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Further Math with Ali Hashir

Summation of Series

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 Paper 1 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/11, 9231/12 • May/June 2015 • Question 1

↑ Back to contents

Mark scheme.

$3 \times \dfrac{13 \times 14 \times 27}{6} - 5 \times \dfrac{13 \times 14}{2} + 13 = 2015$	**M1A1**
$\left[\dfrac{9 \times 10}{2}\right]^2 - 10 = 2015$ (Award M1 for subtracting 9 or 10 here.)	**M1A1**
	(4)
	Total: 4

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Q2

9231/13 • May/June 2015 • Question 4

↑ Back to contents

Mark scheme.

$A = \tan^{-1}(x+1)$; $B = \tan^{-1}(x-1) \Rightarrow \tan(A-B) = \dfrac{(x+1)-(x-1)}{1+(x+1)(x-1)}$	**M1A1**
$\tan(A-B) = \dfrac{2}{x^2} \Rightarrow A - B = \tan^{-1}\left(\dfrac{2}{x^2}\right)$ (AG)	**A1**
	(3)
LHS $= (\tan^{-1}2 - \tan^{-1}0) + (\tan^{-1}3 - \tan^{-1}1) + \cdots + (\tan^{-1}n - \tan^{-1}[n-2])$ $+(\tan^{-1}[n+1] - \tan^{-1}[n-1])$.	**M1A1**
$= \tan^{-1}[n+1] + \tan^{-1}n - \tan^{-1}1 - \tan^{-1}0 = \tan^{-1}[n+1] + \tan^{-1}n - \frac{1}{4}\pi$	**A1A1**
	(4)
	Total: 7

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Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 4

↑ Back to contents

Mark scheme.

$\left(\frac{6}{\sqrt{1}}-\frac{7}{\sqrt{3}}\right)+\left(\frac{7}{\sqrt{3}}-\frac{8}{\sqrt{7}}\right)+\ldots+\left(\frac{35}{\sqrt{871}}-\frac{36}{\sqrt{931}}\right)=6-\frac{36}{\sqrt{931}}$	**M1A1**
$=4.820$ ((i) [3])	**A1**
$6-\frac{n+6}{\sqrt{n^2+n+1}}>4.9 \Rightarrow 0.21n^2-10.79n-34.79(>0)$	**M1A1**
$\Rightarrow n>54.42\ldots$ so 55 terms required. ((ii) [4])	**dM1A1**

Total: 7

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Q4

9231/11, 9231/12 • May/June 2016 • Question 2

↑ Back to contents

Mark scheme.

$\frac{2}{r} - \frac{4}{r+1} + \frac{2}{r+2}$ (Award B2 if written down by cover up rule.)	**M1A1**
$\left(2 - 2 + \frac{2}{3}\right) + \left(1 - \frac{4}{3} + \frac{1}{2}\right) + \cdots + \left(\frac{2}{n-1} - \frac{4}{n} + \frac{2}{n+1}\right) + \left(\frac{2}{n} - \frac{4}{n+1} + \frac{2}{n+2}\right)$	**M1A1**
$= 1 - \frac{2}{n+1} + \frac{2}{n+2}$ (AEF)	**A1**
	(5)
Sum to infinity $= 1$	**B1**
	(1)
	Total: 6

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Q5

9231/13 • May/June 2016 • Question 1

↑ Back to contents

Mark scheme.

$\frac{1}{3}\left(\frac{1}{3r+1}-\frac{1}{3r+4}\right)=\frac{1}{3}\left(\frac{(3r+4)-(3r+1)}{(3r+1)(3r+4)}\right)=\frac{1}{(3r+1)(3r+4)}$ AG	**B1**
	(1)
$S_N = \frac{1}{3}\left[\left(\frac{1}{4}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{10}\right)+\cdots+\left(\frac{1}{3N+1}-\frac{1}{3N+4}\right)\right] = \frac{1}{3}\left(\frac{1}{4}-\frac{1}{3N+4}\right)$	**M1 A1**
$\Rightarrow S=\frac{1}{12}$	**A1**
$\Rightarrow S-S_N=\frac{1}{3(3N+4)}<\frac{1}{10000}$	**M1**
$\Rightarrow 3N+4>\frac{10000}{3}\Rightarrow N>1109\frac{7}{9}$. Thus least N is 1110.	**A1**
	(5)
	Total: 6

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Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 1

↑ Back to contents

Mark scheme.

$\frac{1}{(2r-1)(2r+1)} = \frac{1}{2}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$	**M1A1**
$\sum_{r=1}^{n} \frac{1}{(2r)^2-1} = \frac{1}{2}\left(\left[1-\frac{1}{3}\right] + \left[\frac{1}{3}-\frac{1}{5}\right] + \ldots + \left[\frac{1}{2n-1} - \frac{1}{2n+1}\right]\right) = \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$ (OE)	**M1A1**
	(4)
$\frac{1}{2}\left(1 - \frac{1}{2n+1}\right) = \frac{n}{2n+1} \Rightarrow \sum_{r=1}^{\infty} \frac{1}{(2r)^2-1} = \frac{1}{2}$	**B1✓**
	(1)
	Total: 5

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Q7

9231/13 • May/June 2017 • Question 2

↑ Back to contents

Mark scheme.

RHS $= \frac{1}{2}\left\{\frac{4r^3+8r^2+3r-(4r^2-1)(r+2)}{r(r+1)(r+2)}\right\}$	**M1**
$= \frac{1}{2}\left\{\frac{4r^3+8r^2+3r-(4r^3+8r^2-r-2)}{r(r+1)(r+2)}\right\} = \frac{1}{2}\left\{\frac{(4r+2)}{r(r+1)(r+2)}\right\} = \frac{(2r+1)}{r(r+1)(r+2)}$ (AG)	**A1**
	(2)
Sum to n terms is:	
$\frac{1}{2}\left\{\left[\frac{3.5}{2.3}-\frac{1.3}{1.2}\right]+\left[\frac{5.7}{3.4}-\frac{3.5}{2.3}\right]+\dots + \left[\frac{(2n-1)(2n+1)}{n(n+1)}-\frac{(2n-3)(2n-1)}{n(n-1)}\right] + \left[\frac{(2n+1)(2n+3)}{(n+1)(n+2)}-\frac{(2n-1)(2n+1)}{n(n+1)}\right]\right\}$	**M1**
$= \frac{1}{2}\left\{\frac{(2n+1)(2n+3)}{(n+1)(n+2)}-\frac{3}{2}\right\}$ (AG)	**A1**
$S_\infty = \frac{1}{2}\times 4 - \frac{3}{4} = 1\frac{1}{4}$.	**M1A1**
	(4)
	Total: 6

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Q8

9231/11, 9231/12 • May/June 2017 • Question 1

↑ Back to contents

Mark scheme.

$\sum_{n+1}^{2n} u_r = (2n)^2(4n+3) - n^2(2n+3)$ (Method mark for using $S_{2n} - S_n$)	**M1**
$= 14n^3 + 9n^2$	**A1**
	(2)
$u_r = r^2(2r+3) - (r-1)^2(2r+1)$ (Method mark for using $S_r - S_{r-1}$ OE)	**M1A1**
$= 6r^2 - 1$ (SR: CAO **B1** without wrong working)	**A1**
	(3)
	Total: 5

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Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 1

↑ Back to contents

Mark scheme.

$\sum_{r=1}^{n} u_r = 16\sum_{r=1}^{n} r^2 - 8\sum_{r=1}^{n} r - 3n$	**M1A1**
$= 16 \cdot \frac{n(n+1)(2n+1)}{6} - 8 \cdot \frac{n(n+1)}{2} - 3n$	**M1**
$= \ldots = \frac{n}{3}\left(16n^2 + 12n - 13\right)$ (3 terms)	**A1**

Total: 4

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Q10

9231/11, 9231/12 • May/June 2018 • Question 5

↑ Back to contents

Mark scheme.

$S_{2n} = 1^2 - 2^2 + 3^2 - 4^2 \ldots$ Uses correct difference.	**M1**
$S_{2n} = \sum_{r=1}^{2n} r^2 - 2\sum_{r=1}^{n}(2r)^2 = \sum_{r=1}^{2n} r^2 - 8\sum_{r=1}^{n} r^2$	**A1**
Thus $S_{2n} = \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{8}{6}n(n+1)(2n+1)$	**M1**
Factorising, $S_{2n} = \frac{1}{3}n(2n+1)(4n+1-4n-4) = -n(2n+1)$	**A1**
	(4)
$\lim_{n\to\infty} \frac{S_{2n}}{n^2} = -2$	**B1**
$S_{2n+1} = S_{2n} + (-1)^{2n}(2n+1)^2$	**M1**
So, $S_{2n+1} = -n(2n+1) + (2n+1)^2 = (2n+1)(n+1)$. Uses the result given in (i) or using $\lim_{n\to\infty} \frac{S_{2n}}{n^2}$ and correct sign	**M1**
Thus $\lim_{n\to\infty} \frac{S_{2n+1}}{n^2} = 2$	**A1**
	(4)
	Total: 8

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Q11

9231/13 • May/June 2018 • Question 2

↑ Back to contents

Mark scheme.

$\frac{1}{ne^n} - \frac{1}{(n+1)e^{n+1}} = \frac{(n+1)e - n}{n(n+1)e^{n+1}} = \frac{n(e-1)+e}{n(n+1)e^{n+1}}$. Verifies result (AG).	**B1**
	(1)
$S_N = \sum_{n=1}^{N}\left(\frac{1}{ne^n} - \frac{1}{(n+1)e^{n+1}}\right) = \left(\frac{1}{e} - \frac{1}{2e^2} + \frac{1}{2e^2} - \frac{1}{3e^3} + \ldots - \frac{1}{Ne^N} - \frac{1}{(N+1)e^{N+1}}\right)$ SOI. Uses difference method to sum.	**M1**
$= \frac{1}{e} - \frac{1}{(N+1)e^{N+1}}$	**A1**
	(2)
$S = \frac{1}{e}$. Finds S	**B1**
$(N+1)(S - S_N) < 10^{-3} \Rightarrow \frac{1}{e^{N+1}} < 10^{-3}$. Attempts to find difference between sum and sum to infinity.	**M1**
$\Rightarrow e^{N+1} > 10^3 \Rightarrow$ least such N is 6.	**A1**
	(3)
	Total: 6

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Q12

9231/11, 9231/13 • October/November 2018 • Question 11 (EITHER)

↑ Back to contents

Mark scheme.

$(2r+1)^2 - (2r-1)^2 = 8r$	**B1**
$\Rightarrow 8\sum_{r=1}^{n} r = -1^2 + (2n+1)^2$ (Sums both sides and uses method of differences.)	**M1**
$\Rightarrow \sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$ (AG.)	**A1**
	(3)
$(2r+1)^4 - (2r-1)^4 = \left((2r+1)^2 + (2r-1)^2\right)\left((2r+1)^2 - (2r-1)^2\right)$ (Uses difference of squares or expands.)	**M1**
$= (8r^2+2)(8r) = 16(4r^3+r)$	**A1**
$\Rightarrow 4\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r = -\left(1-\frac{1}{2}\right)^4 + \left(n+\frac{1}{2}\right)^4$ (Sums both sides and uses method of differences.)	**M1**
$4\sum_{r=1}^{n} r^3 = \left(n+\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^4 - \frac{1}{2}n(n+1)$ (Uses formula for $\sum r$)	**M1**
$= \left(\left(n+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right)\left(\left(n+\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right) - \frac{1}{2}n(n+1) = n^2(n+1)^2$ (AG)	**A1**
	(5)
$S = \frac{1}{4}(2N+1)^2(2N+2)^2 = (2N+1)^2(N+1)^2$ (Uses formula for $\sum r^3$. AG.)	**B1**
	(1)
$T = S - \sum_{r=1}^{N}(2r)^3 = (2N+1)^2(N+1)^2 - 2N^2(N+1)^2$ (Eliminates even terms from S.)	**M1**
$(N+1)^2(2N^2+4N+1)$ (Accept $(N+1)^2(2N(N+2)+1)$.)	**A1**
	(2)
$\frac{S}{T} = \frac{(2N+1)^2}{2N^2+4N+1} = \frac{4N^2+4N+1}{2N^2+4N+1}$ (Writes fraction as quadratic in N divided by quadratic in N.)	**M1**
Converges to 2 as $N \to \infty$.	**A1**

(2)

Total: 13

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Q13

9231/12 • October/November 2018 • Question 7

↑ Back to contents

Mark scheme.

$\sum_{r=1}^{N}(3r+1)(3r+4) = 9\sum_{r=1}^{N} r^2 + 15\sum_{r=1}^{N} r + 4N$ (Expands)	**M1**
$9\left(\frac{1}{6}N(N+1)(2N+1)\right)+15\left(\frac{1}{2}N(N+1)\right)+4N$ (Substitutes formulae for $\sum r$ and $\sum r^2$.)	**M1**
$= N\left(\frac{9}{6}(2N^2+3N+1)+\frac{15}{2}N+\frac{15}{2}+4\right)$	**A1**
$= N(3N^2+12N+13)$ (Shows simplification to the given answer (AG).)	
	(3)
$\frac{1}{(3r+1)(3r+4)} = \frac{1}{3}\left(\frac{1}{3r+1}-\frac{1}{3r+4}\right)$ (Finds partial fractions.)	**B1**
$T_N = \frac{1}{3}\left(\frac{1}{4}-\frac{1}{7}+\frac{1}{7}-\frac{1}{10}+\cdots+\frac{1}{3(N-1)+1}-\frac{1}{3N+4}\right)$ (Expresses terms as differences.)	**M1**
$\frac{1}{3}\left(\frac{1}{4}-\frac{1}{3N+4}\right) = \frac{1}{12}-\frac{1}{3(3N+4)}$ (Cancels to given answer (AG).)	**A1**
	(3)
$T_N = \frac{N}{4(3N+4)} \Rightarrow \frac{S_N}{T_N} = 4(3N+4)(3N^2+12N+13)$ (Writes $\frac{S_N}{T_N}$ as a polynomial)	**M1**
So $\frac{S_N}{T_N}$ is an integer because all terms are integers (Justifies expression being integer)	**A1**
	(2)
$\frac{S_N}{N^3T_N} = \frac{4(3N+4)(3N^2+16N+9)}{N^3}$ (Divides expression in (iii) by N^3 and takes limit)	**M1**
$\to 4(3)(3) = 36$	**A1**
	(2)
	Total: 10

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Q14

9231/13 • May/June 2019 • Question 4

↑ Back to contents

Mark scheme.

$\frac{1}{(3r+1)(3r-2)} = \frac{1}{3}\left(\frac{1}{3r-2} - \frac{1}{3r+1}\right)$	**M1A1**
$\sum_{r=1}^{N} \frac{1}{(3r+1)(3r-2)} = \frac{1}{3}\left(\frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \dots \frac{1}{3N-2} - \frac{1}{3N+1}\right)$	**M1**
$= \frac{1}{3}\left(1 - \frac{1}{3N+1}\right)$	**A1**
	(4)
$\sum_{r=N+1}^{N^2} \frac{N}{(3r+1)(3r-2)} = \sum_{r=1}^{N^2} \frac{N}{(3r+1)(3r-2)} - \sum_{r=1}^{N} \frac{N}{(3r+1)(3r-2)}$	**M1**
$= \frac{N}{3} - \frac{N}{3(3N^2+1)} - \left(\frac{N}{3} - \frac{N}{3(3N+1)}\right)$	**M1**
$= \frac{N}{3(3N+1)} - \frac{N}{3(3N^2+1)} = \frac{N^3-N^2}{(3N+1)(3N^2+1)}$	**A1**
$\to \frac{1}{9}$ as $N \to \infty$	**B1**
	(4)
	Total: 8

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Q15

9231/11, 9231/12 • May/June 2019 • Question 2

↑ Back to contents

Mark scheme.

$\dfrac{4\sin\left(n-\frac{1}{2}\right)\sin\frac{1}{2}}{\cos(2n-1)+\cos 1} = \dfrac{2(\cos(n-1)-\cos n)}{2\cos n\cos(n-1)}$	**M1**
$\dfrac{1}{\cos n} - \dfrac{1}{\cos(n-1)}$	**A1**
	(2)
$\displaystyle\sum_{n=1}^{N} \frac{4\sin(n-\frac{1}{2})\sin\frac{1}{2}}{\cos(2n-1)+\cos 1} = \sum_{n=1}^{N} \frac{1}{\cos n} - \frac{1}{\cos(n-1)}$	**M1**
$= \dfrac{1}{\cos 1} - \dfrac{1}{1} + \dfrac{1}{\cos 2} - \dfrac{1}{\cos 1} + \ldots + \dfrac{1}{\cos N} - \dfrac{1}{\cos(N-1)}$	
$-1 + \dfrac{1}{\cos N}$	**A1**
	(2)
$\cos N$ oscillates as $N \to \infty$ so $u_1 + u_2 + u_3 + \ldots$ does not converge.	**B1**
	(1)
	Total: 5

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Q16

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 5

↑ Back to contents

Mark scheme.

$\sum_{r=1}^{N}(5r+1)(5r+6) = 25\sum_{r=1}^{N} r^2 + 35\sum_{r=1}^{N} r + 6N$	**M1**
Expands.	
$25\left(\frac{1}{6}N(N+1)(2N+1)\right) + 35\left(\frac{1}{2}N(N+1)\right) + 6N$	**M1**
Substitutes formulae for $\sum r$ and $\sum r^2$.	
$= N\left(\frac{25}{6}(2N^2+3N+1) + \frac{35}{2}N + \frac{35}{2} + 6\right) = \frac{1}{3}N(25N^2+90N+83)$	**A1**
Simplifies to the given answer (AG).	
	(3)
$\frac{1}{(5r+1)(5r+6)} = \frac{1}{5}\left(\frac{1}{5r+1} - \frac{1}{5r+6}\right)$	**M1A1**
Finds partial fractions.	
$T_N = \frac{1}{5}\left(\frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \cdots + \frac{1}{5N+1} - \frac{1}{5N+6}\right)$	**M1**
Expresses terms as differences.	
$\frac{1}{5}\left(\frac{1}{6} - \frac{1}{5N+6}\right) = \frac{1}{30} - \frac{1}{5(5N+6)}$	**A1**
At least 3 terms including last.	
	(4)
$\frac{S_N}{N^3}T_N \to \frac{25}{3} \times \frac{1}{30} = \frac{5}{18}$	**M1A1**
Divides S_N by N^3 and takes limits as $N \to \infty$.	
	(2)
	Total: 9

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Q17

9231/11, 9231/12 • May/June 2020 • Question 4

↑ Back to contents

Mark scheme.

$\dfrac{1}{r^2-1} = \dfrac{1}{(r-1)(r+1)} = \dfrac{1}{2}\left(\dfrac{1}{r-1} - \dfrac{1}{r+1}\right)$	**M1A1**
$\displaystyle\sum_{r=2}^{n} \frac{1}{r^2-1} = \frac{1}{2}\Big(\frac{1}{1} - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \cdots$ $\displaystyle + \frac{1}{n-2} - \frac{1}{n} + \frac{1}{n-1} - \frac{1}{n+1}\Big)$	**M1**
$= \dfrac{1}{2}\left(1 + \dfrac{1}{2} - \dfrac{1}{n} - \dfrac{1}{n+1}\right)$	**A1**
$= \dfrac{3}{4} - \dfrac{2n+1}{2n(n+1)}$	**A1**
	(5)
$\dfrac{3}{4}$	**B1**
	(1)
$\displaystyle\sum_{r=n+1}^{2n} \frac{n}{r^2-1} = \sum_{r=2}^{2n} \frac{n}{r^2-1} - \sum_{r=2}^{n} \frac{n}{r^2-1}$	**M1**
$= n\left(\dfrac{3}{4} - \dfrac{4n+1}{4n(2n+1)}\right) - n\left(\dfrac{3}{4} - \dfrac{2n+1}{2n(n+1)}\right) = \dfrac{n(2n+1)}{2n(n+1)} - \dfrac{n(4n+1)}{4n(2n+1)}$	**M1A1**
$\to \dfrac{1}{2}$ as $n \to \infty$	**A1**
	(4)
	Total: 10

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Q18

9231/13 • May/June 2020 • Question 3

↑ Back to contents

Mark scheme.

$S_n = \sum_{r=1}^{n}(4r-2)^2 = 16\sum_{r=1}^{n} r^2 - 16\sum_{r=1}^{n} r + 4n$	**M1A1**
$= \frac{8}{3}n(n+1)(2n+1) - 8n(n+1) + 4n$	**M1**
$\frac{4}{3}n(4n^2+6n+2-6n-6+3) = \frac{4}{3}n(4n^2-1)$	**A1 AG**
	(4)
$\frac{n}{S_n} = \frac{3}{4(2n-1)(2n+1)} = \frac{3}{8}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$	**M1A1**
$\sum_{n=1}^{N}\frac{n}{S_n} = \frac{3}{8}\left(1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7}\cdots + \frac{1}{2N-1} - \frac{1}{2N+1}\right)$	**M1**
$= \frac{3}{8}\left(1 - \frac{1}{2N+1}\right)$	**A1**
	(4)
$\frac{3}{8}$	**B1**
	(1)
	Total: 9

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Q19

9231/11, 9231/13 • October/November 2020 • Question 2

↑ Back to contents

Mark scheme.

$\sum_{r=1}^{n}(7r+1)(7r+8) = \sum_{r=1}^{n} 49r^2 + 63r + 8$	**M1**
Expands.	
$= 49\left(\frac{1}{6}n(n+1)(2n+1)\right) + 63\left(\frac{1}{2}n(n+1)\right) + 8n$	**M1**
Substitutes formulae for $\sum r^2$ and $\sum r$.	
$= \frac{49}{3}n^3 + 56n^2 + \frac{143}{3}n$	**A1**
	(3)
$\dfrac{1}{(7r+1)(7r+8)} = \frac{1}{7}\left(\dfrac{1}{7r+1} - \dfrac{1}{7r+8}\right)$	**M1A1**
Finds partial fractions.	
$\sum_{r=1}^{n} \dfrac{1}{(7r+1)(7r+8)} = \frac{1}{7}\left(\frac{1}{8} - \frac{1}{15} + \frac{1}{15} - \frac{1}{22} + \ldots + \frac{1}{7n+1} - \frac{1}{7n+8}\right)$	**M1**
Writes at least three correct terms, including first and last.	
$= \frac{1}{7}\left(\frac{1}{8} - \dfrac{1}{7n+8}\right)$	**A1**
	(4)
$\dfrac{1}{56}$	**B1FT**
	(1)
	Total: 8

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Q20

9231/12 • October/November 2020 • Question 3

↑ Back to contents

Mark scheme.

$\left(x^n - \sqrt{x^{2n}+1}\right)\left(x^n + \sqrt{x^{2n}+1}\right) = x^{2n} - (x^{2n}+1) = -1$	**B1**
Uses the difference of two squares and obtains the given answer. AG	
	(1)
$u_n = x^{n+1} + \sqrt{x^{2n+2}+1} - x^n - \sqrt{x^{2n}+1}$	**B1**
Uses the identity given in part (a).	
$\sum_{n=1}^{N} u_n = x^2 + \sqrt{x^4+1} - x - \sqrt{x^2+1} + x^3 + \sqrt{x^6+1} - x^2 - \sqrt{x^4+1} + \ldots + x^{N+1} + \sqrt{x^{2N+2}+1} - x^N - \sqrt{x^{2N}+1}$	**M1**
Writes at least three complete correct terms, including the first and last.	
$= x^{N+1} + \sqrt{x^{2N+2}+1} - x - \sqrt{x^2+1}$	**A1**
(Don't allow in terms of n.)	
	(3)
$-1 < x < 1$	**B1**
$x^{N+1} \to 0$ as $n \to \infty \Rightarrow u_1 + u_2 + u_3 + \ldots = 1 - x - \sqrt{x^2+1}$	**M1A1**
Finds sum to infinity.	
	(3)
	Total: 7

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Q21

9231/11, 9231/12 • May/June 2021 • Question 2

↑ Back to contents

Mark scheme.

$n - \frac{1}{2}n(n+1) - \frac{1}{6}n(n+1)(2n+1)$	**M1 A1**
$\frac{1}{3}n - n^2 - \frac{1}{3}n^3$	**A1**
	(3)
$\frac{r+1}{(r+1)^2+1} - \frac{r}{r^2+1} = \frac{(r+1)(r^2+1) - r(r^2+2r+2)}{(r^2+2r+2)(r^2+1)} = \frac{1-r-r^2}{(r^2+2r+2)(r^2+1)}$	**M1 A1**
$\sum_{r=1}^{n} \frac{1-r-r^2}{(r^2+2r+2)(r^2+1)} = \sum_{r=1}^{n} \left(\frac{r+1}{(r+1)^2+1} - \frac{r}{r^2+1} \right) = \frac{2}{5} - \frac{1}{2} + \frac{3}{10} - \frac{2}{5} + \frac{4}{17} - \frac{3}{10} + \cdots + \frac{n+1}{(n+1)^2+1} - \frac{n}{n^2+1}$	**M1 A1**
$= -\frac{1}{2} + \frac{n+1}{(n+1)^2+1}$	**A1**
	(5)
$-\frac{1}{2}$	**B1 FT**
	(1)
	Total: 9

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Q22

9231/13 • May/June 2021 • Question 1

↑ Back to contents

Mark scheme.

$\dfrac{\sin(r+1)}{\cos(r+1)} - \dfrac{\sin r}{\cos r} = \dfrac{\sin(r+1)\cos r - \cos(r+1)\sin r}{\cos(r+1)\cos r} = \dfrac{\sin(r+1-r)}{\cos(r+1)\cos r}$ (or alternative route via $\tan(A-B)$)	**M1**
$= \dfrac{\sin 1}{\cos(r+1)\cos r}$ (SC B2 for alternative route completely correct to AG)	**A1**
	(2)
$[\sin 1]\sum_{r=1}^{n} u_r = \tan 2 - \tan 1 + \tan 3 - \tan 2 + \ldots + \tan(n+1) - \tan n$	**M1 A1**
$\sum_{r=1}^{n} u_r = \dfrac{\tan(n+1) - \tan 1}{\sin 1}$ (OE and ISW)	**A1**
	(3)
$\tan(n+1)$ oscillates as $n \to \infty$ so $u_1 + u_2 + u_3 + \ldots$ does not converge.	**B1**
	(1)
	Total: 6

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Q23

9231/11, 9231/13 • October/November 2021 • Question 2

↑ Back to contents

Mark scheme.

$\sum_{r=1}^{n}(r^3+3r^2+2r) = \frac{1}{4}n^2(n+1)^2 + \frac{1}{2}n(n+1)(2n+1) + n(n+1)$	**M1A1**
At least two formulae used for M1.	
$= \frac{1}{4}n(n+1)\left(n^2+n+4n+2+4\right) = \frac{1}{4}n(n+1)(n+2)(n+3)$	**A1**
	(3)
$\frac{1}{r(r+1)(r+2)} = \frac{1}{2r} - \frac{1}{r+1} + \frac{1}{2(r+2)}$	**M1A1**
$\sum_{r=1}^{n}\frac{1}{r(r+1)(r+2)} = \frac{1}{2}f(1) - f(2) + \frac{1}{2}f(3)$	**M1A1**
$+\frac{1}{2}f(2) - f(3) + \frac{1}{2}f(4)$	
$+\frac{1}{2}f(3) - f(4) + \frac{1}{2}f(5)$	
$\vdots$	
$+\frac{1}{2}f(n-1) - f(n) + \frac{1}{2}f(n+1)$	
$+\frac{1}{2}f(n) - f(n+1) + \frac{1}{2}f(n+2)$	
Shows enough terms for cancelation to be clear. Where $f(x) = \frac{1}{r}$.	
$= \frac{1}{4} - \frac{1}{2(n+1)} + \frac{1}{2(n+2)}$	**A1**
ISW	
	(5)
$\frac{1}{4}$	**B1 FT**
FT from *their* answer to part (b).	
	(1)
	Total: 9

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Q24

9231/12 • October/November 2021 • Question 3

↑ Back to contents

Mark scheme.

$\sum_{r=1}^{n} (\ln r - 2\ln(r+1) + \ln(r+2))$	**B1**
Separates logarithms into correct form using a difference. Or as logarithm of product.	
$\ln 1 - 2\ln 2 + \ln 3$	**M1A1**
$\ln 2 - 2\ln 3 + \ln 4$	
$\ln 3 - 2\ln 4 + \ln 5$	
$\vdots$	
$\ln(n-1) - 2\ln n + \ln(n+1)$	
$\ln n - 2\ln(n+1) + \ln(n+2)$	
Shows enough terms to make cancellation clear.	
$[\ln 1] - \ln 2 - \ln(n+1) + \ln(n+2) = \ln \frac{n+2}{2(n+1)}$	**A1**
AG	
	(4)
$S = -\ln 2$	**B1**
States sum to infinity. AEF	
$S_n - S = \ln\left(\frac{n+2}{n+1}\right) < 0.01$ leading to $n+2 < e^{0.01}(n+1)$	**M1**
Least value of n is 99	**A1**
CAO	
	(3)
	Total: 7

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Q25

9231/11, 9231/12 • May/June 2022 • Question 1

↑ Back to contents

Mark scheme.

$\dfrac{1}{(ar+1)(ar+a+1)} = \dfrac{1}{a}\left(\dfrac{1}{ar+1} - \dfrac{1}{ar+a+1}\right)$	**M1 A1**
$\displaystyle\sum_{r=1}^{n} \dfrac{1}{(ar+1)(ar+a+1)} = \dfrac{1}{a}\left(\dfrac{1}{a+1} - \dfrac{1}{2a+1} + \dfrac{1}{2a+1} - \dfrac{1}{3a+1} + \ldots + \dfrac{1}{an+1} - \dfrac{1}{an+a+1}\right)$	**M1**
$\displaystyle\sum_{r=1}^{n} \dfrac{1}{(ar+1)(ar+a+1)} = \dfrac{1}{a}\left(\dfrac{1}{a+1} - \dfrac{1}{an+a+1}\right)$	**A1**
	(4)
$\displaystyle\sum_{r=1}^{\infty} \dfrac{1}{(ar+1)(ar+a+1)} = \dfrac{1}{a^2+a} = \dfrac{1}{6}$ leading to $a^2+a-6=0$	**M1 A1**
$a=2$	**A1**
	(3)
	Total: 7

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Q26

9231/13 • May/June 2022 • Question 4

↑ Back to contents

Mark scheme.

$\sum_{r=1}^{n} e^{rx}(e^{2x} - 2e^{x} + 1) = e^{3x} - 2e^{2x} + e^{x}$	
$+e^{4x} - 2e^{3x} + e^{2x}$	
$+e^{5x} - 2e^{4x} + e^{3x}$	
$\vdots$	
$+e^{(n+1)x} - 2e^{nx} + e^{(n-1)x}$	
$+e^{(n+2)x} - 2e^{(n+1)x} + e^{nx}$	**M1 A1**
$= e^{x} - e^{2x} - e^{(n+1)x} + e^{(n+2)x}$	**A1**
	(3)
$x < 0$	**B1**
$e^{nx} \to 0$ as $n \to \infty$ leading to $u_1 + u_2 + u_3 + \ldots = e^{x} - e^{2x}$	**M1 A1**
	(3)
$\sum_{r=1}^{n} \ln u_r = \sum_{r=1}^{n} (rx + \ln(e^{x} - 1)^2)$	**M1***
$\sum_{r=1}^{n} \ln u_r = \frac{1}{2}xn(n+1) + n\ln(e^{x} - 1)^2$	**dM1 A1**
	(3)
	Total: 9

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Q27

9231/11, 9231/13 • October/November 2022 • Question 3

↑ Back to contents

Mark scheme.

$8r^3 + 12r^2 + 6r + 1 - (8r^3 - 12r^2 + 6r - 1) = 24r^2 + 2$	**M1**
$24\sum_{r=1}^{n} r^2 + 2n = (2n+1)^3 - 1$	**M1 A1** **A1**
$24\sum_{r=1}^{n} r^2 = 8n^3 + 12n^2 + 4n = 4n(n+1)(2n+1)$	**A1**
	(5)
$S_{2n} = \sum_{r=1}^{2n} r^2 + 2\sum_{r=1}^{n}(2r)^2 = \sum_{r=1}^{2n} r^2 + 8\sum_{r=1}^{n} r^2$	**M1**
$S_{2n} = \frac{1}{6}(2n)(2n+1)(4n+1) + \frac{8}{6}n(n+1)(2n+1)$	**M1**
$S_{2n} = \frac{1}{3}n(2n+1)(4n+1+4n+4) = \frac{1}{3}n(2n+1)(8n+5)$	**A1**
	(3)
$\frac{16}{3}$	**B1FT**
	(1)
	Total: 9

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Q28

9231/12 • October/November 2022 • Question 1

↑ Back to contents

Mark scheme.

$\sum_{r=1}^{n} r(r+2) = \sum_{r=1}^{n} r^2 + 2r = \frac{1}{6}n(n+1)(2n+1) + n(n+1)$	**M1**
$\frac{1}{6}n(n+1)(2n+7)$	**A1**
	(2)
$\dfrac{1}{r(r+2)} = \dfrac{1}{2}\left(\dfrac{1}{r} - \dfrac{1}{r+2}\right)$	**M1A1**
$\sum_{r=1}^{n} \dfrac{2}{r(r+2)} = 1 - \dfrac{1}{3} + \dfrac{1}{2} - \dfrac{1}{4} + \dfrac{1}{3} - \dfrac{1}{5} + \dfrac{1}{4} - \dfrac{1}{6} + \cdots + \dfrac{1}{n-1} - \dfrac{1}{n+1} + \dfrac{1}{n} - \dfrac{1}{n+2}$	**M1**
$\sum_{r=1}^{n} \dfrac{1}{r(r+2)} = \dfrac{1}{2}\left(\dfrac{3}{2} - \dfrac{1}{n+1} - \dfrac{1}{n+2}\right)$	**A1**
	(4)
$\frac{3}{4}$	**B1FT**
	(1)
	Total: 7

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Q29

9231/11, 9231/12 • May/June 2023 • Question 2(b)

↑ Back to contents

Mark scheme.

$(\alpha+r)^2=\alpha^2+2\alpha r+r^2$	**B1**
$\sum_{r=1}^{n}\left((\alpha+r)^2+(\beta+r)^2+(\gamma+r)^2\right)=\sum_{r=1}^{n}\left(4+2(-4)r+3r^2\right)$	**M1A1**
Collects like terms and uses $\alpha+\beta+\lambda=-4$ and *their* result from part (a).	
$4n-4n(n+1)+\frac{1}{2}n(n+1)(2n+1)$	**M1**
Applies formulae from MF19.	
$-4n^2+\frac{1}{2}n(n+1)(2n+1)=n\left(-4n+\frac{1}{2}(2n^2+3n+1)\right)=n\left(n^2-\frac{5}{2}n+\frac{1}{2}\right)$	**M1A1**
	Total: 6

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Q30

9231/11, 9231/12 • May/June 2023 • Question 3

↑ Back to contents

Mark scheme.

$\dfrac{1}{(kr+1)(kr-k+1)} = \dfrac{1}{k}\left(\dfrac{1}{k(r-1)+1} - \dfrac{1}{kr+1}\right)$	**M1A1**
Finds partial fractions.	
$\displaystyle\sum_{r=1}^{n} \frac{1}{(kr+1)(kr-k+1)} = \frac{1}{k}\left(1 - \frac{1}{k+1} + \frac{1}{k+1} - \frac{1}{2k+1} + \ldots + \frac{1}{k(n-1)+1} - \frac{1}{kn+1}\right)$	**M1**
Writes at least three complete terms, including last.	
$\dfrac{1}{k}\left(1 - \dfrac{1}{kn+1}\right)$ OE e.g. $\dfrac{n}{kn+1}$	**A1**
	(4)
$\dfrac{1}{k}$	**B1**
	(1)
$\displaystyle\sum_{r=n}^{n^2} \frac{1}{(kr+1)(kr-k+1)} = \sum_{r=1}^{n^2} \frac{1}{(kr+1)(kr-k+1)} - \sum_{r=1}^{n-1} \frac{1}{(kr+1)(kr-k+1)}$	**M1**
$\dfrac{1}{k}\left(1 - \dfrac{1}{kn^2+1} - \left(1 - \dfrac{1}{k(n-1)+1}\right)\right) = \dfrac{1}{k}\left(\dfrac{1}{k(n-1)+1} - \dfrac{1}{kn^2+1}\right)$ OE e.g. $\dfrac{n^2}{kn^2+1} - \dfrac{(n-1)}{k(n-1)+1}$	**A1**
	(2)
	Total: 7

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Q31

9231/13 • May/June 2023 • Question 2

↑ Back to contents

Mark scheme.

$6\left(\frac{1}{6}n(n+1)(2n+1)\right)+6\left(\frac{1}{2}n(n+1)\right)\ [-5n]$	**M1**
Substitutes formulae for $\sum r^2$ and $\sum r$.	
$2n^3+6n^2-n$	**A1**
	(2)
$\frac{6r^2+6r-5}{r^2+r}=6-\frac{5}{r(r+1)}$	**B1**
Divides by denominator.	
$\frac{1}{r(r+1)}=\frac{1}{r}-\frac{1}{r+1}$	**B1**
Finds partial fractions.	
$\sum_{r=1}^{n}\frac{1}{r(r+1)}=1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\ldots+\frac{1}{n}-\frac{1}{n+1}$	**M1**
Writes at least three complete terms, including the last term, to show cancelation.	
$\sum_{r=1}^{n}\frac{6r^2+6r-5}{r^2+r}=6n-5+\frac{5}{n+1}$ OE e.g. $\frac{6n^2+n}{n+1}$	**A1**
	(4)
$\sum_{r=1}^{2n}\frac{6r^2+6r-5}{r^2+r}-\sum_{r=1}^{n}\frac{6r^2+6r-5}{r^2+r}=12n-5+\frac{5}{2n+1}-6n+5-\frac{5}{n+1}$	**M1**
Or uses method of differences again.	
$12n-5+\frac{5}{2n+1}-6n+5-\frac{5}{n+1}=6n+\frac{5}{2n+1}-\frac{5}{n+1}$ Or $6n-\frac{5n}{(n+1)(2n+1)}$. OE, like terms collected.	**A1**
	(2)
	Total: 8

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Q32

9231/11, 9231/13 • October/November 2023 • Question 1

↑ Back to contents

Mark scheme.

$r^2 + 2r + 1 - r^2 = 2r + 1$	**B1**
$2\sum_{r=1}^{n} r + n = (n+1)^2 - 1^2$	**M1A1**
$\Rightarrow 2\sum_{r=1}^{n} r = n^2 + n = n(n+1)$ ((a) Total: 4)	**A1**
$\sum_{r=1}^{n}(r+a) = \sum_{r=1}^{n} r + an$	**M1**
$\frac{1}{2}n(n+1) + an = n$	**M1**
$a = \frac{1}{2}(1-n)$ ((b) Total: 3)	**A1**
	Total: 7

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Q33

9231/12 • October/November 2023 • Question 1

↑ Back to contents

Mark scheme.

$\frac{1}{2}n(n+1)(2n+1)+\frac{3}{2}n(n+1)+n$	**M1A1**
n^3+3n^2+3n ((a) Total: 3)	**A1**
$\dfrac{1}{r^3}-\dfrac{1}{(r+1)^3}=\dfrac{(r+1)^3-r^3}{r^3(r+1)^3}=\dfrac{r^3+3r^2+3r+1-r^3}{r^3(r+1)^3}=\dfrac{3r^2+3r+1}{r^3(r+1)^3}$	**M1A1**
$\sum_{r=1}^{n}\dfrac{3r^2+3r+1}{r^3(r+1)^3}=\sum_{r=1}^{n}\left(\dfrac{1}{r^3}-\dfrac{1}{(r+1)^3}\right)$	**M1A1**
$=1-\dfrac{1}{2^3}+\dfrac{1}{2^3}-\dfrac{1}{3^3}+\cdots+\dfrac{1}{n^3}-\dfrac{1}{(n+1)^3}$	
$1-\dfrac{1}{(n+1)^3}$ ((b) Total: 5)	**A1**
1 ((c) Total: 1)	**B1FT**
	Total: 9

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Q34

9231/11, 9231/12 • May/June 2024 • Question 3

↑ Back to contents

Mark scheme.

$\sum_{r=1}^{N} r(r+1)(3r+4) = 3\sum_{r=1}^{N} r^3 + 7\sum_{r=1}^{N} r^2 + 4\sum_{r=1}^{N} r$	**B1**
Expands.	
$\frac{3}{4}N^2(N+1)^2 + \frac{7}{6}N(N+1)(2N+1) + 2N(N+1)$	**M1**
Substitutes formulae from MF19.	
$\frac{1}{12}N\left(9N(N^2+2N+1) + 14(2N^2+3N+1) + 24N + 24\right)$	**A1**
$\frac{1}{12}N\left(9N^3 + 46N^2 + 75N + 38\right) = \frac{1}{12}N(N+1)(N+2)(9N+19)$	**AG**
	(3)
$\frac{3r+4}{r(r+1)} = \frac{4}{r} - \frac{1}{r+1}$	**M1 A1**
Finds partial fractions.	
$\sum_{r=1}^{N} \frac{3r+4}{r(r+1)}\left(\frac{1}{4}\right)^{r+1} = \left(\frac{1}{4^2}\right)\left(\frac{4}{1}-\frac{1}{2}\right) + \left(\frac{1}{4^3}\right)\left(\frac{4}{2}-\frac{1}{3}\right) + \ldots + \left(\frac{1}{4^{N+1}}\right)\left(\frac{4}{N}-\frac{1}{N+1}\right)$	**M1 Writes at least three terms, including last.**
$\frac{1}{4} - \frac{1}{4^{N+1}(N+1)}$	**A1**
	(4)
$\frac{1}{4}$	**B1**
CAO	
	(1)
	Total: 8

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Q35

9231/13 • May/June 2024 • Question 4

↑ Back to contents

Mark scheme.

$1^2 = \frac{1}{6}(1)(2)(3)$ so H_1 is true.	**B1**
Checks base case.	
Assume that $\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$.	**B1**
States inductive hypothesis.	
$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	**M1**
Considers sum to $k+1$.	
$\frac{1}{6}(k+1)\left(2k^2+k+6k+6\right) = \frac{1}{6}(k+1)\left(2k^2+7k+6\right) = \frac{1}{6}(k+1)(k+2)(2k+3)$	**A1**
So H_{k+1} is true. By induction, H_n is true for all positive integers n.	**A1**
States conclusion.	
	(5)
$3^5 - 1^5 + 5^5 - 3^5 + 7^5 - 5^5 \ldots + (2n+1)^5 - (2n-1)^5$	**M1**
Applies method of differences to LHS, writes complete terms for at least three values of r including the first and last.	
$(2n+1)^5 - 1$	**A1**
SCB1 for insufficient complete terms shown.	
$160S_n + \frac{40}{3}n(n+1)(2n+1) + 2n$	**M1 A1**
Sums RHS and applies standard formulae.	
$160S_n = (2n+1)^5 - \frac{40}{3}n(n+1)(2n+1) - (2n+1)$	**M1**
Makes S_n the subject and factorises.	
$160S_n = (2n+1)\left((2n+1)^4 - \frac{40}{3}n(n+1) - 1\right)$	

$(2n+1)\left(16n^4+32n^3+\frac{32}{3}n^2-\frac{16}{3}n\right)=\frac{16}{3}n(2n+1)\left(3n^3+6n^2+2n-1\right)$	**A1**
AG	
$\Rightarrow S_n=\frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$	
	(6)
$n^{-5}S_n=\frac{1}{30}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\left(3+\frac{3}{n}-\frac{1}{n^2}\right)$	**M1**
Divides by n^5.	
$\frac{1}{5}$	**A1**
	(2)
	Total: 13

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Q36

9231/11, 9231/13 • October/November 2024 • Question 4

↑ Back to contents

Mark scheme.

$\dfrac{5k}{(5r+k)(5r+5+k)} = k\left(\dfrac{1}{5r+k} - \dfrac{1}{5r+5+k}\right)$	**M1A1**
$\displaystyle\sum_{r=1}^{n} \dfrac{5k}{(5r+k)(5r+5+k)} = k\left(\dfrac{1}{5+k} - \dfrac{1}{10+k} + \dfrac{1}{10+k} - \dfrac{1}{15+k} + \ldots + \dfrac{1}{5n+k} - \dfrac{1}{5n+5+k}\right)$	**M1**
$= k\left(\dfrac{1}{5+k} - \dfrac{1}{5n+5+k}\right)$ ((a) Total: 4)	**A1**
$\dfrac{k}{5+k} = \dfrac{1}{3} \Rightarrow 3k = 5+k \Rightarrow k = \dfrac{5}{2}$ ((b) Total: 2)	**M1A1**
$\displaystyle\sum_{r=n}^{n^2} \dfrac{5k}{(5r+k)(5r+5+k)} = \sum_{r=1}^{n^2} \dfrac{5k}{(5r+k)(5r+5+k)} - \sum_{r=1}^{n-1} \dfrac{5k}{(5r+k)(5r+5+k)}$	**M1**
$= k\left(\dfrac{1}{5+k} - \dfrac{1}{5n^2+5+k}\right) - k\left(\dfrac{1}{5+k} - \dfrac{1}{5n+k}\right) = k\left(\dfrac{1}{5n+k} - \dfrac{1}{5n^2+5+k}\right)$	
$= \dfrac{1}{2n+1} - \dfrac{1}{2n^2+3}$ ((c) Total: 2)	**A1FT**
	Total: 8

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Q37

9231/12 • October/November 2024 • Question 5

↑ Back to contents

Mark scheme.

$x^{f(1)} - x^{f(2)} + x^{f(2)} - x^{f(3)} + \cdots + x^{f(n)} - x^{f(n+1)}$	**M1**
$= x^{f(1)} - x^{f(n+1)}$ ((a) Total: 2)	**A1**
$[0 <] x \leq 1$	**B1**
$\sum_{r=1}^{\infty} u_r = 1$ [for $x < 1$]	**B1**
$\sum_{r=1}^{\infty} u_r = 0$ for $x = 1$ ((b) Total: 3)	**B1**
Uses $x^{2\log_x r} = r^2$	**M1**
$S_n = 1 - (n+1)^2 = -n^2 - 2n$	**A1**
$\sum_{n=1}^{N} -n^2 - 2n = -\frac{1}{6}N(N+1)(2N+1) - N(N+1)$	**M1**
$-\frac{1}{6}N(N+1)(2N+7)$ ((c) Total: 4)	**A1**
	Total: 9

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Q38

9231/11, 9231/12 • May/June 2025 • Question 1

↑ Back to contents

Mark scheme.

$(2-3r)(5-3r) = 9r^2 - 21r + 10$	**B1**
$9\left(\frac{1}{6}n(n+1)(2n+1)\right) - 21\left(\frac{1}{2}n(n+1)\right) + 10n$	**M1**
Substitutes formulae for $\sum r^2$ and $\sum r$.	
$= 3n^3 - 6n^2 + n$	**A1**
	(3)
$\dfrac{1}{(2-3r)(5-3r)} = \dfrac{1}{3}\left(\dfrac{1}{2-3r} - \dfrac{1}{5-3r}\right)$	**B1**
Finds partial fractions.	
$\displaystyle\sum_{r=1}^{n} \frac{1}{(2-3r)(5-3r)} = \frac{1}{3}\left(\frac{1}{-1} - \frac{1}{2}\right) + \frac{1}{3}\left(\frac{1}{-4} - \frac{1}{-1}\right) + \frac{1}{3}\left(\frac{1}{-7} - \frac{1}{-4}\right) + \ldots + \frac{1}{3}\left(\frac{1}{2-3n} - \frac{1}{5-3n}\right)$	**M1A1**
Writes at least three terms, including first and last, to demonstrate cancelation.	
$= -\dfrac{1}{6} + \dfrac{1}{3(2-3n)}$	**A1**
	(4)
$\displaystyle\sum_{r=1}^{\infty} \frac{1}{(2-3r)(5-3r)} = -\frac{1}{6}$	**B1**
Allow FT from (b) if of similar form.	
	(1)
	Total: 8

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Q39

9231/13 • May/June 2025 • Question 4

↑ Back to contents

Mark scheme.

$w_{r+1} - w_r = (r+1)(r+2)\ldots(r+10) - r(r+1)(r+2)\ldots(r+9)$	**M1**
Correct expression seen for the difference.	
$(r+1)(r+2)\ldots(r+9)(r+10-r) = 10(r+1)(r+2)\ldots(r+9)$	**A1**
Must see the $(r+10-r)$ AG.	
	(2)
$\frac{1}{10}(w_2 - w_1) + \frac{1}{10}(w_3 - w_2) + \ldots + \frac{1}{10}(w_{n+1} - w_n)$	**M1A1**
Writes at least three terms, including last. Can be implied by correct final answer.	
$= \frac{1}{10}(w_{n+1} - w_1) = \frac{1}{10}(n+1)(n+2)\ldots(n+10) - \frac{10!}{10} = \frac{1}{10}(n+1)(n+2)\ldots(n+10) - 9!$	**A1**
ISW	
	(3)
$v_1 + v_2 + \ldots + v_n = x^{w_2} - x^{w_1} + x^{w_3} - x^{w_2} + \ldots + x^{w_{n+1}} - x^{w_n} = x^{w_{n+1}} - x^{w_1}$	**M1**
For $-1 < x < 1$, sum to infinity is $-x^{w_1} = -x^{10!}$	**A1**
For $x = \pm 1$, sum to infinity is 0.	**A1**
	(3)
	Total: 8

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Q40

9231/11, 9231/13 • October/November 2025 • Question 1

↑ Back to contents

Mark scheme.

$2n^2(n+1)^2 + 2n(n+1)(2n+1) + 2n(n+1) + 3n$	**M1A1**
$= 2n^4 + 8n^3 + 10n^2 + 7n$	**A1**
	(3)
$\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{(r+1)^4 - r^4}{r^4(r+1)^4} = \frac{r^4 + 4r^3 + 6r^2 + 4r + 1 - r^4}{r^4(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$, AG.	**M1A1**
$\sum_{r=1}^{n} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4} = \sum_{r=1}^{n} \left(\frac{1}{r^4} - \frac{1}{(r+1)^4}\right) = 1 - \frac{1}{2^4} + \frac{1}{2^4} - \frac{1}{3^4} + \ldots + \frac{1}{n^4} - \frac{1}{(n+1)^4}$	**M1A1**
Shows three complete terms, including first and last.	
$= 1 - \frac{1}{(n+1)^4}$	**A1**
	(5)
1	**B1FT**
FT from their answer to part (b).	
	(1)
	Total: 9

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Q41

9231/12 • October/November 2025 • Question 1

↑ Back to contents

Mark scheme.

$\sum_{r=1}^{n}(r^3 - r) = \frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$	**M1A1**
Substitutes formulae. For the 1st A mark allow any letter for n.	
$= \frac{1}{4}n(n+1)(n-1)(n+2)$	**A1**
	(3)
$\frac{r+3}{r^3-r} = \frac{2}{r-1} - \frac{3}{r} + \frac{1}{r+1}$	**M1A1**
Finds partial fractions. M mark is for an equation that would lead to values for A, B and C.	
$\sum_{r=2}^{n} \frac{r+3}{(r-1)r(r+1)} = 2f(1) - 3f(2) + f(3) + 2f(2) - 3f(3) + f(4) + 2f(3) - 3f(4) + f(5) + \cdots + 2f(n-2) - 3f(n-1) + f(n) + 2f(n-1) - 3f(n) + f(n+1)$	**M1A1**
Shows five complete terms including first two and last two.	
$= \frac{3}{2} - \frac{2}{n} + \frac{1}{n+1}$	**A1**
SCB1 if insufficient terms to show all of the cancelling, but must start at $r = 2$.	
	(5)
$\frac{3}{2}$	**B1FT**
FT from *their* part (b). Needs at least two terms in part (b) with one involving a number and a term in n.	
	(1)
	Total: 9

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Q42

9231/14 • October/November 2025 • Question 3

↑ Back to contents

Mark scheme.

$\sum_{r=1}^{n}(r^3+6r^2+11r+6) = \frac{1}{4}n^2(n+1)^2 + n(n+1)(2n+1) + \frac{11}{2}n(n+1) + 6n$	**M1A1**
Expands and substitutes formulae.	
$= \frac{1}{4}n\left(n^3+10n^2+35n+50\right)$	**A1**
	(3)
$\frac{2}{(r+1)(r+2)(r+3)} = \frac{1}{r+1} - \frac{2}{r+2} + \frac{1}{r+3}$	**M1A1**
Finds partial fractions.	
$\sum_{r=1}^{n}\frac{2}{(r+1)(r+2)(r+3)} = f(1)-2f(2)+f(3)$ $+f(2)-2f(3)+f(4)$ $+f(3)-2f(4)+f(5)$ $\vdots$ $+f(n-1)-2f(n)+f(n+1)$ $+f(n)-2f(n+1)+f(n+2)$	**M1A1**
Shows enough complete terms for cancellation to be clear and any terms that form part of the final answer. $f(r) = \frac{1}{r+1}$.	
$= \frac{1}{6} - \frac{1}{n+2} + \frac{1}{n+3}$	**A1**
Accept $\frac{1}{6} - \frac{1}{(n+2)(n+3)}$	
	(5)
$\frac{1}{6}$ (FT their result of similar form)	**B1**
	(1)
	Total: 9

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Written by Ali Hashir.

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