Further Math with Ali Hashir

Polar Coordinates

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics Paper 1 past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/11, 9231/12 • May/June 2015 • Question 5

The curves C_1 and C_2 have polar equations

$$C_1: \quad r = \frac{1}{\sqrt{2}}, \quad \text{for } 0 \leqslant \theta < 2\pi,$$

$$C_2: \quad r = \sqrt{\left(\sin \tfrac{1}{2}\theta\right)}, \quad \text{for } 0 \leqslant \theta \leqslant \pi.$$

Find the polar coordinates of the point of intersection of C_1 and C_2. [2]

Sketch C_1 and C_2 on the same diagram. [3]

Find the exact value of the area of the region enclosed by C_1, C_2 and the half-line $\theta = 0$. [4]

Q2

9231/13 • May/June 2015 • Question 2

The curve C has polar equation $r = \mathrm{e}^{4\theta}$ for $0 \leqslant \theta \leqslant \alpha$, where α is measured in radians. The length of C is 2015. Find the value of α. [6]

Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 11 (OR)

Answer only **one** of the following two alternatives.

The curve C has polar equation $r = a(1 - \cos\theta)$ for $0 \leqslant \theta < 2\pi$. Sketch C. [2]

Find the area of the region enclosed by the arc of C for which $\frac{1}{2}\pi \leqslant \theta \leqslant \frac{3}{2}\pi$, the half-line $\theta = \frac{1}{2}\pi$ and the half-line $\theta = \frac{3}{2}\pi$. [5]

Show that

$$\left(\frac{\mathrm{d}s}{\mathrm{d}\theta}\right)^2 = 4a^2 \sin^2\left(\tfrac{1}{2}\theta\right),$$

where s denotes arc length, and find the length of the arc of C for which $\frac{1}{2}\pi \leqslant \theta \leqslant \frac{3}{2}\pi$. [7]

Q4

9231/11, 9231/12 • May/June 2016 • Question 4

A curve C has polar equation $r^2 = 8\operatorname{cosec} 2\theta$ for $0 < \theta < \frac{1}{2}\pi$. Find a cartesian equation of C. [3]

Sketch C. [2]

Determine the exact area of the sector bounded by the arc of C between $\theta = \frac{1}{6}\pi$ and $\theta = \frac{1}{3}\pi$, the half-line $\theta = \frac{1}{6}\pi$ and the half-line $\theta = \frac{1}{3}\pi$. [3]

[It is given that $\displaystyle\int \operatorname{cosec} x \, dx = \ln\left|\tan \tfrac{1}{2}x\right| + c$.]

Q5

9231/13 • May/June 2016 • Question 7

A curve has polar equation $r = \dfrac{1}{1-\cos\theta}$, for $0 < \theta < 2\pi$. Find, in the form $y^2 = f(x)$, the cartesian equation of the curve. [3]

Hence sketch the curve, and shade the region whose area is given by $\dfrac{1}{2}\displaystyle\int_{\frac{1}{2}\pi}^{\frac{3}{2}\pi} \dfrac{1}{(1-\cos\theta)^2}\,d\theta$. [3]

Using the cartesian equation of the curve, find the area of this region. [3]

Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 11 (OR)

Answer only **one** of the following two alternatives.

OR

A curve C has parametric equations

$$x = 1 - 3t^2, \qquad y = t(1 - 3t^2), \qquad \text{for } 0 \leqslant t \leqslant \frac{1}{\sqrt{3}}.$$

Show that $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (1 + 9t^2)^2$. [2]

Hence find

(i) the arc length of C, [2]

(ii) the surface area generated when C is rotated through 2π radians about the x-axis. [3]

Use the fact that $t = \frac{y}{x}$ to find a cartesian equation of C. Hence show that the polar equation of C is $r = \sec\theta(1 - 3\tan^2\theta)$, and state the domain of θ. [4]

Find the area of the region enclosed between C and the initial line. [3]

Q7

9231/11, 9231/12 • May/June 2017 • Question 11

The curve C has polar equation $r = a(1 + \sin\theta)$ for $-\pi < \theta \leqslant \pi$, where a is a positive constant.

(i) Sketch C. [2]

(ii) Find the area of the region enclosed by C. [4]

(iii) Show that the length of the arc of C from the pole to the point furthest from the pole is given by

$$s = (\sqrt{2})a \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \sqrt{(1 + \sin\theta)}\,\mathrm{d}\theta.$$

[3]

(iv) Show that the substitution $u = 1 + \sin\theta$ reduces this integral for s to $(\sqrt{2})a \int_0^2 \frac{1}{\sqrt{(2-u)}}\,\mathrm{d}u$. Hence evaluate s. [4]

Q8

9231/13 • May/June 2017 • Question 11 (EITHER)

Answer only **one** of the following two alternatives.

EITHER

A curve C has polar equation $r = 2a\cos\left(2\theta + \frac{1}{2}\pi\right)$ for $0 \leqslant \theta < 2\pi$, where a is a positive constant.

(i) Show that $r = -2a\sin 2\theta$ and sketch C. [4]

(ii) Deduce that the cartesian equation of C is

$$(x^2 + y^2)^{\frac{3}{2}} = -4axy.$$

[2]

(iii) Find the area of one loop of C. [5]

(iv) Show that, at the points (other than the pole) at which a tangent to C is parallel to the initial line,

$$2\tan\theta = -\tan 2\theta.$$

[3]

Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 11 (OR)

OR

The polar equation of a curve C is $r = a(1 + \cos\theta)$ for $0 \leqslant \theta < 2\pi$, where a is a positive constant.

(i) Sketch C. [2]

(ii) Show that the cartesian equation of C is

$$x^2 + y^2 = a\left(x + \sqrt{(x^2 + y^2)}\right).$$

[2]

(iii) Find the area of the sector of C between $\theta = 0$ and $\theta = \frac{1}{3}\pi$. [4]

(iv) Find the arc length of C between the point where $\theta = 0$ and the point where $\theta = \frac{1}{3}\pi$. [5]

Q10

9231/11, 9231/12 • May/June 2018 • Question 3

The curve C has polar equation $r = \cos 2\theta$, for $-\frac{1}{4}\pi \leqslant \theta \leqslant \frac{1}{4}\pi$.

(i) Sketch C. [2]

(ii) Find the area of the region enclosed by C, showing full working. [3]

(iii) Find a cartesian equation of C. [3]

Q11

9231/13 • May/June 2018 • Question 8

The curves C_1 and C_2 have polar equations, for $0 \leqslant \theta \leqslant \pi$, as follows:

$$C_1 : r = a,$$

$$C_2 : r = 2a|\cos\theta|,$$

where a is a positive constant. The curves intersect at the points P_1 and P_2.

(i) Find the polar coordinates of P_1 and P_2. [2]

(ii) In a single diagram, sketch C_1, C_2 and their line of symmetry. [3]

(iii) The region R enclosed by C_1 and C_2 is bounded by the arcs OP_1, P_1P_2 and P_2O, where O is the pole. Find the area of R, giving your answer in exact form. [5]

Q12

9231/11, 9231/13 • October/November 2018 • Question 9

The curve C has polar equation

$$r = 5\sqrt{(\cot\theta)},$$

where $0.01 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Find the area of the finite region bounded by C and the line $\theta = 0.01$, showing full working. Give your answer correct to 1 decimal place. [3]

Let P be the point on C where $\theta = 0.01$.

(ii) Find the distance of P from the initial line, giving your answer correct to 1 decimal place. [2]

(iii) Find the maximum distance of C from the initial line. [3]

(iv) Sketch C. [2]

Q13

9231/12 • October/November 2018 • Question 3

The curve C has polar equation $r = a\cos 3\theta$, for $-\frac{1}{6}\pi \leqslant \theta \leqslant \frac{1}{6}\pi$, where a is a positive constant.

(i) Sketch C. [2]

(ii) Find the area of the region enclosed by C, showing full working. [3]

(iii) Using the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$, find a cartesian equation of C. [3]

Q14

9231/11, 9231/12 • May/June 2019 • Question 11 (EITHER)

Answer only **one** of the following two alternatives.

The curve C_1 has polar equation $r^2 = 2\theta$, for $0 \le \theta \le \frac{1}{2}\pi$.

(i) The point on C_1 furthest from the line $\theta = \frac{1}{2}\pi$ is denoted by P. Show that, at P,

$$2\theta \tan\theta = 1$$

and verify that this equation has a root between 0.6 and 0.7. [5]

The curve C_2 has polar equation $r^2 = \theta \sec^2\theta$, for $0 \le \theta < \frac{1}{2}\pi$. The curves C_1 and C_2 intersect at the pole, denoted by O, and at another point Q.

(ii) Find the exact value of θ at Q. [2]

(iii) The diagram below shows the curve C_2. Sketch C_1 on this diagram. [2]

[Diagram: axes labelled $\theta = \frac{1}{2}\pi$ (vertical) and $\theta = 0$ (horizontal), with curve C_2 shown emanating from pole O along $\theta = 0$ and curving up toward the line $\theta = \frac{1}{2}\pi$.]

(iv) Find, in exact form, the area of the region OPQ enclosed by C_1 and C_2. [5]

Q15

9231/13 • May/June 2019 • Question 2

The curve C has polar equation $r^2 = \ln(1 + \theta)$, for $0 \leq \theta \leq 2\pi$.

(i) Sketch C. [2]

(ii) Using the substitution $u = 1 + \theta$, or otherwise, find the area of the region bounded by C and the initial line, leaving your answer in an exact form. [5]

Q16

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 11 (OR)

OR

The curves C_1 and C_2 have polar equations, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$, as follows:

$$C_1 : r = 2(\mathrm{e}^{\theta} + \mathrm{e}^{-\theta}), \qquad C_2 : r = \mathrm{e}^{2\theta} - \mathrm{e}^{-2\theta}.$$

The curves intersect at the point P where $\theta = \alpha$.

(i) Show that $\mathrm{e}^{2\alpha} - 2\mathrm{e}^{\alpha} - 1 = 0$. Hence find the exact value of α and show that the value of r at P is $4\sqrt{2}$. [6]

(ii) Sketch C_1 and C_2 on the same diagram. [3]

(iii) Find the area of the region enclosed by C_1, C_2 and the initial line, giving your answer correct to 3 significant figures. [5]

Q17

9231/11, 9231/12 • May/June 2020 • Question 7

The curve C_1 has polar equation $r = \theta \cos \theta$, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(a) The point on C_1 furthest from the line $\theta = \frac{1}{2}\pi$ is denoted by P. Show that, at P,

$$2\theta \tan \theta - 1 = 0$$

and verify that this equation has a root between 0.6 and 0.7. [5]

The curve C_2 has polar equation $r = \theta \sin \theta$, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. The curves C_1 and C_2 intersect at the pole, denoted by O, and at another point Q.

(b) Find the polar coordinates of Q, giving your answers in exact form. [2]

(c) Sketch C_1 and C_2 on the same diagram. [3]

(d) Find, in terms of π, the area of the region bounded by the arc OQ of C_1 and the arc OQ of C_2. [7]

Q18

9231/13 • May/June 2020 • Question 5

The curve C has polar equation $r = a\tan\theta$, where a is a positive constant and $0 \leqslant \theta \leqslant \frac{1}{4}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole. [2]

(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{4}\pi$. [4]

(c) Show that C has Cartesian equation $y = \dfrac{x^2}{\sqrt{a^2 - x^2}}$. [3]

(d) Using your answer to part (b), deduce the exact value of $\displaystyle\int_0^{\frac{1}{2}a\sqrt{2}} \frac{x^2}{\sqrt{a^2 - x^2}}\,\mathrm{d}x$. [2]

Q19

9231/11, 9231/13 • October/November 2020 • Question 7

(a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^{\frac{5}{2}} = 4xy(x^2 - y^2)$$

has polar equation $r = \sin 4\theta$. [4]

The curve C has polar equation $r = \sin 4\theta$, for $0 \leqslant \theta \leqslant \frac{1}{4}\pi$.

(b) Sketch C and state the equation of the line of symmetry. [3]

(c) Find the exact value of the area of the region enclosed by C. [4]

(d) Using the identity $\sin 4\theta \equiv 4\sin\theta\cos^3\theta - 4\sin^3\theta\cos\theta$, find the maximum distance of C from the line $\theta = \frac{1}{2}\pi$. Give your answer correct to 2 decimal places. [6]

Q20

9231/12 • October/November 2020 • Question 5

The curve C has polar equation $r = \ln(1 + \pi - \theta)$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C and state the polar coordinates of the point of C furthest from the pole. [3]

(b) Using the substitution $u = 1 + \pi - \theta$, or otherwise, show that the area of the region enclosed by C and the initial line is

$$\tfrac{1}{2}(1 + \pi)\ln(1 + \pi)\big(\ln(1 + \pi) - 2\big) + \pi.$$

[6]

(c) Show that, at the point of C furthest from the initial line,

$$(1 + \pi - \theta)\ln(1 + \pi - \theta) - \tan\theta = 0$$

and verify that this equation has a root between 1.2 and 1.3. [5]

Q21

9231/11, 9231/12 • May/June 2021 • Question 5

The curve C has polar equation $r = a\cot\left(\frac{1}{3}\pi - \theta\right)$, where a is a positive constant and $0 \leq \theta \leq \frac{1}{6}\pi$.

It is given that the greatest distance of a point on C from the pole is $2\sqrt{3}$.

(a) Sketch C and show that $a = 2$. [3]

(b) Find the exact value of the area of the region bounded by C, the initial line and the half-line $\theta = \frac{1}{6}\pi$. [4]

(c) Show that C has Cartesian equation $2\left(x + y\sqrt{3}\right) = \left(x\sqrt{3} - y\right)\sqrt{x^2 + y^2}$. [3]

Q22

9231/13 • May/June 2021 • Question 5

The curve C has polar equation $r = \dfrac{1}{\pi - \theta} - \dfrac{1}{\pi}$, where $0 \leq \theta \leq \frac{1}{2}\pi$.

(a) Sketch C. [3]

(b) Show that the area of the region bounded by the half-line $\theta = \frac{1}{2}\pi$ and C is $\dfrac{3 - 4\ln 2}{4\pi}$. [6]

Q23

9231/11, 9231/13 • October/November 2021 • Question 6

The curve C has polar equation $r = 2\cos\theta(1+\sin\theta)$, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(a) Find the polar coordinates of the point on C that is furthest from the pole. [5]

(b) Sketch C. [2]

(c) Find the area of the region bounded by C and the initial line, giving your answer in exact form. [6]

Q24

9231/12 • October/November 2021 • Question 5

The curve C has polar equation $r = 3 + 2\sin\theta$, for $-\pi < \theta \leqslant \pi$.

(a) The diagram shows part of C. Sketch the rest of C on the diagram. [1]

The straight line l has polar equation $r\sin\theta = 2$.

(b) Add l to the diagram in part (a) and find the polar coordinates of the points of intersection of C and l. [5]

The region R is enclosed by C and l, and contains the pole.

(c) Find the area of R, giving your answer in exact form. [6]

Q25

9231/11, 9231/12 • May/June 2022 • Question 6

The curve C has polar equation $r^2 = \tan^{-1}\left(\frac{1}{2}\theta\right)$, where $0 \le \theta \le 2$.

(a) Sketch C and state, in exact form, the greatest distance of a point on C from the pole. [3]

(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = 2$. [5]

Now consider the part of C where $0 \le \theta \le \frac{1}{2}\pi$.

(c) Show that, at the point furthest from the half-line $\theta = \frac{1}{2}\pi$,

$$(\theta^2 + 4)\tan^{-1}\left(\frac{1}{2}\theta\right)\sin\theta - \cos\theta = 0$$

and verify that this equation has a root between 0.6 and 0.7. [5]

Q26

9231/13 • May/June 2022 • Question 6

The curve C has Cartesian equation $x^2 + xy + y^2 = a$, where a is a positive constant.

(a) Show that the polar equation of C is $r^2 = \dfrac{2a}{2 + \sin 2\theta}$. [3]

(b) Sketch the part of C for $0 \leq \theta \leq \frac{1}{4}\pi$. [2]

The region R is enclosed by this part of C, the initial line and the half-line $\theta = \frac{1}{4}\pi$.

(c) It is given that $\sin 2\theta$ may be expressed as $\dfrac{2\tan\theta}{1 + \tan^2\theta}$. Use this result to show that the area of R is

$$\frac{1}{2}a \int_0^{\frac{1}{4}\pi} \frac{1 + \tan^2\theta}{1 + \tan\theta + \tan^2\theta}\, \mathrm{d}\theta$$

and use the substitution $t = \tan\theta$ to find the exact value of this area. [8]

Q27

9231/11, 9231/13 • October/November 2022 • Question 6

(a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^2 = 36(x^2 - y^2)$$

has polar equation $r^2 = 36\cos 2\theta$. [3]

The curve C has polar equation $r^2 = 36\cos 2\theta$, for $-\frac{1}{4}\pi \leqslant \theta \leqslant \frac{1}{4}\pi$.

(b) Sketch C and state the maximum distance of a point on C from the pole. [3]

(c) Find the area of the region enclosed by C. [2]

(d) Find the maximum distance of a point on C from the initial line, giving the answer in exact form. [6]

Q28

9231/12 • October/November 2022 • Question 5

The curve C has polar equation $r = a\sec^2\theta$, where a is a positive constant and $0 \leqslant \theta \leqslant \frac{1}{4}\pi$.

(a) Sketch C, stating the polar coordinates of the point of intersection of C with the initial line and also with the half-line $\theta = \frac{1}{4}\pi$. [3]

(b) Find the maximum distance of a point of C from the initial line. [2]

(c) Find the area of the region enclosed by C, the initial line and the half-line $\theta = \frac{1}{4}\pi$. [4]

(d) Find, in the form $y = \mathrm{f}(x)$, the Cartesian equation of C. [3]

Q29

9231/11, 9231/12 • May/June 2023 • Question 5

The curve C has polar equation $r^2 = \dfrac{1}{\theta^2 + 1}$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C and state the polar coordinates of the point of C furthest from the pole. [3]

(b) Find the area of the region enclosed by C, the initial line, and the half-line $\theta = \pi$. [4]

(c) Show that, at the point of C furthest from the initial line,

$$\left(\theta + \frac{1}{\theta}\right)\cot\theta - 1 = 0$$

and verify that this equation has a root between 1.1 and 1.2. [5]

Q30

9231/13 • May/June 2023 • Question 5

(a) Show that the curve with Cartesian equation

$$x^2 - y^2 = a,$$

where a is a positive constant, has polar equation $r^2 = a \sec 2\theta$. [3]

The curve C has polar equation $r^2 = a \sec 2\theta$, where a is a positive constant, for $0 \leqslant \theta < \frac{1}{4}\pi$.

(b) Sketch C and state the minimum distance of C from the pole. [3]

(c) Find, in terms of a, the exact value of the area of the region enclosed by C, the initial line, and the half-line $\theta = \frac{1}{12}\pi$. [You may use any result from the list of formulae (MF19) without proof.] [4]

Q31

9231/11, 9231/13 • October/November 2023 • Question 6

(a) Show that the curve with Cartesian equation

$$\left(x - \tfrac{1}{2}\right)^2 + y^2 = \tfrac{1}{4}$$

has polar equation $r = \cos\theta$. [3]

The curves C_1 and C_2 have polar equations

$$r = \cos\theta \qquad \text{and} \qquad r = \sin 2\theta$$

respectively, where $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. The curves C_1 and C_2 intersect at the pole and at another point P.

(b) Find the polar coordinates of P. [3]

(c) In a single diagram sketch C_1 and C_2, clearly identifying each curve, and mark the point P. [3]

(d) The region R is enclosed by C_1 and C_2 and includes the line OP.

Find, in exact form, the area of R. [6]

Q32

9231/12 • October/November 2023 • Question 6

The curve C has polar equation $r = e^{-\theta} - e^{-\frac{1}{2}\pi}$, where $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(a) Sketch C and state, in exact form, the greatest distance of a point on C from the pole. [3]

(b) Find the exact value of the area of the region bounded by C and the initial line. [5]

(c) Show that, at the point on C furthest from the initial line,

$$1 - e^{\theta - \frac{1}{2}\pi} - \tan\theta = 0$$

and verify that this equation has a root between 0.56 and 0.57. [5]

Q33

9231/11, 9231/12 • May/June 2024 • Question 7

The curve C has polar equation $r^2 = (\pi - \theta)\tan^{-1}(\pi - \theta)$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C and state the polar coordinates of the point of C furthest from the pole. [3]

(b) Using the substitution $u = \pi - \theta$, or otherwise, find the area of the region enclosed by C and the initial line. [7]

(c) Show that, at the point of C furthest from the initial line,

$$2(\pi - \theta)\tan^{-1}(\pi - \theta)\cot\theta - \frac{\pi - \theta}{1 + (\pi - \theta)^2} - \tan^{-1}(\pi - \theta) = 0$$

and verify that this equation has a root for θ between 1.2 and 1.3. [5]

Q34

9231/13 • May/June 2024 • Question 7

The curve C has polar equation $r^2 = \sin 2\theta \cos\theta$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C and state the equation of the line of symmetry. [3]

(b) Find a Cartesian equation for C. [3]

(c) Find the total area enclosed by C. [4]

(d) Find the greatest distance of a point on C from the pole. [6]

Q35

9231/11, 9231/13 • October/November 2024 • Question 5

(a) Show that the curve with Cartesian equation

$$\left(x^2 + y^2\right)^2 = 6xy$$

has polar equation $r^2 = 3\sin 2\theta$. [2]

The curve C has polar equation $r^2 = 3\sin 2\theta$, for $0 \leq \theta \leq \frac{1}{2}\pi$.

(b) Sketch C and state the maximum distance of a point on C from the pole. [3]

(c) Find the area of the region enclosed by C. [2]

(d) Find the maximum distance of a point on C from the initial line. [6]

Q36

9231/12 • October/November 2024 • Question 7

The curve C_1 has polar equation $r = a(\cos\theta + \sin\theta)$ for $-\frac{1}{4}\pi \leq \theta \leq \frac{3}{4}\pi$, where a is a positive constant.

(a) Find a Cartesian equation for C_1 and show that it represents a circle, stating its radius and the Cartesian coordinates of its centre. [4]

(b) Sketch C_1 and state the greatest distance of a point on C_1 from the pole. [3]

The curve C_2 with polar equation $r = a\theta$ intersects C_1 at the pole and the point with polar coordinates $(a\phi, \phi)$.

(c) Verify that $1.25 < \phi < 1.26$. [2]

(d) Show that the area of the smaller region enclosed by C_1 and C_2 is equal to

$$\frac{1}{2}a^2\left(\frac{3}{4}\pi + \frac{1}{3}\phi^3 - \phi + \frac{1}{2}\cos 2\phi\right)$$

and deduce, in terms of a and ϕ, the area of the larger region enclosed by C_1 and C_2. [7]

Q37

9231/11, 9231/12 • May/June 2025 • Question 5

The curve C has polar equation $r = \theta e^{\frac{1}{8}\theta}$, for $0 \leqslant \theta \leqslant 2\pi$.

(a) Sketch C. [2]

(b) Find the area of the region bounded by C and the initial line, giving your answer in the form $\left(p\pi^2 + q\pi + r\right) e^{\frac{1}{2}\pi} + s$, where p, q, r and s are integers to be determined. [6]

(c) Show that, at the point of C furthest from the initial line,

$$\theta\cos\theta + \left(\tfrac{1}{8}\theta + 1\right)\sin\theta = 0$$

and verify that this equation has a root between 5 and 5.05. [5]

Q38

9231/13 • May/June 2025 • Question 7

The curve C has polar equation $r^2 = e^{\sin\theta}\cos\theta$, for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$.

(a) Find the polar coordinates of the point on C that is furthest from the pole, giving your answers correct to 3 decimal places. [5]

(b) Find the polar coordinates of the point on C that is furthest from the half-line $\theta = \frac{1}{2}\pi$, giving your answers correct to 3 decimal places. [5]

(c) Sketch C. [3]

(d) Find the area of the region bounded by C, giving your answer in exact form. [3]

Q39

9231/11, 9231/13 • October/November 2025 • Question 6

The curve C has polar equation $r = \sin 3\theta$, for $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.

(a) Sketch C and state the equation of the line of symmetry. [3]

(b) Find the exact value of the area of the region enclosed by C. [4]

In parts (c) and (d) you may use the identity $\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$.

(c) Find the maximum distance of a point on C from the initial line. [5]

(d) Find a Cartesian equation for C. [3]

Q40

9231/12 • October/November 2025 • Question 5

The curve C has polar equation $r^2 = \tan 2\theta$, where $0 \leqslant \theta \leqslant \frac{1}{8}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole. [2]

(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{8}\pi$. [4]

(c) Show that C has Cartesian equation $x^4 - 2xy - y^4 = 0$ given that $0 \leqslant x \leqslant \cos\left(\frac{1}{8}\pi\right)$ and $0 \leqslant y \leqslant \sin\left(\frac{1}{8}\pi\right)$. [4]

(d) Using your answer to (b), deduce the exact value of the area bounded by C, the x-axis and the line $x = \cos\left(\frac{1}{8}\pi\right)$. [2]

Q41

9231/14 • October/November 2025 • Question 6

The curve C has polar equation $r = \cos\frac{1}{2}\theta$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C. [2]

In parts **(b)** and **(c)** you may use the identities $\sin\theta \equiv 2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta$ and $\cos\theta \equiv 2\cos^2\frac{1}{2}\theta - 1$.

(b) Find the exact value of the area of the region enclosed by C and the initial line. [4]

(c) Find the maximum distance of a point on C from the initial line. Give your answer in the form $p\sqrt{q}$, where p and q are rational numbers to be determined. [6]

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Answers available in the companion mark scheme booklet.