Further Math with Ali Hashir

Roots of Polynomials

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 Paper 1 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/11, 9231/12 • May/June 2015 • Question 4

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Mark scheme.

$\alpha+\beta+\gamma=7,\ \alpha\beta+\beta\gamma+\gamma\alpha=2,\ \alpha\beta\gamma=3$ (Stated or implied by working.)	**B1**
(i) $\dfrac{1}{(\alpha\beta)(\beta\gamma)(\gamma\alpha)}=\dfrac{1}{(\alpha\beta\gamma)^2}=\dfrac{1}{9}$	**B1**
(ii) $\dfrac{1}{\alpha\beta}+\dfrac{1}{\beta\gamma}+\dfrac{1}{\gamma\alpha}=\dfrac{\alpha+\beta+\gamma}{\alpha\beta\gamma}=\dfrac{7}{3}$	**M1A1**
(iii) $\dfrac{1}{\alpha^2\beta\gamma}+\dfrac{1}{\alpha\beta^2\gamma}+\dfrac{1}{\alpha\beta\gamma^2}=\dfrac{1}{\alpha\beta\gamma}\left(\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}\right)=\dfrac{1}{\alpha\beta\gamma}\left(\dfrac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}\right)=\dfrac{2}{9}$	**M1A1**
	(6)
$\Rightarrow x^3-\dfrac{7}{3}x^2+\dfrac{2}{9}x-\dfrac{1}{9}=0\Rightarrow 9x^3-21x^2+2x-1=0$	**M1A1**$\surd$
	(2)
	Total: 8

Q2

9231/13 • May/June 2015 • Question 1

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Mark scheme.

(i) $2\alpha + 2\beta = 0$ (ii) $\alpha^2 + 4\alpha\beta + \beta^2 = -p$ (iii) $2\alpha^2\beta + 2\alpha\beta^2 = -q$ (iv) $\alpha^2\beta^2 = -r$	
All four correct (Any two correct)	**B2 (B1)**
Use of $\beta = -\alpha$ in ii, iii or iv	**M1**
$\Rightarrow p = 2\alpha^2$ and $\alpha^4 = -r \Rightarrow p^2 + 4r = 0$ (AG)	**M1A1**
and $q = 0$ (CAO)	**B1**
	(6)
	Total: 6

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Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 5

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Mark scheme.

$\alpha+\beta+\gamma=-p=15\Rightarrow p=-15$	**B1**
$2(\alpha\beta+\beta\gamma+\gamma\alpha)=(\alpha+\beta+\gamma)^2-(\alpha^2+\beta^2+\gamma^2)=2q$	**M1**
$\Rightarrow q=\frac{1}{2}(225-83)=71$	**A1**
	(3)
$\frac{36}{\alpha}=15-\alpha\quad(=[\beta+\gamma])$	**M1**
$\Rightarrow \alpha^2-15\alpha+36=0\Rightarrow\alpha=3,\ \alpha\neq 12$, **e.g.** since $12^2>83$ or other reason	**M1A1**
$\beta\gamma=71-36=35$	**B1**
$\Rightarrow r=-\alpha\beta\gamma=-3\times 35=-105$ (extra answer penalised)	**A1**
	(5)
	Total: 8

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Q4

9231/11, 9231/12 • May/June 2016 • Question 1

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Mark scheme.

$y = 1 + \frac{1}{x} \Rightarrow x = \frac{1}{y-1}$	**M1**
$\frac{2}{(y-1)^3} + \frac{1}{(y-1)^2} - 7 = 0 \Rightarrow 2 + (y-1) - 7(y-1)^3 = 0$	**A1**
$\Rightarrow 7(y^3 - 3y^2 + 3y - 1) - y + 1 - 2 = 0 \Rightarrow 7y^3 - 21y^2 + 20y - 8 = 0$	**M1A1**
	(4)
ALT METHOD: $\sum\alpha$, $\sum\alpha\beta$, $\alpha\beta\gamma$ M1 A1, $\sum(1 + 1/\alpha)$ etc M1 A1	
	Total: 4

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Q5

9231/13 • May/June 2016 • Question 8

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Mark scheme.

Let S_n denote $\alpha^n + \beta^n + \gamma^n$.	
$S_2 = S_1^2 - 2\sum\alpha\beta = 1^2 - 2 \times (-1) = 3$	**M1A1**
$S_3 = S_2 + S_1 + 15 = 3 + 1 + 15 = 19$ (AG)	**M1A1**
	(4)
Alt method: $\sum\alpha = 1$, $\sum\alpha\beta = -1$, and $\alpha\beta\gamma = 5$	**M1, A1**
$S_3 = S_1^3 - 3\sum\alpha\sum\alpha\beta + 3\alpha\beta\gamma = 19$	**M1, A1**
$S_4 = S_3 + S_2 + 5S_1 = 19 + 3 + 5 = 27$	**M1A1**
	(2)
$x = \dfrac{z-1}{z} = 1 - \dfrac{1}{z} \Rightarrow \dfrac{1}{z} = 1 - x \Rightarrow z = \dfrac{1}{1-x}$	**B1**
Hence $\dfrac{1}{(1-x)^3} - \dfrac{1}{(1-x)^2} - \dfrac{1}{(1-x)} - 5 = 0$ is the required equation.	**M1**
$\Rightarrow 1 - (1-x) - (1-x)^2 - 5(1-x)^3 = 0 \Rightarrow \cdots \Rightarrow 5x^3 - 16x^2 + 18x - 6 = 0$	**M1A1**
	(4)
	Total: 10

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Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 2

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Mark scheme.

$2\sum\alpha\beta = 9 - 1 \Rightarrow \sum\alpha\beta = 4$	**M1A1**
Use of, e.g.: $\sum\alpha^3 - 3\alpha\beta\gamma = \sum\alpha\left(\sum\alpha^2 - \sum\alpha\beta\right)$ or $\left(\sum\alpha\right)^3 = \sum\alpha^3 + 3\sum\alpha\sum\alpha\beta - 3\alpha\beta\gamma$	**M1**
Correct substitution in formula	**A1**
$\Rightarrow \alpha\beta\gamma = -7$	**A1**
Required cubic equation is $x^3 - 3x^2 + 4x + 7 = 0$ *must see final equation*	**A1**
ALT METHOD: $S_3 - 3S_2 + 4S_1 + 3r = 0$ **M1** $3r = 30 + 3 \times 1 - 4 \times 3$ **A1** $r = 7$ **A1**	
Total: 6	

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Q7

9231/13 • May/June 2017 • Question 1

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Mark scheme.

$y = \frac{1}{x^2} \Rightarrow x = \frac{1}{\sqrt{y}}$ (Rearranging to make x the subject)	**M1**
$\frac{1}{y\sqrt{y}} + \frac{2}{y} - 3 = 0 \Rightarrow \frac{1}{y\sqrt{y}} = 3 - \frac{2}{y}$ (Substituting and squaring)	**M1**
$\Rightarrow 9y^3 - 12y^2 + 4y - 1 = 0$ (OE). SR **B1** for finding cubic by manipulating roots	**A1**
	(3)
$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{12}{9}$ or $\frac{4}{3}$	**B1FT**
	(1)
$\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2} = \frac{4}{9}$	**B1FT**
	(1)
	Total: 5

Q8

9231/11, 9231/12 • May/June 2017 • Question 7

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Mark scheme.

$2\sum\alpha\beta = 1 - 29 \Rightarrow \sum\alpha\beta = -14$	**M1A1**
$\dfrac{\sum\alpha\beta}{\alpha\beta\gamma} = \dfrac{-14}{\alpha\beta\gamma} = -1 \Rightarrow \alpha\beta\gamma = 14$	**M1A1 FT**
$\Rightarrow x^3 + x^2 - 14x - 14 = 0$	**A1**
$\Rightarrow (x+1)(x^2 - 14)$ (Attempt to factorise cubic)	**M1A1**
Solution is -1, $\pm\sqrt{14}$ any order. Accept ± 3.74 (awrt). SR: **B1** for correct roots without working	**A1**
	Total: 8

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Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 4

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Mark scheme.

$\alpha+\beta+\gamma=\frac{3}{2}$, $\alpha\beta+\beta\gamma+\gamma\alpha=2$, $\alpha\beta\gamma=5$ (SOI (can be awarded in (ii) if not seen here))	**B1**
$(\alpha+1)(\beta+1)(\gamma+1)=\alpha\beta\gamma+(\alpha\beta+\beta\gamma+\gamma\alpha)+(\alpha+\beta+\gamma)+1$ (Multiply out and group for M1)	**M1A1**
$=5+2+1\frac{1}{2}+1=9\frac{1}{2}$	**A1FT**
Alt method: let $x=y-1$; substitute and expand to give $2y^3-9y^2+16y-19=0$; product of roots $=\frac{19}{2}$	
	(4)
$(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)=\left(1\frac{1}{2}-\alpha\right)\left(1\frac{1}{2}-\beta\right)\left(1\frac{1}{2}-\gamma\right)$	**M1**
$=\frac{27}{8}-\frac{9}{4}(\alpha+\beta+\gamma)+\frac{3}{2}(\alpha\beta+\beta\gamma+\gamma\alpha)-\alpha\beta\gamma$	**A1**
$=\frac{27}{8}-\frac{9}{4}\times\frac{3}{2}+\frac{3}{2}\times2-5=-2$	**M1A1**
	(4)
	Total: 8

Q10

9231/11, 9231/12 • May/June 2018 • Question 4

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Mark scheme.

$\alpha\beta\gamma = a^3 = 216 \Rightarrow a = 6$	**M1A1**
$a + ar + ar^{-1} = 21$	**M1**
$6\left(1 + r + r^{-1}\right) = 21$	
$2r^2 - 5r + 2 = 0 \Rightarrow r = 2$ or $r = 0.5$	**M1A1**
Roots are $6, 12, 3$	**A1**
	(6)
$k = \alpha\beta + \alpha\gamma + \beta\gamma = 6(12) + 6(3) + 12(3) = 126$	**M1A1**
	(2)
	Total: 8

Q11

9231/13 • May/June 2018 • Question 6

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Mark scheme.

Substitutes $x = \dfrac{y+1}{3}$	**M1**
Obtains the given result	**A1**
	(2)
$S_3 = 2S_1 + 7 \times 3$	**M1**
$= 21$	**A1**
	(2)
$S_{-1} = \dfrac{(3\alpha-1)(3\beta-1)+(3\alpha-1)(3\gamma-1)+(3\beta-1)(3\gamma-1)}{(3\alpha-1)(3\beta-1)(3\gamma-1)} = \dfrac{-2}{7}$	**M1A1**
$7S_{-2} = S_1 - 2S_{-1}$	**M1**
$S_{-2} = \dfrac{4}{49}$	**A1**
	(4)
	Total: 8

Q12

9231/11, 9231/13 • October/November 2018 • Question 2

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Mark scheme.

$\alpha + 2\alpha + 4\alpha = -p$ (Sum of roots.)	**B1**
$2\alpha^2 + 4\alpha^2 + 8\alpha^2 = q$ (Sum of products in pairs.)	**B1**
$\frac{14\alpha^2}{7\alpha} = -\frac{q}{p}$ (Combines equations.)	**M1**
$\Rightarrow 2p\alpha + q = 0$ (Verifies result (AG).)	**A1**
	(4)
$8\alpha^3 = -r$ (Product of roots.)	**B1**
$\Rightarrow r = \frac{q^3}{p^3} \Rightarrow p^3r - q^3 = 0$ (Verifies result (AG).)	**B1**
	(2)
	Total: 6

Q13

9231/12 • October/November 2018 • Question 1

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Mark scheme.

$\alpha+\beta+\gamma=5,\ \alpha\beta+\alpha\gamma+\beta\gamma=13$ (Sum of roots and $\alpha\beta+\alpha\gamma+\beta\gamma$. SOI)	**B1**
$\alpha^2+\beta^2+\gamma^2=5^2-2(13)$ (Uses $\sum\alpha^2=(\sum\alpha)^2-2(\sum\alpha\beta)$)	**M1**
$=-1$ (www)	**A1**
	(3)
$\alpha^3+\beta^3+\gamma^3=5(\alpha^2+\beta^2+\gamma^2)-13(\alpha+\beta+\gamma)+12$ (Uses $\alpha^3=5\alpha^2-13\alpha+4$.)	**M1**
$=5(-1)-13(5)+12=-58$	**A1**
Alt method: Use formula e.g. $\sum\alpha^3=(\sum\alpha)(\sum\alpha^2-\sum\alpha\beta)+3\alpha\beta\gamma$ or $(\sum\alpha)^3-3(\sum\alpha)(\sum\alpha\beta)+3\alpha\beta\gamma$	
	(2)
	Total: 5

Q14

9231/11, 9231/12 • May/June 2019 • Question 6

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Mark scheme.

$y = (y+1)^3$	**M1**
$y = y^3 + 3y^2 + 3y + 1 \Rightarrow y^3 + 3y^2 + 2y + 1 = 0$	**A1**
$S_3 = -3$	**B1**
	(3)
$S_{-3} = \dfrac{\alpha^3\beta^3 + \beta^3\gamma^3 + \alpha^3\gamma^3}{\alpha^3\beta^3\gamma^3} = \dfrac{2}{-1} = -2$	**M1A1**
	(2)
$S_6 = (-3)^2 - 2(2) = 5$	**M1A1**
$S_9 = -3S_6 - 2S_3 - 3 = -3(5) - 2(-3) - 3 = -12$	**M1A1**
	(4)
	Total: 9

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Q15

9231/13 • May/June 2019 • Question 9

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Mark scheme.

$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\alpha\beta+\beta\gamma+\alpha\gamma}{\alpha\beta\gamma}\Rightarrow c=\alpha\beta+\beta\gamma+\alpha\gamma=5$	**M1A1**
$d=12$	**B1**
	(3)
$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$	**M1**
$(\alpha+\beta+\gamma)^2-10=b^2-10$	**A1**
	(2)
$x^3+bx^2+5x+12=0 \quad (-b=\alpha+\beta+\gamma)$	**M1**
$90+b(b^2-10)-5b+36=0$	**M1A1**
$b^3-15b+126=0$	**A1**
	(4)
Real root is $b=-6$	**M1A1**
	(2)
	Total: 11

Q16

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 7

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Mark scheme.

$\sqrt{-7y}(-7y) + 2(-7y) + \sqrt{-7y} + 7 = 0$	**M1**
$\Rightarrow \sqrt{-7y}(-7y+1) = 14y - 7 \Rightarrow -7y(-7y+1)^2 = (14y-7)^2$	
Uses given substitution and eliminates radical.	
$\Rightarrow 49y^3 + 14y^2 - 27y + 7 = 0$	**A1**
AG	
$y = \dfrac{x^2}{-7} = \dfrac{x^2}{\alpha\beta\gamma}$	**M1**
Uses $\alpha\beta\gamma = -7$.	
So roots are $\dfrac{\alpha^2}{\alpha\beta\gamma} = \dfrac{\alpha}{\beta\gamma}$, $\dfrac{\beta^2}{\alpha\beta\gamma} = \dfrac{\beta}{\gamma\alpha}$, $\dfrac{\gamma^2}{\alpha\beta\gamma} = \dfrac{\gamma}{\alpha\beta}$	**A1**
AG	
	(4)
$\dfrac{\alpha}{\beta\gamma} + \dfrac{\beta}{\gamma\alpha} + \dfrac{\gamma}{\alpha\beta} = -\dfrac{2}{7}$, $\dfrac{1}{\gamma^2} + \dfrac{1}{\beta^2} + \dfrac{1}{\alpha^2} = -\dfrac{27}{49}$	**B1**
States sum of roots and $\alpha'\beta' + \alpha'\gamma' + \beta'\gamma'$.	
$\dfrac{\alpha^2}{\beta^2\gamma^2} + \dfrac{\beta^2}{\alpha^2\gamma^2} + \dfrac{\gamma^2}{\alpha^2\beta^2} = \left(-\dfrac{2}{7}\right)^2 - 2\left(-\dfrac{27}{49}\right) = \dfrac{58}{49}$	**M1A1**
Uses $\alpha'^2 + \beta'^2 + \gamma'^2 = (\alpha' + \beta' + \gamma')^2 - 2(\alpha'\beta' + \alpha'\gamma' + \beta'\gamma')$, AG.	
	(3)
$49\left(\dfrac{\alpha^3}{\beta^3\gamma^3} + \dfrac{\beta^3}{\alpha^3\gamma^3} + \dfrac{\gamma^3}{\alpha^3\beta^3}\right) = -14\left(\dfrac{58}{49}\right) + 27\left(-\dfrac{2}{7}\right) - 21$	**M1**
Uses $49\alpha'^3 = -14\alpha'^2 + 27\alpha' - 7$.	
$\Rightarrow \dfrac{\alpha^3}{\beta^3\gamma^3} + \dfrac{\beta^3}{\alpha^3\gamma^3} + \dfrac{\gamma^3}{\alpha^3\beta^3} = -\dfrac{317}{343}$	**A1**
	(2)
	Total: 9

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Q17

9231/11, 9231/12 • May/June 2020 • Question 2

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Mark scheme.

$y = x^2$	**B1**
$6y^{\frac{3}{2}} + py - 3y^{\frac{1}{2}} - 5 = 0 \Rightarrow y^{\frac{1}{2}}(6y - 3) = -py + 5$	**M1**
$y(6y-3)^2 = (-py+5)^2 \Rightarrow y(36y^2 - 36y + 9) = p^2y^2 - 10py + 25$	
$36y^3 - (p^2 + 36)y^2 + (10p + 9)y - 25 = 0$	**A1**
	(3)
$\alpha^2 + \beta^2 + \gamma^2 = \dfrac{p^2 + 36}{36}$	**B1**
$\dfrac{p^2 + 36}{36} = -\dfrac{2p}{6} \Rightarrow p^2 + 12p + 36 = 0$	**M1**
$p = -6$	**A1**
	(3)
$6(\alpha^3 + \beta^3 + \gamma^3) = 6(\alpha^2 + \beta^2 + \gamma^2) + 3(\alpha + \beta + \gamma) + 15$	**M1**
$\alpha^3 + \beta^3 + \gamma^3 = 5$	**A1**
	(2)
	Total: 8

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Q18

9231/13 • May/June 2020 • Question 1

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Mark scheme.

$y = x^{-1}$	**B1**
$7y^{-3} + 3y^{-2} + 5y^{-1} + 1 = 0 \Rightarrow y^3 + 5y^2 + 3y + 7 = 0$	**M1A1**
	(3)
$\alpha^{-2} + \beta^{-2} + \gamma^{-2} = (-5)^2 - 2(3) = 19$	**M1A1**
	(2)
$\alpha^{-3} + \beta^{-3} + \gamma^{-3} = -5(19) - 3(-5) - 21 = -101$	**M1A1**
	(2)
	Total: 7

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Q19

9231/11, 9231/13 • October/November 2020 • Question 3

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Mark scheme.

$y = x^3 \Rightarrow x = y^{\frac{1}{3}}$	**B1**
$y + cy^{\frac{1}{3}} + 1 = 0 \Rightarrow -c^3y = (y+1)^3 = y^3 + 3y^2 + 3y + 1$	**M1**
$y^3 + 3y^2 + (3 + c^3)y + 1 = 0$	**A1**
	(3)
$\alpha^3 + \beta^3 + \gamma^3 = -3, \quad \alpha^3\beta^3 + \beta^3\gamma^3 + \gamma^3\alpha^3 = 3 + c^3$	**B1 FT**
Using *their* answer to (a).	
$\alpha^6 + \beta^6 + \gamma^6 = (-3)^2 - 2(3 + c^3)$	**M1**
$= 3 - 2c^3$	**A1**
	(3)
$\alpha^3\beta^3\gamma^3 = -1$	**B1**
If using *their* answer to (a) FT	
$\begin{vmatrix} 1 & \alpha^3 & \beta^3 \\ \alpha^3 & 1 & \gamma^3 \\ \beta^3 & \gamma^3 & 1 \end{vmatrix} = 1 - (\alpha^6 + \beta^6 + \gamma^6) + 2\alpha^3\beta^3\gamma^3 = 2c^3 - 4$	**M1A1**
Evaluates determinant.	
$2c^3 - 4 = 0$	**M1**
Sets determinant equal to zero.	
$c = \sqrt[3]{2}$	**A1**
	(5)
	Total: 11

Q20

9231/12 • October/November 2020 • Question 1

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Mark scheme.

$d = 1$	**B1**
	(1)
$y = x + 1 \Rightarrow x = y - 1$	**B1**
Uses correct substitution.	
$y^3 + (b-3)y^2 + (c-2b+3)y + b - c = 0$	**M1A1**
Substitutes and expands.	
Alternative method for question 1(b)	
$(\alpha+1)(\beta+1) + (\alpha+1)(\gamma+1) + (\beta+1)(\gamma+1) = c - 2b + 3$	**B1**
$(\alpha+1+\beta+1+\gamma+1) = 3 - b,\ (\alpha+1)(\beta+1)(\gamma+1) = c - b$	**M1**
Using these relationships.	
$y^3 + (b-3)y^2 + (c-2b+3)y + b - c = 0)$	**A1**
	(3)
$\beta + 1 = -(b-3)$	**B1**
Uses sum of roots.	
$-(\alpha+1)(\alpha+1) = c - 2b + 3$	**B1**
Uses sum of products in pairs.	
$-(\alpha+1)(\beta+1)(\alpha+1) = -(b-c)$	**M1**
Applies product of roots.	
$\Rightarrow (c-2b+3)(b-3) = b - c$	**A1**
AG	

(4)

Total: 8

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Q21

9231/11, 9231/12 • May/June 2021 • Question 3

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Mark scheme.

$y = x^3$	**B1**
$y^{\frac{4}{3}} - 2y - 1 = 0 \Rightarrow y^4 = (2y+1)^3 = 8y^3 + 12y^2 + 6y + 1$	**M1**
$y^4 - 8y^3 - 12y^2 - 6y - 1 = 0$	**A1**
$\alpha^3 + \beta^3 + \gamma^3 + \delta^3 = 8$	**B1 FT**
	(4)
$\frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3} + \frac{1}{\delta^3} = \frac{\alpha^3\beta^3\delta^3 + \alpha^3\beta^3\gamma^3 + \beta^3\gamma^3\delta^3 + \alpha^3\gamma^3\delta^3}{\alpha^3\beta^3\gamma^3\delta^3} = \frac{6}{-1}$	**M1 A1 FT**
-6	**A1**
	(3)
$\alpha^4 + \beta^4 + \gamma^4 + \delta^4 = 2(\alpha^3 + \beta^3 + \gamma^3 + \delta^3) + 4$	**M1**
$= 20$	**A1**
	(2)
	Total: 9

Q22

9231/13 • May/June 2021 • Question 2

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Mark scheme.

$S_1 = 2$	**B1**
$S_2 = S_1^2 - 2(0)$	**M1**
$= 4$	**A1**
	(3)
$S_{n+3} = 2S_{n+2} - \frac{3}{2}S_n$	**B1**
	(1)
$S_4 = 2S_3 - \frac{3}{2}S_1 = 2(2S_2 - \frac{3}{2}S_0) - \frac{3}{2}S_1 \left[S_3 = \frac{7}{2}\right]$	**M1**
$= 4$	**A1**
	(2)
$x = 2 - y$	**B1**
$2(2-y)^3 - 4(2-y)^2 + 3 = 0$	**M1**
$2y^3 - 8y^2 + 8y - 3 = 0$	**A1**
	(3)
$\frac{\frac{8}{2}}{\frac{-3}{2}}$ OR use $2S_2 - 8S_1 + 8S_0 - 3S_{-1} = 0$ with substitution of their values	**M1**
$= \frac{8}{3}$	**A1 FT**
	(2)
	Total: 11

Q23

9231/11, 9231/13 • October/November 2021 • Question 1

↑ Back to contents

Mark scheme.

$b = -(\alpha + \beta + \gamma) = -3$	**B1**
$5 = 3^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$	**M1A1**
$c = 2$	**A1**
$6 - 3(5) + 2(3) + 3d = 0$	**M1**
[Equation is $x^3 - 3x^2 + 2x + 1 = 0$] $\quad d = 1$	**A1**

Total: 6

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Q24

9231/12 • October/November 2021 • Question 4

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Mark scheme.

$(-2)^2 - 2(3)$	**M1**
-2	**A1**
	(2)
$\alpha^3 + \beta^3 + \gamma^3 = -2(-2) - 3(-2) - 3(3)$	**M1**
1	**A1**
AG	
	(2)
$(\alpha + r)^3 = \alpha^3 + 3\alpha^2 r + 3\alpha r^2 + r^3$	**B1**
$\sum_{r=1}^{n} \left((\alpha + r)^3 + (\beta + r)^3 + (\gamma + r)^3\right) = \sum_{r=1}^{n} \left(1 + 3(-2)r + 3(-2)r^2 + 3r^3\right)$	**M1A1**
Collects like terms and uses results from parts (a) and (b).	
$n - 6\left(\frac{1}{2}n(n+1)\right) - 6\left(\frac{1}{6}n(n+1)(2n+1)\right) + \frac{3}{4}n^2(n+1)^2$	**M1**
$n - 3n(n+1) - n(n+1)(2n+1) + \frac{3}{4}n^2(n+1)^2$	
$n + \frac{1}{4}n(n+1)\left(-12 - 4(2n+1) + 3n(n+1)\right)$	**M1A1**
$n + \frac{1}{4}n(n+1)\left(3n^2 - 5n - 16\right)$	
	(6)
	Total: 10

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Q25

9231/11, 9231/12 • May/June 2022 • Question 4

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Mark scheme.

$y = x^{-3} \Rightarrow x = y^{-\frac{1}{3}}$	**B1**
$\Rightarrow 2y^{-1} + 5y^{-\frac{2}{3}} - 6 = 0$ leading to $5y^{-\frac{2}{3}} = 6 - 2y^{-1} \Rightarrow 125y = (6y-2)^3$	**M1**
$216y^3 - 216y^2 - 53y - 8 = 0$	**A1**
	(3)
$\alpha^{-3} + \beta^{-3} + \gamma^{-3} = 1 \quad \alpha^{-3}\beta^{-3} + \beta^{-3}\gamma^{-3} + \gamma^{-3}\alpha^{-3} = -\frac{53}{216}$	**B1 FT**
$\alpha^{-6} + \beta^{-6} + \gamma^{-6} = 1^2 - 2\left(-\frac{53}{216}\right)$	**M1**
$\alpha^{-6} + \beta^{-6} + \gamma^{-6} = \frac{161}{108}$	**A1**
	(3)
$216S_{-9} = 216S_{-6} + 53S_{-3} + 24$	**M1**
$S_{-9} = \frac{399}{216} = \frac{133}{72}$	**A1**
	(2)
	Total: 8

Q26

9231/13 • May/June 2022 • Question 2

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Mark scheme.

$\alpha\beta + \beta\gamma + \gamma\alpha = 10$	**B1**
$\alpha^2 + \beta^2 + \gamma^2 = 5^2 - 20 = 5$	**M1 A1**
	(3)
$\begin{vmatrix} 1 & \alpha & \beta \\ \alpha & 1 & \gamma \\ \beta & \gamma & 1 \end{vmatrix} = \begin{vmatrix} 1 & \gamma \\ \gamma & 1 \end{vmatrix} - \alpha \begin{vmatrix} \alpha & \gamma \\ \beta & 1 \end{vmatrix} + \beta \begin{vmatrix} \alpha & 1 \\ \beta & \gamma \end{vmatrix} = 1 - \alpha^2 - \beta^2 - \gamma^2 + 2\alpha\beta\gamma$	**M1 A1**
$1 - \alpha^2 - \beta^2 - \gamma^2 + 2\alpha\beta\gamma = 1 - 5 + 2(2) = 0$	**M1 A1**
	(4)
	Total: 7

Q27

9231/11, 9231/13 • October/November 2022 • Question 1

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Mark scheme.

$2\alpha + \gamma = -b,\ \alpha^2 + 2\alpha\gamma = 0$	**B1**
$\alpha = -2\gamma$ leading to $-4\gamma + \gamma = -b$	**M1**
$\gamma = \frac{1}{3}b,\ \alpha = -\frac{2}{3}b$	**A1**
$\alpha^2\gamma = -d$ leading to $\frac{4}{27}b^3 = -d$ leading to $4b^3 + 27d = 0$	**M1A1**
	(5)
$3b = b^2$ leading to $b = 3$	**M1A1**
$d = -4$	**A1**
	(3)
	Total: 8

Q28

9231/12 • October/November 2022 • Question 2

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Mark scheme.

$y = x^{-2}$ leading to $x = y^{-\frac{1}{2}}$	**B1**
$y^{-2} + 3y^{-1} + 2y^{-\frac{1}{2}} + 6 = 0$ leading to $1 + 3y + 2y^{\frac{3}{2}} + 6y^2 = 0$	**M1**
$36y^4 + 32y^3 + 21y^2 + 6y + 1 = 0$	**A1**
$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2} = -\frac{32}{36} = -\frac{8}{9}$	**B1FT**
	(4)
$\alpha\beta\gamma\delta = 6$	**B1**
$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2} = \frac{\alpha^2\beta^2\delta^2 + \alpha^2\beta^2\gamma^2 + \beta^2\gamma^2\delta^2 + \alpha^2\gamma^2\delta^2}{\alpha^2\beta^2\gamma^2\delta^2}$	**M1**
$\beta^2\gamma^2\delta^2 + \alpha^2\gamma^2\delta^2 + \alpha^2\beta^2\delta^2 + \alpha^2\beta^2\gamma^2 = -32$	**A1**
	(3)
$\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} + \frac{1}{\delta^4} = \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}\right)^2 - 2\left(\alpha^{-2}\beta^{-2} + \alpha^{-2}\gamma^{-2} + \alpha^{-2}\delta^{-2} + \beta^{-2}\gamma^{-2} + \beta^{-2}\delta^{-2} + \gamma^{-2}\delta^{-2}\right) = \left(-\frac{8}{9}\right)^2 - 2\left(\frac{21}{36}\right)$	**M1**
$-\frac{61}{162}$	**A1**
	(2)
	Total: 9

Q29

9231/11, 9231/12 • May/June 2023 • Question 2(a)

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Mark scheme.

$(-4)^2 - 2(6)$	**M1**
4	**A1**
	Total: 2

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Q30

9231/13 • May/June 2023 • Question 3

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Mark scheme.

$y = x^2$ so, $x = y^{\frac{1}{2}}$	**B1**
$y^2 - y + 2y^{\frac{1}{2}} + 5 = 0$ so, $\left(2y^{\frac{1}{2}}\right)^2 = \left(-y^2 + y - 5\right)^2$	**M1**
$y^4 - 2y^3 + 11y^2 - 14y + 25 = 0$	**A1**
$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 2$	**B1FT**
	(4)
$\alpha^2\beta^2\gamma^2\delta^2 = 25$	**B1FT**
$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2} = \frac{\alpha^2\beta^2\delta^2 + \alpha^2\beta^2\gamma^2 + \beta^2\gamma^2\delta^2 + \alpha^2\gamma^2\delta^2}{\alpha^2\beta^2\gamma^2\delta^2}$	**M1**
$\frac{14}{25}$	**A1**
	(3)
$\alpha^4 + \beta^4 + \gamma^4 + \delta^4 = \left(\alpha^2 + \beta^2 + \gamma^2 + \delta^2\right)^2 - 2\left(\alpha^2\beta^2 + \alpha^2\gamma^2 + \alpha^2\delta^2 + \beta^2\gamma^2 + \beta^2\delta^2 + \gamma^2\delta^2\right) = 2^2 - 2(11)$	**M1**
-18	**A1**
	(2)
	Total: 9

Q31

9231/11, 9231/13 • October/November 2023 • Question 3

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Mark scheme.

$b = -(\alpha+\beta+\gamma+\delta) = -3$	**B1**
$5 = (-3)^2 - 2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)$	**M1A1**
$c = 2$	**A1**
$6 = \dfrac{\alpha\beta\gamma+\beta\gamma\delta+\gamma\delta\alpha+\delta\alpha\beta}{\alpha\beta\gamma\delta} = \dfrac{-d}{-2}$	**M1**
$d = 12$	**A1**
Equation is $x^4 - 3x^3 + 2x^2 + 12x - 2 = 0$. ((a) Total: 6)	
$\alpha^4+\beta^4+\gamma^4+\delta^4 = 3(-27) - 2(5) - 12(3) + 2(4)$	**M1**
-119 ((b) Total: 2)	**A1**
	Total: 8

Q32

9231/12 • October/November 2023 • Question 4

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Mark scheme.

$y = 3x + 1 \Rightarrow x = \frac{1}{3}(y-1)$	**B1**
$\Rightarrow 27\left(\frac{y-1}{3}\right)^3 + 18\left(\frac{y-1}{3}\right)^2 + 6\left(\frac{y-1}{3}\right) - 1 = 0$	
$\Rightarrow (y-1)^3 + 2(y-1)^2 + 2(y-1) - 1 = 0$	**M1**
$\Rightarrow y^3 - 3y^2 + 3y - 1 + 2y^2 - 4y + 2 + 2y - 2 - 1 = 0$	
$y^3 - y^2 + y - 2 = 0$ ((a) Total: 3)	**A1**
$S_2 = 1^2 - 2(1) = -1$	**M1A1**
$S_3 = (3\alpha+1)^3 + (3\beta+1)^3 + (3\gamma+1)^3 = -1 - (1) + 6$	**M1**
4 ((b) Total: 4)	**A1**
$S_{-1} = \dfrac{(3\alpha+1)(3\beta+1) + (3\beta+1)(3\gamma+1) + (3\gamma+1)(3\alpha+1)}{(3\alpha+1)(3\beta+1)(3\gamma+1)} = \dfrac{1}{2}$	**B1**
$2S_{-2} = S_1 - 3 + S_{-1} = 1 - 3 + \frac{1}{2}$	**M1**
$S_{-2} = -\frac{3}{4}$ ((c) Total: 3)	**A1**
	Total: 10

Q33

9231/11, 9231/12 • May/June 2024 • Question 1

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Mark scheme.

$-\frac{1}{2}p$	**B1**
	(1)
$\alpha\beta^2\gamma + \alpha^2\beta\gamma + \alpha\beta\gamma^2 = \alpha\beta\gamma(\beta+\gamma+\alpha) = \frac{5}{2}\left(-\frac{1}{2}\right)$	**M1**
Factorises.	
$-\frac{5}{4}$	**A1**
	(2)
$4z^3 + 2pz^2 - 5z - 25 = 0$	**B1 FT**
OE. FT on *their* value of $\alpha\beta + \beta\gamma + \gamma\alpha$ (in terms of p).	
	(1)
$\frac{1}{3} = \frac{1}{4} + p$	**M1**
Uses $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)$	
$p = \frac{1}{12}$	**A1**
CAO	
	(2)
	Total: 6

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Q34

9231/13 • May/June 2024 • Question 2

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Mark scheme.

$y = x^2 + 1$	**B1**
Uses correct substitution. May use a different letter.	
$(y-1)^{\frac{3}{2}} + 2(y-1) + 3(y-1)^{\frac{1}{2}} + 1 = 0 \Rightarrow (y-1)^{\frac{1}{2}}\left((y-1)+3\right) = -2(y-1) - 1$	**M1 Substitutes and obtains an equation not involving radicals.**
$(y-1)(y+2)^2 = (-2y+1)^2 \Rightarrow (y-1)(y^2+4y+4) = 4y^2 - 4y + 1$	
OR $(x^3+3x)^2 = (-2x^2-1)^2 \Rightarrow x^6 + 2x^4 + 5x^2 - 1 = 0$	
$(y-1)^3 + 2(y-1)^2 + 5(y-1) - 1 = 0$	
$y^3 - y^2 + 4y - 5 = 0$	**A1**
OE	
	(3)
$\left(\alpha^2+1\right)^2 + \left(\beta^2+1\right)^2 + \left(\gamma^2+1\right)^2 = 1 - 2(4)$	**M1**
Uses $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ on *their* answer for part (a).	
-7	**A1**
CAO	
	(2)
$\left(\alpha^2+1\right)^3 + \left(\beta^2+1\right)^3 + \left(\gamma^2+1\right)^3 = -7 - 4 + 5(3)$	**M1**
Uses *their* equation from part (a).	

4	**A1**
CAO	
Alternative method:	
Uses $\sum(\alpha^2+1)^3 = \left(\sum(\alpha^2+1)\right)^3 - 3\left(\sum(\alpha^2+1)\right)\left(\sum(\alpha^2+1)(\beta^2+1)\right) + 3\left((\alpha^2+1)(\beta^2+1)(\gamma^2+1)\right)$	**M1**
Uses *their* equation from part (a) in a valid formula.	
$= 1^3 - 3\times 1\times 4 + 3\times 5$	
4	**A1**
CAO	
	(2)
	Total: 7

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Q35

9231/11, 9231/13 • October/November 2024 • Question 3

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Mark scheme.

$y = x^4$	**B1**
$y + 2y^{\frac{3}{4}} - 1 = 0 \Rightarrow 16y^3 = (1-y)^4$	**M1**
Uses binomial expansion.	**M1**
$16y^3 = 1 - 4y + 6y^2 - 4y^3 + y^4$	
$y^4 - 20y^3 + 6y^2 - 4y + 1 = 0$	**A1**
$\alpha^4 + \beta^4 + \gamma^4 + \delta^4 = 20$ ((a) Total: 5)	**B1**
$\alpha + \beta + \gamma + \delta = -2$	**B1**
$x^5 + 2x^4 - x = 0 \Rightarrow \alpha^5 + \beta^5 + \gamma^5 + \delta^5 = -2(20) + (-2)$	**M1**
-42 ((b) Total: 3)	**A1**
$\alpha^8 + \beta^8 + \gamma^8 + \delta^8 = 20^2 - 2(6)$	**M1**
388 ((c) Total: 2)	**A1**
	Total: 10

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Q36

9231/12 • October/November 2024 • Question 3

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Mark scheme.

$\alpha^2+\beta^2+\gamma^2+\delta^2=(\alpha+\beta+\gamma+\delta)^2-2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)$	**M1**
$3=2^2-2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)$	
$\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta=\frac{1}{2}$ ((a) Total: 2)	**A1**
$\alpha^2(\beta+\gamma+\delta)+\beta^2(\alpha+\gamma+\delta)+\gamma^2(\alpha+\beta+\delta)+\delta^2(\alpha+\beta+\gamma)$	**M1A1**
$=\alpha^2(2-\alpha)+\beta^2(2-\beta)+\gamma^2(2-\gamma)+\delta^2(2-\delta)$	
$2(3)-4=2$ ((b) Total: 3)	**A1**
$6S_4-12S_3+3S_2+2S_1+24=0\Rightarrow 6S_4-12(4)+3(3)+2(2)+24=0$	**M1**
A1	
$S_4=\frac{11}{6}$ ((c)(i) Total: 3)	**A1**
$6S_5-12S_4+3S_3+2S_2+6S_1=0\Rightarrow 6S_5-12\left(\frac{11}{6}\right)+3(4)+2(3)+12=0$	**M1**
$S_5=-\frac{4}{3}$ ((c)(ii) Total: 2)	**A1**
	Total: 10

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Q37

9231/11, 9231/12 • May/June 2025 • Question 2

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Mark scheme.

$y = x^3 - 1$	**B1**
Uses correct substitution.	
$(y+1)^{\frac{1}{3}} + 2(y+1)^{\frac{1}{3}} + 1 = 0 \Rightarrow 2(y+1)^{\frac{1}{3}} = -y - 2$	**M1**
$8(y+1) = -(y+2)^3 \Rightarrow 8y + 8 = -y^3 - 6y^2 - 12y - 8$	
Substitutes and obtains an equation not involving radicals.	
$y^3 + 6y^2 + 20y + 16 = 0$	**A1**
	(3)
$(\alpha^3 - 1)^2 + (\beta^3 - 1)^2 + (\gamma^3 - 1)^2 = 6^2 - 2(20)$	**M1**
Uses $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$.	
$= -4$	**A1**
FT	
	(2)
$(\alpha^3 - 1)^3 + (\beta^3 - 1)^3 + (\gamma^3 - 1)^3 = -6(-4) - 20(-6) - 16(3)$	**M1**
Uses their equation from (a) or a valid formula for the sum of cubes.	
$= 96$	**A1**
	(2)
	Total: 7

Q38

9231/13 • May/June 2025 • Question 3

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Mark scheme.

$0 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(7)$	**M1**
Uses $(\alpha + \beta + \gamma + \delta)^2 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$	
$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = -14$	**A1**
	(2)
$S_4 + 7S_2 + 3S_1 + 88 = 0$	**M1**
Uses given equation.	
$S_4 = 10$	**A1FT**
FT from (a)	
	(2)
$(r + \alpha^2)^2 = r^2 + 2\alpha^2 r + \alpha^4$	**B1**
Expands.	
$\sum_{r=1}^{10} \left((r + \alpha^2)^2 + (r + \beta^2)^2 + (r + \gamma^2)^2 + (r + \delta^2)^2 \right)$	**M1**
Collects like terms and uses results from parts (a) and (b).	
$= \sum_{r=1}^{10} (4r^2 - 28r + 10)$	**A1FT**
FT from (a) and (b) evaluated coefficients.	
$= 4\left(\frac{10}{6}(10 + 1)(2(10) + 1)\right) - 14(10)(10 + 1) + 10(10)$	**M1**
Applies formulae from MF19.	
$= 100$	**A1**
Evaluated. CAO	
	(5)

Total: 9

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Q39

9231/11, 9231/13 • October/November 2025 • Question 4

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Mark scheme.

$y = 2x + 1 \Rightarrow x = \frac{1}{2}(y-1)$	**B1**
Correct substitution used in equation.	
$\frac{1}{16}(y-1)^4 + \frac{1}{8}(y-1)^3 + \frac{1}{4}(y-1)^2 + \frac{1}{2}(y-1) + 1 = 0$	**M1A1**
$\frac{1}{16}\left(y^4 - 4y^3 + 6y^2 - 4y + 1\right) + \frac{1}{8}\left(y^3 - 3y^2 + 3y - 1\right) + \frac{1}{4}\left(y^2 - 2y + 1\right) + \frac{1}{2}(y-1) + 1 = 0$	
Expands.	
$\frac{1}{16}y^4 - \frac{1}{8}y^3 + \frac{1}{4}y^2 + \frac{1}{8}y + \frac{11}{16} = 0 \Rightarrow y^4 - 2y^3 + 4y^2 + 2y + 11 = 0$	**A1**
Simplifies, AG.	
	(4)
$S_2 = 2^2 - 2(4)$	**M1**
Uses $(\alpha+\beta+\gamma+\delta)^2 = \alpha^2+\beta^2+\gamma^2+\delta^2+2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)$	
$S_2 = -4$	**A1**
	(2)
$S_4 - 2S_3 + 4S_2 + 2S_1 + 44 = 0$	**M1**
Sums over roots.	
$S_4 = -76$	**A1**
	(2)
	Total: 8

Q40

9231/12 • October/November 2025 • Question 2

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Mark scheme.

$b = -(\alpha + \beta + \gamma) = -2$	**B1**
$3 = (-2)^2 - 2c$	**M1**
Uses formula for sum of squares.	
$c = \frac{1}{2}$	**A1**
	(3)
$S_3 + bS_2 + cS_1 + 3d = 0$	**M1**
Uses original equation or formula for sum of cubes.	
$S_3 = -3b - 2c - 3d = 5 - 3d$	**A1FT**
Allow FT from part (a) for their value.	
$S_4 + bS_3 + cS_2 + dS_1 = 0$	**M1**
Uses original equation.	
$5 + (-2)(5 - 3d) + \left(\frac{1}{2}\right)(3) + 2d = 0 \Rightarrow 8d - \frac{7}{2} = 0$	**A1FT**
Allow FT from part (a) for their value and for their value of S_3.	
$d = \frac{7}{16}$	**A1**
CAO	
	(5)
	Total: 8

Q41

9231/14 • October/November 2025 • Question 2

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Mark scheme.

$\alpha = -d$ SOI	**B1**
$\alpha + \beta + \dfrac{1}{\beta} = -b$	**M1**
Uses $\alpha + \beta + \gamma = -b$ and replaces γ with $\dfrac{1}{\beta}$. OE	
$\beta + \dfrac{1}{\beta} = d - b$ (AG)	**A1**
	(3)
$-d\beta + 1 - \dfrac{d}{\beta} = c$	**M1**
Uses $\alpha\beta + \beta\gamma + \gamma\alpha = c$.	
$\beta + \dfrac{1}{\beta} = \dfrac{1-c}{d}$ (AG)	**A1**
	(2)
$d - 3 = \dfrac{1+3}{d} \Rightarrow d^2 - 3d - 4 = 0$	**M1**
Equates expressions given in (a) and (b), with correct use of coefficients.	
$d = 4$ (only, for final answer)	**A1**
	(2)
$\alpha^2 + \beta^2 + \gamma^2 = b^2 - 2c = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$	**M1**
$= 15$	**A1**
	(2)
	Total: 9

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