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Further Math with Ali Hashir

Roots of Polynomials

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics Paper 1 past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/11, 9231/12 • May/June 2015 • Question 4

The roots of the cubic equation $x^3 - 7x^2 + 2x - 3 = 0$ are α, β and γ. Find the values of

(i) $\dfrac{1}{(\alpha\beta)(\beta\gamma)(\gamma\alpha)}$,

(ii) $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\gamma\alpha}$,

(iii) $\dfrac{1}{\alpha^2\beta\gamma} + \dfrac{1}{\alpha\beta^2\gamma} + \dfrac{1}{\alpha\beta\gamma^2}$.

[6]

Deduce a cubic equation, with integer coefficients, having roots $\dfrac{1}{\alpha\beta}$, $\dfrac{1}{\beta\gamma}$ and $\dfrac{1}{\gamma\alpha}$. [2]

Q2

9231/13 • May/June 2015 • Question 1

The quartic equation $x^4 - px^2 + qx - r = 0$, where p, q and r are real constants, has two pairs of equal roots. Show that $p^2 + 4r = 0$ and state the value of q. [6]

Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 5

The cubic equation $x^3 + px^2 + qx + r = 0$, where p, q and r are integers, has roots α, β and γ, such that

$$\alpha + \beta + \gamma = 15,$$

$$\alpha^2 + \beta^2 + \gamma^2 = 83.$$

Write down the value of p and find the value of q. [3]

Given that α, β and γ are all real and that $\alpha\beta + \alpha\gamma = 36$, find α and hence find the value of r. [5]

Q4

9231/11, 9231/12 • May/June 2016 • Question 1

The roots of the cubic equation $2x^3 + x^2 - 7 = 0$ are α, β and γ. Using the substitution $y = 1 + \frac{1}{x}$, or otherwise, find the cubic equation whose roots are $1 + \frac{1}{\alpha}$, $1 + \frac{1}{\beta}$ and $1 + \frac{1}{\gamma}$, giving your answer in the form $ay^3 + by^2 + cy + d = 0$, where a, b, c and d are constants to be found. [4]

Q5

9231/13 • May/June 2016 • Question 8

The cubic equation

$$z^3 - z^2 - z - 5 = 0$$

has roots α, β and γ. Show that the value of $\alpha^3 + \beta^3 + \gamma^3$ is 19. [4]

Find the value of $\alpha^4 + \beta^4 + \gamma^4$. [2]

Show that the cubic equation with roots $\frac{\alpha - 1}{\alpha}$, $\frac{\beta - 1}{\beta}$ and $\frac{\gamma - 1}{\gamma}$ may be found using the substitution $z = \frac{1}{1 - x}$, and find this equation, giving your answer in the form $px^3 + qx^2 + rx + s = 0$, where p, q, r and s are constants to be determined. [4]

Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 2

Find the cubic equation with roots α, β and γ such that

$$\alpha + \beta + \gamma = 3, \qquad \alpha^2 + \beta^2 + \gamma^2 = 1, \qquad \alpha^3 + \beta^3 + \gamma^3 = -30,$$

giving your answer in the form $x^3 + px^2 + qx + r = 0$, where p, q and r are integers to be found. [6]

Q7

9231/13 • May/June 2017 • Question 1

The roots of the cubic equation $x^3 + 2x^2 - 3 = 0$ are α, β and γ.

(i) By using the substitution $y = \frac{1}{x^2}$, find the cubic equation with roots $\frac{1}{\alpha^2}$, $\frac{1}{\beta^2}$ and $\frac{1}{\gamma^2}$. [3]

(ii) Hence find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$. [1]

(iii) Find also the value of $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2}$. [1]

Q8

9231/11, 9231/12 • May/June 2017 • Question 7

By finding a cubic equation whose roots are α, β and γ, solve the set of simultaneous equations

$$\alpha + \beta + \gamma = -1,$$

$$\alpha^2 + \beta^2 + \gamma^2 = 29,$$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -1.$$

[8]

Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 4

The cubic equation $2x^3 - 3x^2 + 4x - 10 = 0$ has roots α, β and γ.

(i) Find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. [4]

(ii) Find the value of $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$. [4]

Q10

9231/11, 9231/12 • May/June 2018 • Question 4

It is given that the equation

$$x^3 - 21x^2 + kx - 216 = 0,$$

where k is a constant, has real roots a, ar and ar^{-1}.

(i) Find the numerical values of the roots. [6]

(ii) Deduce the value of k. [2]

Q11

9231/13 • May/June 2018 • Question 6

The equation

$$9x^3 - 9x^2 + x - 2 = 0$$

has roots α, β, γ.

(i) Use the substitution $y = 3x - 1$ to show that $3\alpha - 1$, $3\beta - 1$, $3\gamma - 1$ are the roots of the equation

$$y^3 - 2y - 7 = 0.$$

[2]

The sum $(3\alpha - 1)^n + (3\beta - 1)^n + (3\gamma - 1)^n$ is denoted by S_n.

(ii) Find the value of S_3. [2]

(iii) Find the value of S_{-2}. [4]

Q12

9231/11, 9231/13 • October/November 2018 • Question 2

The roots of the equation

$$x^3 + px^2 + qx + r = 0$$

are α, 2α, 4α, where p, q, r and α are non-zero real constants.

(i) Show that

$$2p\alpha + q = 0.$$

[4]

(ii) Show that

$$p^3 r - q^3 = 0.$$

[2]

Q13

9231/12 • October/November 2018 • Question 1

The roots of the cubic equation

$$x^3 - 5x^2 + 13x - 4 = 0$$

are α, β, γ.

(i) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [3]

(ii) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. [2]

Q14

9231/11, 9231/12 • May/June 2019 • Question 6

The equation

$$x^3 - x + 1 = 0$$

has roots α, β, γ.

(i) Use the relation $x = y^{\frac{1}{3}}$ to show that the equation

$$y^3 + 3y^2 + 2y + 1 = 0$$

has roots α^3, β^3, γ^3. Hence write down the value of $\alpha^3 + \beta^3 + \gamma^3$. [3]

Let $S_n = \alpha^n + \beta^n + \gamma^n$.

(ii) Find the value of S_{-3}. [2]

(iii) Show that $S_6 = 5$ and find the value of S_9. [4]

Q15

9231/13 • May/June 2019 • Question 9

A cubic equation $x^3 + bx^2 + cx + d = 0$ has real roots α, β and γ such that

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -\frac{5}{12},$$

$$\alpha\beta\gamma = -12,$$

$$\alpha^3 + \beta^3 + \gamma^3 = 90.$$

(i) Find the values of c and d. [3]

(ii) Express $\alpha^2 + \beta^2 + \gamma^2$ in terms of b. [2]

(iii) Show that $b^3 - 15b + 126 = 0$. [4]

(iv) Given that $3 + \mathrm{i}\sqrt{(12)}$ is a root of $y^3 - 15y + 126 = 0$, deduce the value of b. [2]

Q16

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 7

The equation $x^3 + 2x^2 + x + 7 = 0$ has roots α, β, γ.

(i) Use the relation $x^2 = -7y$ to show that the equation

$$49y^3 + 14y^2 - 27y + 7 = 0$$

has roots $\frac{\alpha}{\beta\gamma}, \frac{\beta}{\gamma\alpha}, \frac{\gamma}{\alpha\beta}$. [4]

(ii) Show that $\frac{\alpha^2}{\beta^2\gamma^2} + \frac{\beta^2}{\gamma^2\alpha^2} + \frac{\gamma^2}{\alpha^2\beta^2} = \frac{58}{49}$. [3]

(iii) Find the exact value of $\frac{\alpha^3}{\beta^3\gamma^3} + \frac{\beta^3}{\gamma^3\alpha^3} + \frac{\gamma^3}{\alpha^3\beta^3}$. [2]

Q17

9231/11, 9231/12 • May/June 2020 • Question 2

The cubic equation $6x^3 + px^2 - 3x - 5 = 0$, where p is a constant, has roots α, β, γ.

(a) Find a cubic equation whose roots are α^2, β^2, γ^2. [3]

(b) It is given that $\alpha^2 + \beta^2 + \gamma^2 = 2(\alpha + \beta + \gamma)$.

(i) Find the value of p. [3]

(ii) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. [2]

Q18

9231/13 • May/June 2020 • Question 1

The cubic equation $7x^3 + 3x^2 + 5x + 1 = 0$ has roots α, β, γ.

(a) Find a cubic equation whose roots are α^{-1}, β^{-1}, γ^{-1}. [3]

(b) Find the value of $\alpha^{-2} + \beta^{-2} + \gamma^{-2}$. [2]

(c) Find the value of $\alpha^{-3} + \beta^{-3} + \gamma^{-3}$. [2]

Q19

9231/11, 9231/13 • October/November 2020 • Question 3

The cubic equation $x^3 + cx + 1 = 0$, where c is a constant, has roots α, β, γ.

(a) Find a cubic equation whose roots are α^3, β^3, γ^3. [3]

(b) Show that $\alpha^6 + \beta^6 + \gamma^6 = 3 - 2c^3$. [3]

(c) Find the real value of c for which the matrix $\begin{pmatrix} 1 & \alpha^3 & \beta^3 \\ \alpha^3 & 1 & \gamma^3 \\ \beta^3 & \gamma^3 & 1 \end{pmatrix}$ is singular. [5]

Q20

9231/12 • October/November 2020 • Question 1

The cubic equation $x^3 + bx^2 + cx + d = 0$, where b, c and d are constants, has roots α, β, γ. It is given that $\alpha\beta\gamma = -1$.

(a) State the value of d. [1]

(b) Find a cubic equation, with coefficients in terms of b and c, whose roots are $\alpha+1$, $\beta+1$, $\gamma+1$. [3]

(c) Given also that $\gamma + 1 = -\alpha - 1$, deduce that $(c - 2b + 3)(b - 3) = b - c$. [4]

Q21

9231/11, 9231/12 • May/June 2021 • Question 3

The equation $x^4 - 2x^3 - 1 = 0$ has roots α, β, γ, δ.

(a) Find a quartic equation whose roots are α^3, β^3, γ^3, δ^3 and state the value of $\alpha^3 + \beta^3 + \gamma^3 + \delta^3$. [4]

(b) Find the value of $\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3} + \dfrac{1}{\gamma^3} + \dfrac{1}{\delta^3}$. [3]

(c) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

Q22

9231/13 • May/June 2021 • Question 2

The cubic equation $2x^3 - 4x^2 + 3 = 0$ has roots α, β, γ. Let $S_n = \alpha^n + \beta^n + \gamma^n$.

(a) State the value of S_1 and find the value of S_2. [3]

(b) (i) Express S_{n+3} in terms of S_{n+2} and S_n. [1]

(ii) Hence, or otherwise, find the value of S_4. [2]

(c) Use the substitution $y = S_1 - x$, where S_1 is the numerical value found in part (a), to find and simplify an equation whose roots are $\alpha + \beta$, $\beta + \gamma$, $\gamma + \alpha$. [3]

(d) Find the value of $\dfrac{1}{\alpha+\beta} + \dfrac{1}{\beta+\gamma} + \dfrac{1}{\gamma+\alpha}$. [2]

Q23

9231/11, 9231/13 • October/November 2021 • Question 1

It is given that

$$\alpha + \beta + \gamma = 3, \quad \alpha^2 + \beta^2 + \gamma^2 = 5, \quad \alpha^3 + \beta^3 + \gamma^3 = 6.$$

The cubic equation $x^3 + bx^2 + cx + d = 0$ has roots α, β, γ.

Find the values of b, c and d. [6]

Q24

9231/12 • October/November 2021 • Question 4

The cubic equation $x^3 + 2x^2 + 3x + 3 = 0$ has roots α, β, γ.

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

(b) Show that $\alpha^3 + \beta^3 + \gamma^3 = 1$. [2]

(c) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n} \left((\alpha + r)^3 + (\beta + r)^3 + (\gamma + r)^3\right) = n + \tfrac{1}{4}n(n+1)\left(an^2 + bn + c\right),$$

where a, b and c are constants to be determined. [6]

Q25

9231/11, 9231/12 • May/June 2022 • Question 4

The cubic equation $2x^3 + 5x^2 - 6 = 0$ has roots α, β, γ.

(a) Find a cubic equation whose roots are $\frac{1}{\alpha^3}, \frac{1}{\beta^3}, \frac{1}{\gamma^3}$. [3]

(b) Find the value of $\frac{1}{\alpha^6} + \frac{1}{\beta^6} + \frac{1}{\gamma^6}$. [3]

(c) Find also the value of $\frac{1}{\alpha^9} + \frac{1}{\beta^9} + \frac{1}{\gamma^9}$. [2]

Q26

9231/13 • May/June 2022 • Question 2

The cubic equation $x^3 + 5x^2 + 10x - 2 = 0$ has roots α, β, γ.

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [3]

(b) Show that the matrix $\begin{pmatrix} 1 & \alpha & \beta \\ \alpha & 1 & \gamma \\ \beta & \gamma & 1 \end{pmatrix}$ is singular. [4]

Q27

9231/11, 9231/13 • October/November 2022 • Question 1

The cubic equation $x^3 + bx^2 + d = 0$ has roots α, β, γ, where $\alpha = \beta$ and $d \neq 0$.

(a) Show that $4b^3 + 27d = 0$. [5]

(b) Given that $2\alpha^2 + \gamma^2 = 3b$, find the values of b and d. [3]

Q28

9231/12 • October/November 2022 • Question 2

The equation $x^4 + 3x^2 + 2x + 6 = 0$ has roots α, β, γ, δ.

(a) Find a quartic equation whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}, \frac{1}{\delta^2}$ and state the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}$. [4]

(b) Find the value of $\beta^2\gamma^2\delta^2 + \alpha^2\gamma^2\delta^2 + \alpha^2\beta^2\delta^2 + \alpha^2\beta^2\gamma^2$. [3]

(c) Find the value of $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} + \frac{1}{\delta^4}$. [2]

Q29

9231/11, 9231/12 • May/June 2023 • Question 2(a)

The cubic equation $x^3 + 4x^2 + 6x + 1 = 0$ has roots α, β, γ.

Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

Q30

9231/13 • May/June 2023 • Question 3

The equation $x^4 - x^2 + 2x + 5 = 0$ has roots α, β, γ, δ.

(a) Find a quartic equation whose roots are α^2, β^2, γ^2, δ^2 and state the value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$. [4]

(b) Find the value of $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} + \dfrac{1}{\gamma^2} + \dfrac{1}{\delta^2}$. [3]

(c) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

Q31

9231/11, 9231/13 • October/November 2023 • Question 3

The quartic equation $x^4 + bx^3 + cx^2 + dx - 2 = 0$ has roots α, β, γ, δ. It is given that

$$\alpha + \beta + \gamma + \delta = 3, \qquad \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 5, \qquad \alpha^{-1} + \beta^{-1} + \gamma^{-1} + \delta^{-1} = 6.$$

(a) Find the values of b, c and d. [6]

(b) Given also that $\alpha^3 + \beta^3 + \gamma^3 + \delta^3 = -27$, find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

Q32

9231/12 • October/November 2023 • Question 4

The cubic equation $27x^3 + 18x^2 + 6x - 1 = 0$ has roots α, β, γ.

(a) Show that a cubic equation with roots $3\alpha + 1$, $3\beta + 1$, $3\gamma + 1$ is

$$y^3 - y^2 + y - 2 = 0.$$

[3]

The sum $(3\alpha + 1)^n + (3\beta + 1)^n + (3\gamma + 1)^n$ is denoted by S_n.

(b) Find the values of S_2 and S_3. [4]

(c) Find the values of S_{-1} and S_{-2}. [3]

Q33

9231/11, 9231/12 • May/June 2024 • Question 1

The cubic equation $2x^3 + x^2 - px - 5 = 0$, where p is a positive constant, has roots α, β, γ.

(a) State, in terms of p, the value of $\alpha\beta + \beta\gamma + \gamma\alpha$. [1]

(b) Find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$. [2]

(c) Deduce a cubic equation whose roots are $\alpha\beta, \beta\gamma, \alpha\gamma$. [1]

(d) Given that $\alpha^2 + \beta^2 + \gamma^2 = \frac{1}{3}$, find the value of p. [2]

Q34

9231/13 • May/June 2024 • Question 2

The cubic equation $x^3 + 2x^2 + 3x + 1 = 0$ has roots α, β, γ.

(a) Find a cubic equation whose roots are $\alpha^2 + 1, \beta^2 + 1, \gamma^2 + 1$. [3]

(b) Find the value of $\left(\alpha^2 + 1\right)^2 + \left(\beta^2 + 1\right)^2 + \left(\gamma^2 + 1\right)^2$. [2]

(c) Find the value of $\left(\alpha^2 + 1\right)^3 + \left(\beta^2 + 1\right)^3 + \left(\gamma^2 + 1\right)^3$. [2]

Q35

9231/11, 9231/13 • October/November 2024 • Question 3

The quartic equation $x^4 + 2x^3 - 1 = 0$ has roots α, β, γ, δ.

(a) Find a quartic equation whose roots are $\alpha^4, \beta^4, \gamma^4, \delta^4$ and state the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [5]

(b) Find the value of $\alpha^5 + \beta^5 + \gamma^5 + \delta^5$. [3]

(c) Find the value of $\alpha^8 + \beta^8 + \gamma^8 + \delta^8$. [2]

Q36

9231/12 • October/November 2024 • Question 3

It is given that

$$\alpha+\beta+\gamma+\delta=2,$$
$$\alpha^2+\beta^2+\gamma^2+\delta^2=3,$$
$$\alpha^3+\beta^3+\gamma^3+\delta^3=4.$$

(a) Find the value of $\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta$. [2]

(b) Find the value of $\alpha^2\beta+\alpha^2\gamma+\alpha^2\delta+\beta^2\alpha+\beta^2\gamma+\beta^2\delta+\gamma^2\alpha+\gamma^2\beta+\gamma^2\delta+\delta^2\alpha+\delta^2\beta+\delta^2\gamma$. [3]

(c) It is given that α, β, γ, δ are the roots of the equation

$$6x^4-12x^3+3x^2+2x+6=0.$$

(i) Find the value of $\alpha^4+\beta^4+\gamma^4+\delta^4$. [3]

(ii) Find the value of $\alpha^5+\beta^5+\gamma^5+\delta^5$. [2]

Q37

9231/11, 9231/12 • May/June 2025 • Question 2

The cubic equation $x^3 + 2x + 1 = 0$ has roots α, β, γ.

(a) Find a cubic equation whose roots are $\alpha^3 - 1$, $\beta^3 - 1$, $\gamma^3 - 1$. [3]

(b) Find the value of $(\alpha^3 - 1)^2 + (\beta^3 - 1)^2 + (\gamma^3 - 1)^2$. [2]

(c) Find the value of $(\alpha^3 - 1)^3 + (\beta^3 - 1)^3 + (\gamma^3 - 1)^3$. [2]

Q38

9231/13 • May/June 2025 • Question 3

The quartic equation $x^4 + 7x^2 + 3x + 22 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$. [2]

(b) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

(c) Use standard results from the list of formulae (MF19) to find the value of

$$\sum_{r=1}^{10} \left((\alpha^2 + r)^2 + (\beta^2 + r)^2 + (\gamma^2 + r)^2 + (\delta^2 + r)^2\right).$$

[5]

Q39

9231/11, 9231/13 • October/November 2025 • Question 4

The quartic equation $x^4 + x^3 + x^2 + x + 1 = 0$ has roots α, β, γ, δ.

(a) Show that a quartic equation with roots $2\alpha + 1, 2\beta + 1, 2\gamma + 1, 2\delta + 1$ is

$$y^4 - 2y^3 + 4y^2 + 2y + 11 = 0.$$

[4]

The sum $(2\alpha + 1)^n + (2\beta + 1)^n + (2\gamma + 1)^n + (2\delta + 1)^n$ is denoted by S_n.

(b) Find the value of S_2. [2]

(c) Given that $S_3 = -22$, find the value of S_4. [2]

Q40

9231/12 • October/November 2025 • Question 2

The cubic equation $x^3 + bx^2 + cx + d = 0$, where b, c and d are constants, has roots α, β and γ. It is given that

$$\alpha + \beta + \gamma = 2,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 3,$$
$$\alpha^4 + \beta^4 + \gamma^4 = 5.$$

(a) Find the values of b and c. [3]

(b) Find the value of d. [5]

Q41

9231/14 • October/November 2025 • Question 2

The cubic equation $x^3 + bx^2 + cx + d = 0$, where $d \neq 0$, has roots α, β, γ such that $\gamma = \dfrac{1}{\beta}$.

(a) Show that $\beta + \dfrac{1}{\beta} = d - b$. [3]

(b) Show also that $\beta + \dfrac{1}{\beta} = \dfrac{1-c}{d}$. [2]

(c) It is given that $b = 3$, $c = -3$ and $d > 0$.

(i) Find the value of d. [2]

(ii) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

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Written by Ali Hashir.

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