ in terms of n and k, where k is a positive constant. [4]

It is given that $\displaystyle\sum_{r=1}^{\infty} \frac{5k}{(5r+k)(5r+5+k)} = \frac{1}{3}$.

(b) Find the value of k. [2]

(c) Hence find $\displaystyle\sum_{r=n}^{n^2} \frac{5k}{(5r+k)(5r+5+k)}$ in terms of n. [2]

Q37

9231/12 • October/November 2024 • Question 5

It is given that $S_n = \sum_{r=1}^{n} u_r$, where $u_r = x^{f(r)} - x^{f(r+1)}$ and $x > 0$.

(a) Find S_n in terms of n, x and the function f. [2]

(b) Given that $f(r) = \ln r$, find the set of values of x for which the infinite series

$$u_1 + u_2 + u_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

(c) Given instead that $f(r) = 2\log_x r$ where $x \neq 1$, use standard results from the List of formulae (MF19) to find $\sum_{n=1}^{N} S_n$ in terms of N. Fully factorise your answer. [4]

Q38

9231/11, 9231/12 • May/June 2025 • Question 1

(a) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n}(2-3r)(5-3r) = an^3 + bn^2 + cn,$$

where a, b and c are integers to be determined. [3]

(b) Use the method of differences to find $\displaystyle\sum_{r=1}^{n}\frac{1}{(2-3r)(5-3r)}$ in terms of n. [4]

(c) Deduce the value of $\displaystyle\sum_{r=1}^{\infty}\frac{1}{(2-3r)(5-3r)}$. [1]

Q39

9231/13 • May/June 2025 • Question 4

Let $w_r = r(r+1)(r+2)\dots(r+9)$.

(a) Show that

$$w_{r+1} - w_r = 10(r+1)(r+2)\dots(r+9).$$

[2]

(b) Given that $u_r = (r+1)(r+2)\dots(r+9)$, find $\sum_{r=1}^{n} u_r$ in terms of n. [3]

(c) Given that $v_r = x^{w_{r+1}} - x^{w_r}$, find the set of values of x for which the infinite series

$$v_1 + v_2 + v_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

Q40

9231/11, 9231/13 • October/November 2025 • Question 1

(a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n}\left(8r^3+12r^2+4r+3\right)$ in terms of n, simplifying your answer. [3]

(b) Show that

$$\frac{1}{r^4}-\frac{1}{(r+1)^4}=\frac{4r^3+6r^2+4r+1}{r^4(r+1)^4}$$

and hence use the method of differences to find $\sum_{r=1}^{n}\frac{4r^3+6r^2+4r+1}{r^4(r+1)^4}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty}\frac{4r^3+6r^2+4r+1}{r^4(r+1)^4}$. [1]

Q41

9231/12 • October/November 2025 • Question 1

(a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n}(r^3 - r)$ in terms of n, fully factorising your answer. [3]

(b) Express $\frac{r+3}{r^3-r}$ in the form $\frac{A}{r-1} + \frac{B}{r} + \frac{C}{r+1}$, where A, B and C are constants to be determined, and hence use the method of differences to find $\sum_{r=2}^{n}\frac{r+3}{r^3-r}$. [5]

(c) Deduce the value of $\sum_{r=2}^{\infty}\frac{r+3}{r^3-r}$. [1]

Q42

9231/14 • October/November 2025 • Question 3

(a) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n}(r+1)(r+2)(r+3) = \tfrac{1}{4}n\left(n^3 + bn^2 + cn + d\right),$$

where b, c and d are integers to be determined. [3]

(b) Express $\dfrac{2}{(r+1)(r+2)(r+3)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^{n}\frac{2}{(r+1)(r+2)(r+3)}.$$

[5]

(c) State the value of $\displaystyle\sum_{r=1}^{\infty}\frac{2}{(r+1)(r+2)(r+3)}$. [1]

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Written by Ali Hashir.

Answers available in the companion mark scheme booklet.

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