Further Math with Ali Hashir

Polar Coordinates

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 Paper 1 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/11, 9231/12 • May/June 2015 • Question 5

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Mark scheme.

$\sin\frac{1}{2}\theta = \frac{1}{2} \Rightarrow \theta = \frac{1}{3}\pi$; Intersection at $\left(\frac{1}{\sqrt{2}}, \frac{1}{3}\pi\right)$ (Accept 1.05 for $\frac{1}{3}\pi$.)	**M1A1**
	(2)
C_1: Circle centre at pole and radius $1/\sqrt{2}$	**B1**
C_2: Curve, approx. correct orientation, from $(0,0)$ to $(1,\pi)$.	**B1**
Completely correct correct shape.	**B1**
	(3)
$\frac{1}{6} \times \pi \times \frac{1}{2} - \frac{1}{2}\int_0^{\frac{1}{3}\pi} \sin\frac{1}{2}\theta \, \mathrm{d}\theta$	**M1A1**
$= \frac{1}{12}\pi \times \frac{1}{2}\Big[-2\cos\frac{1}{2}\theta\Big]_0^{\frac{1}{3}\pi} = \frac{1}{12}\pi + \frac{\sqrt{3}}{2} - 1$	**M1A1**
	(4)
	Total: 9

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Q2

9231/13 • May/June 2015 • Question 2

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Mark scheme.

$s = \int_0^\alpha \sqrt{e^{8\theta} + 16e^{8\theta}}\,d\theta = \int_0^\alpha \sqrt{17}\,e^{4\theta}\,d\theta$ (AEF) (LNR)	**M1A1**
$= \left[\frac{\sqrt{17}}{4}e^{4\theta}\right]_0^\alpha \Rightarrow \frac{\sqrt{17}}{4}e^{4\alpha} - \frac{\sqrt{17}}{4} = 2015$	**dM1A1**
$\Rightarrow e^{4\alpha} = 1955.837\ldots \Rightarrow \alpha = 1.89$ CAO	**dM1A1**
	(6)
	Total: 6

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Q3

9231/11, 9231/12, 9231/13 • October/November 2015 • Question 11 (OR)

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Mark scheme.

Closed curve through pole with correct orientation.	**B1**
Completely correct.	**B1**
	(2)
$2 \times \frac{1}{2}a^2 \int_{\frac{1}{2}\pi}^{\pi} (1 - 2\cos\theta + \cos^2\theta)\mathrm{d}\theta = a^2 \int_{\frac{1}{2}\pi}^{\pi} \left(\frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos 2\theta\right) \mathrm{d}\theta$	**M1M1**
$= a^2 \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_{\frac{1}{2}\pi}^{\pi}$	**M1A1**
$= a^2 \left(\frac{3}{4}\pi + 2\right)$	**A1**
	(5)
$\left(\frac{\mathrm{d}s}{\mathrm{d}\theta}\right)^2 = a^2(1 - 2\cos\theta + \cos^2\theta + \sin^2\theta)$	**B1**
$= 2a^2(1 - \cos\theta) = 2a^2.2\sin^2\frac{1}{2}\theta = 4a^2\sin^2\frac{1}{2}\theta$ (AG)	**M1A1**
$s = 2 \times \int_{\frac{1}{2}\pi}^{\pi} 2a\sin\frac{1}{2}\theta\,\mathrm{d}\theta$	**M1**
$= 4a\left[-2\cos\frac{1}{2}\theta\right]_{\frac{1}{2}\pi}^{\pi}$	**A1**
$= 4\sqrt{2}a$	**M1A1**
	(7)
	Total: 14

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Q4

9231/11, 9231/12 • May/June 2016 • Question 4

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Mark scheme.

Using $x = r\cos\theta$ and $y = r\sin\theta$	**B1**
$r^2 = 8\,\text{cosec}\,2\theta \Rightarrow r^2 = \dfrac{4}{\sin\theta\cos\theta}$	**M1**
$\Rightarrow r\cos\theta . r\sin\theta = 4 \Rightarrow xy = 4$ (in simple form)	**A1**
	(3)
Sketch: Curve in 1st quadrant with correct concavity, asymptotic to **both** axes.	**B1B1**
	(2)
$\dfrac{1}{2}\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 8\,\text{cosec}\,2\theta\,d\theta = [2\ln\lvert\tan\theta\rvert]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}$	**M1A1**
$= 2\left\{\ln\left\lvert\sqrt{3}\right\rvert - \ln\left\lvert\dfrac{1}{\sqrt{3}}\right\rvert\right\} = 2\ln 3$ or $\ln 9$	**A1**
	(3)
	Total: 8

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Q5

9231/13 • May/June 2016 • Question 7

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Mark scheme.

$\sqrt{x^2+y^2} = \dfrac{1}{1-\dfrac{x}{\sqrt{x^2+y^2}}} \Rightarrow \sqrt{x^2+y^2} - x = 1$	**M1A1**
$\Rightarrow y^2 = 1 + 2x$ or equivalent RHS	**A1**
	(3)
Sketch: Parabola symmetrical about x-axis. Intercepts at $\left(-\dfrac{1}{2}, 0\right)$ and $(0, \pm 1)$.	**B1 B1**
Shading correct area	**B1**
	(3)
Recognises $\dfrac{1}{2}\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \dfrac{1}{(1-\cos\theta)^2}\, d\theta$ as the area of the region between parabola and y-axis.	**M1**
$= 2 \times \displaystyle\int_{-\frac{1}{2}}^{0} (1+2x)^{\frac{1}{2}}\, dx$	
$= 2\left[\dfrac{1}{3}(1+2x)^{\frac{3}{2}}\right]_{-\frac{1}{2}}^{0} = \dfrac{2}{3}$	**M1A1**
	(4)
	Total: 10

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Q6

9231/11, 9231/12, 9231/13 • October/November 2016 • Question 11 (OR)

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Mark scheme.

$\dot{x}^2+\dot{y}^2 = 36t^2+1-18t^2+81t^4 = 1+18t^2+81t^4 = \left(1+9t^2\right)^2$ (AG)	**M1A1**
	(2)
$s = \int_0^{\frac{1}{\sqrt{3}}} \left(1+9t^2\right) \mathrm{d}t = \left[t+3t^3\right]_0^{\frac{1}{\sqrt{3}}} = \frac{2}{\sqrt{3}}$	**M1A1**
	(2)
$S = 2\pi \int_0^{\frac{1}{\sqrt{3}}} t(1-3t^2)(1+9t^2)\,\mathrm{d}t = 2\pi \int_0^{\frac{1}{\sqrt{3}}} (t+6t^3-27t^5)\,\mathrm{d}t$	***M1**
$= 2\pi \left[\frac{1}{2}t^2+\frac{3}{2}t^4-\frac{9}{2}t^6\right]_0^{\frac{1}{\sqrt{3}}} = \frac{\pi}{3}$	**DM1A1**
	(3)
Substitute $t=\frac{y}{x}$ in one of the parametric equations to obtain e.g. $x = 1-3\frac{y^2}{x^2}$.	**M1**
Uses $x=r\cos\theta$ and $y=r\sin\theta \Rightarrow r\cos\theta = 1-3\tan^2\theta \Rightarrow r=\sec\theta(1-3\tan^2\theta)$.	**M1A1**
States domain for θ: $0 \leqslant \theta \leqslant \frac{\pi}{6}$,	**B1**
	(4)
$A = \frac{1}{2}\int_0^{\frac{\pi}{6}} \sec^2\theta\left(1-6\tan^2\theta+9\tan^4\theta\right)\mathrm{d}\theta$	**M1**
$= \frac{1}{2}\left[\tan\theta-2\tan^3\theta+\frac{9}{5}\tan^5\theta\right]_0^{\frac{\pi}{6}} = \frac{4}{45}\sqrt{3}$	**A1A1**
	(3)
	Total: 14

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Q7

9231/11, 9231/12 • May/June 2017 • Question 11

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Mark scheme.

Sketch of a cardioid	**B1**
Correct orientation and labelled	**B1**
	(2)
$A = \dfrac{a^2}{2}\displaystyle\int_{-\pi}^{\pi} \left(1 + 2\sin\theta + \sin^2\theta\right) \mathrm{d}\theta$	**M1**
$= \left(\dfrac{a^2}{2}\right)\displaystyle\int_{-\pi}^{\pi} \left(1 + 2\sin\theta + \dfrac{1}{2} - \dfrac{1}{2}\cos 2\theta\right) \mathrm{d}\theta$	**M1**
$= \left(\dfrac{a^2}{2}\right)\left[\dfrac{3\theta}{2} - 2\cos\theta - \dfrac{1}{4}\sin 2\theta\right]_{-\pi}^{\pi} = \dfrac{3\pi a^2}{2}$	**M1A1**
	(4)
Show that when $r = 0$, $\theta = -\dfrac{\pi}{2}$, when $r = 2a$ $\theta = \dfrac{\pi}{2}$ and that $\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = a\cos\theta$	**B1**
$s = \displaystyle\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \sqrt{a^2(1 + 2\sin\theta + a^2\sin^2\theta) + a^2\cos^2\theta}\, \mathrm{d}\theta = \sqrt{2}a \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \sqrt{1 + \sin\theta}\, \mathrm{d}\theta$ (Uses correct formula for arc length; AG)	**M1A1**
	(3)
$u = 1 + \sin\theta \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}\theta} = \cos\theta = \sqrt{1 - (u-1)^2} = \sqrt{2u - u^2}$	**M1**
so $s = \sqrt{2}a \displaystyle\int_0^2 \dfrac{1}{\sqrt{2-u}}\, \mathrm{d}u$ AG (Including limits)	**A1**
$= \sqrt{2}a \left[-2(2-u)^{\frac{1}{2}}\right]_0^2 = 4a$	**M1A1**
	(4)
	Total: 13

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Q8

9231/13 • May/June 2017 • Question 11 (EITHER)

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Mark scheme.

$r = 2a\left(\cos 2\theta \cos \frac{1}{2}\pi - \sin 2\theta \sin \frac{1}{2}\pi\right) = 2a(0 - \sin 2\theta.1) = -2a\sin 2\theta$ (AG)	**M1A1**
[Sketch: loops in 2nd and 4th quadrant]	**B1**
Symmetry and shape correct	**B1**
	(4)
$r = -4a\sin\theta\cos\theta = -4a.\frac{y}{r}.\frac{x}{r} \Rightarrow r^3 = -4axy \Rightarrow \left((x^2+y^2)^{\frac{3}{2}} = -4axy\right)$ (AG)	**M1A1**
	(2)
Area of one loop is $\frac{1}{2}\int_{\frac{1}{2}\pi}^{\pi} 4a^2\sin^2 2\theta\,\mathrm{d}\theta = a^2\int_{\frac{1}{2}\pi}^{\pi} 2\sin^2 2\theta\,\mathrm{d}\theta$ (OE)	**M1**
$= a^2\int_{\frac{1}{2}\pi}^{\pi}(1-\cos 4\theta)\,\mathrm{d}\theta$	**M1A1**
$= a^2\left[\theta - \frac{1}{4}\sin 4\theta\right]_{\frac{1}{2}\pi}^{\pi} = \frac{1}{2}\pi a^2$	**M1A1**
	(5)
$y = r\sin\theta = -2a\sin 2\theta\sin\theta$	**B1**
$\frac{\mathrm{d}y}{\mathrm{d}\theta} = -2a(2\cos 2\theta\sin\theta + \sin 2\theta\cos\theta) = 0$	**M1**
$2\cos 2\theta\sin\theta = -\sin 2\theta\cos\theta$ $(\Rightarrow 2\tan\theta = -\tan 2\theta)$ (AG)	**A1**
	(3)
	Total: 14

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Q9

9231/11, 9231/12, 9231/13 • October/November 2017 • Question 11 (OR)

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Mark scheme.

Closed curve (cardioid) starting and ending at pole, in approximately correct location.	**B1**
Cardioid with indication of correct scale.	**B1**
	(2)
$r = a(1+\cos\theta) \Rightarrow \sqrt{x^2+y^2} = a\left(1+\dfrac{x}{\sqrt{x^2+y^2}}\right)$	**M1**
$x^2+y^2 = a\left(x+\sqrt{(x^2+y^2)}\right)$ (Substitutes for r and $\cos(\theta)$)	**A1**
	(2)
Sector area $= \dfrac{a^2}{2}\displaystyle\int_0^{\frac{1}{3}\pi}\left(1+2\cos\theta+\cos^2\theta\right)\,\mathrm{d}\theta$	**M1A1**
$= \dfrac{a^2}{2}\displaystyle\int_0^{\frac{1}{3}\pi}\left(\dfrac{3}{2}+2\cos\theta+\dfrac{\cos 2\theta}{2}\right)\,\mathrm{d}\theta$	
$= \dfrac{a^2}{2}\left[\dfrac{3\theta}{2}+2\sin\theta+\dfrac{\sin 2\theta}{4}\right]_0^{\frac{1}{3}\pi}$	**M1**
$= \dfrac{a^2}{16}\left(4\pi+9\sqrt{3}\right)$	**A1**
	(4)
Arc length $= \displaystyle\int_0^{\frac{1}{3}\pi}\sqrt{a^2(1+2\cos\theta+\cos^2\theta)+a^2(-\sin\theta)^2}\,\mathrm{d}\theta$	**M1A1**
$= a\displaystyle\int_0^{\frac{1}{3}\pi}\sqrt{2+2\cos\theta}\,\mathrm{d}\theta$	**A1**
$= a\left[4\sin\dfrac{\theta}{2}\right]_0^{\frac{1}{3}\pi} = 2a$	**M1A1**
	(5)
	Total: 13

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Q10

9231/11, 9231/12 • May/June 2018 • Question 3

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Mark scheme.

Correct position including label of 1 on initial line, and symmetric about initial	**B1**
Single correct loop.	**B1**
	(2)
$\frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 2\theta \, d\theta$. Correct integral	**M1**
$= \frac{1}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 4\theta + 1 \, d\theta =$. Correct use of double angle formula	**M1**
$\frac{1}{4}\left[\frac{1}{4}\sin 4\theta + \theta\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{8} = 0.393$	**A1**
	(3)
$r = \cos^2\theta - \sin^2\theta$ OE. Uses trig identity	**B1**
Thus $\left(x^2 + y^2\right)^{\frac{3}{2}} = x^2 - y^2$. Uses $x = r\cos\theta$ or $y = r\sin\theta$ or both, AEF	**M1A1**
	(3)
	Total: 8

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Q11

9231/13 • May/June 2018 • Question 8

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Mark scheme.

$a = 2a\lvert\cos\theta\rvert \Rightarrow \cos\theta = \pm\frac{1}{2}$. Eliminates r.	**M1**
$\left(a, \frac{\pi}{3}\right)$ and $\left(a, \frac{2\pi}{3}\right)$. Both points needed for A1.	**A1**
	(2)
Semicircle for C_1 including $r = a$.	**B1**
Half of C_2 including $r = 2a$.	**B1**
Other half of C_2 and line of symmetry.	**B1**
	(3)
$4a^2 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$. Finds area of segment OP_1 and OP_2 of C_2	**M1**
$= 2a^2 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos 2\theta + 1\, d\theta$. Uses $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$	**M1**
$= 2a^2 \left[\frac{1}{2}\sin 2\theta + \theta\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = 2a^2\left(\frac{\pi}{2} - \left(\frac{\sqrt{3}}{4} + \frac{\pi}{3}\right)\right) = a^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$. Integrates correctly.	**A1**
Area $= \frac{\pi a^2}{6} - a^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) = -\frac{\pi a^2}{6} + \frac{a^2\sqrt{3}}{2}$. M1 for subtracting 'their' OP_1P_2 from $\frac{\pi a^2}{6}$	**M1A1FT**
	(5)
	Total: 10

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Q12

9231/11, 9231/13 • October/November 2018 • Question 9

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Mark scheme.

$\dfrac{25}{2}\displaystyle\int_{0.01}^{\frac{\pi}{2}} \cot\theta\,\mathrm{d}\theta$ (Uses $\frac{1}{2}\int r^2\,\mathrm{d}\theta$.)	**M1**
$= \dfrac{25}{2}\left[\ln\sin\theta\right]_{0.01}^{\frac{\pi}{2}}$	**A1**
$= -\dfrac{25}{2}\ln\sin 0.01 \approx 57.6$	**A1**
	(3)
$y = 5\cos^{\frac{1}{2}}\theta\sin^{\frac{1}{2}}\theta = \dfrac{5}{\sqrt{2}}\sin^{\frac{1}{2}} 2\theta$ (Uses $y = r\sin\theta$.)	**M1**
$\theta = 0.01 \Rightarrow y \approx 0.5$	**A1**
	(2)
$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \dfrac{5}{\sqrt{2}}\sin^{-\frac{1}{2}} 2\theta\cos 2\theta = 0$ or $\max(\sin 2\theta) = 1$ (Sets $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = 0$ or considers max (AEF).)	**M1A1**
$\Rightarrow y = \dfrac{5\sqrt{2}}{2}$ $(= 3.54)$	**A1**
	(3)
[Sketch: curve rising steeply from origin to a peak then decreasing asymptotically, intersecting the initial line only when $x = 0$ and $y = 0$.] (Intersecting the initial line only when $x = 0$ and $y = 0$.)	**B1**
Correct shape.	**B1**
	(2)
	Total: 10

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Q13

9231/12 • October/November 2018 • Question 3

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Mark scheme.

[Sketch: single loop, correct shape at extremities, symmetric about the initial line, passing through the pole with tangents at angle $\pm\frac{1}{6}\pi$, labelled point $(a, 0)$.] (Just one loop, correct shape at extremities)	**B1**
Correct position including $(a, 0)$ labelled or in table.	**B1**
	(2)
$\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} a^2\cos^2 3\theta\,\mathrm{d}\theta$ (For using correct formula)	**M1**
$\frac{a^2}{4}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (\cos 6\theta + 1)\,\mathrm{d}\theta$ (Using double angle formula correctly)	**M1**
$= \frac{a^2}{4}\left[\frac{1}{6}\sin 6\theta + \theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}} = \frac{\pi a^2}{12}$	**A1**
	(3)
$r = a\cos\theta(4\cos^2\theta - 3) \Rightarrow r = a\left(\frac{x}{r}\right)\left(4\left(\frac{x}{r}\right)^2 - 3\right)$ (Uses $x = r\cos\theta$ and $x^2 + y^2 = r^2$.)	**B1**
$\Rightarrow r^4 = ax(4x^2 - 3r^2) \Rightarrow (x^2 + y^2)^2 = ax\left(4x^2 - 3(x^2 + y^2)\right)$ (For eliminating θ)	**M1**
$\Rightarrow (x^2+y^2)^2 = ax(x^2-3y^2)$ (Any equivalent cartesian form without fractions.)	**A1**
	(3)
	Total: 8

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Q14

9231/11, 9231/12 • May/June 2019 • Question 11 (EITHER)

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Mark scheme.

$x = \sqrt{2}\theta^{\frac{1}{2}}\cos\theta$	**M1**
$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sqrt{2}\theta^{\frac{1}{2}}\cos\theta\right) = \sqrt{2}\left(-\theta^{\frac{1}{2}}\sin\theta + \frac{1}{2}\theta^{-\frac{1}{2}}\cos\theta\right) = 0$	**M1A1**
$-\theta^{\frac{1}{2}}\sin\theta + \frac{1}{2}\theta^{-\frac{1}{2}}\cos\theta = 0 \Rightarrow \cos\theta = 2\theta\sin\theta \Rightarrow 2\theta\tan\theta = 1$	**A1**
$2(0.6)\tan(0.6) - 1 = -0.179$ and $2(0.7)\tan(0.7) - 1 = 0.179$	**B1**
	(5)
$2\theta = \theta\sec^2\theta \Rightarrow \theta^{\frac{1}{2}}(\sec\theta - \sqrt{2}) = 0 \Rightarrow \theta = \frac{\pi}{4}$	**M1A1**
	(2)
[Sketch: correct shape]	**B1**
[Sketch: intersection correct]	**B1**
	(2)
$\frac{1}{2}\int_0^{\pi/4} 2\theta\,\mathrm{d}\theta - \frac{1}{2}\int_0^{\pi/4}\theta\sec^2\theta\,\mathrm{d}\theta = \frac{1}{2}\int_0^{\pi/4}\theta(2-\sec^2\theta)\,\mathrm{d}\theta$	**M1**
$\frac{1}{2}\left[\theta(2\theta - \tan\theta)\right]_0^{\pi/4} - \frac{1}{2}\int_0^{\pi/4}(2\theta - \tan\theta)\,\mathrm{d}\theta$	**M1A1**
$\frac{1}{2}\left[\theta(2\theta - \tan\theta)\right]_0^{\pi/4} - \frac{1}{2}\left[\theta^2 + \ln\cos\theta\right]_0^{\pi/4}$	**A1**
$\frac{\pi}{8}\left(\frac{\pi}{2} - 1\right) - \frac{1}{2}\left(\left(\frac{\pi}{4}\right)^2 + \frac{1}{2}\ln 2\right) = \frac{1}{4}\ln 2 + \frac{\pi}{8}\left(\frac{\pi}{4} - 1\right).$	**A1**
	(5)
	Total: 14

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Q15

9231/13 • May/June 2019 • Question 2

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Mark scheme.

[Sketch: correct shape, domain and orientation]	**B1**
[Sketch: the initial line is tangential to C at the pole]	**B1**
	(2)
$A = \dfrac{1}{2}\displaystyle\int_0^{2\pi} \ln(1+\theta)\,\mathrm{d}\theta$	**M1**
$\displaystyle\int_0^{2\pi} \ln(1+\theta)\,\mathrm{d}\theta = \int_1^{2\pi+1} \ln u\,\mathrm{d}u$	**M1**
$\displaystyle\int_1^{2\pi+1} \ln u\,\mathrm{d}u = [u\ln u]_1^{2\pi+1} - [u]_1^{2\pi+1}$	**M1A1**
$A = \left(\pi + \dfrac{1}{2}\right)\ln(2\pi+1) - \pi$	**A1**
	(5)
	Total: 7

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Q16

9231/11, 9231/12, 9231/13 • October/November 2019 • Question 11 (OR)

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Mark scheme.

$e^{2\alpha} - e^{-2\alpha} = 2(e^{\alpha} + e^{-\alpha}) \Rightarrow e^{\alpha} - e^{-\alpha} = 2$	**M1**
Sets equations equal and divides by $e^{\alpha} + e^{-\alpha}$.	
$e^{2\alpha} - 2e^{\alpha} - 1 = 0 \Rightarrow e^{\alpha} = 1 + \sqrt{2}$	**M1A1**
Forms quadratic in e^{α}, AG.	
$\alpha = \ln(1 + \sqrt{2})$	**A1**
Must be exact.	
$r = 2\left(1 + \sqrt{2} + \sqrt{2} - 1\right) = 4\sqrt{2}$	**M1A1**
Substitutes to find r.	
	(6)
C_1 has correct shape.	**B1**
C_2 has correct shape.	**B1**
Intersection points positioned correctly.	**B1**
	(3)
$2\int_0^{\ln(1+\sqrt{2})} (e^{\theta} + e^{-\theta})^2 \, d\theta - \frac{1}{2}\int_0^{\ln(1+\sqrt{2})} (e^{2\theta} - e^{-2\theta})^2 \, d\theta$	**M1A1**
Uses $\frac{1}{2}\int r^2 \, d\theta$ to formulate correct area.	
$= \int_0^{\ln(1+\sqrt{2})} 5 + 2e^{2\theta} + 2e^{-2\theta} - \frac{1}{2}e^{4\theta} - \frac{1}{2}e^{-4\theta} \, d\theta$	
$= \left[5\theta + e^{2\theta} - e^{-2\theta} - \frac{1}{8}e^{4\theta} + \frac{1}{8}e^{-4\theta}\right]_0^{\ln(1+\sqrt{2})}$	**M1A1**
Expands and integrates.	
$= 5\ln(1+\sqrt{2}) + (1+\sqrt{2})^2 - (1+\sqrt{2})^{-2} - \frac{1}{8}\left((1+\sqrt{2})^4 - (1+\sqrt{2})^{-4}\right) = 5.82$	**A1**
	(5)

Total: 14

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Q17

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Mark scheme.

$x = \theta\cos^2\theta$	**B1**
$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = -2\theta\cos\theta\sin\theta + \cos^2\theta = 0$	**M1A1**
$\cos\theta \neq 0 \Rightarrow 2\theta\tan\theta - 1 = 0$	**A1**
$2(0.6)\tan 0.6 - 1 = -0.179$ and $2(0.7)\tan 0.7 - 1 = 0.179$	**B1**
	(5)
$\theta\cos\theta = \theta\sin\theta \Rightarrow \tan\theta = 1$	**M1**
$\left(\frac{1}{8}\pi\sqrt{2}, \frac{1}{4}\pi\right)$	**A1**
	(2)
Sketch: initial line drawn and C_1 correct	**B1**
Sketch: C_2 correct	**B1**
Sketch: intersection correct	**B1**
	(3)
$\dfrac{1}{2}\displaystyle\int_0^{\frac{1}{4}\pi} \theta^2\left(\cos^2\theta - \sin^2\theta\right)\mathrm{d}\theta$	**M1**
$= \dfrac{1}{2}\displaystyle\int_0^{\frac{1}{4}\pi} \theta^2\cos 2\theta\,\mathrm{d}\theta$	**M1**
$= \dfrac{1}{2}\left(\left[\dfrac{1}{2}\theta^2\sin 2\theta\right]_0^{\frac{1}{4}\pi} - \displaystyle\int_0^{\frac{1}{4}\pi}\theta\sin 2\theta\,\mathrm{d}\theta\right)$	**M1A1**
$= \dfrac{1}{2}\left(\left[\dfrac{1}{2}\theta^2\sin 2\theta\right]_0^{\frac{1}{4}\pi} - \left[-\dfrac{1}{2}\theta\cos 2\theta\right]_0^{\frac{1}{4}\pi} - \dfrac{1}{2}\displaystyle\int_0^{\frac{1}{4}\pi}\cos 2\theta\,\mathrm{d}\theta\right)$	**M1**
$= \dfrac{1}{4}\left[\theta^2\sin 2\theta + \theta\cos 2\theta - \dfrac{1}{2}\sin 2\theta\right]_0^{\frac{1}{4}\pi}$	**A1**
$= \dfrac{1}{64}\pi^2 - \dfrac{1}{8}$	**A1**
	(7)
	Total: 17

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Q18

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Mark scheme.

Sketch: C correct with initial line	**B1**
Maximum distance of C from the pole is a.	**B1**
	(2)
$\frac{1}{2}a^2\int_0^{\frac{1}{4}\pi}\tan^2\theta\,\mathrm{d}\theta$	**M1**
$=\frac{1}{2}a^2\int_0^{\frac{1}{4}\pi}\sec^2\theta-1\,\mathrm{d}\theta=\frac{1}{2}a^2\left[\tan\theta-\theta\right]_0^{\frac{1}{4}\pi}$	**M1A1**
$=\frac{1}{2}a^2\left(1-\frac{1}{4}\pi\right)$	**A1**
	(4)
$\sqrt{x^2+y^2}=a\frac{y}{x}$	**B1**
$x^2(x^2+y^2)=a^2y^2\Rightarrow y^2(a^2-x^2)=x^4$	**M1**
$y=\frac{x^2}{\sqrt{a^2-x^2}}$	**A1 AG**
	(3)
$\frac{1}{2}\left(a\cos\frac{1}{4}\pi\right)\left(a\sin\frac{1}{4}\pi\right)-\frac{1}{2}a^2\left(1-\frac{1}{4}\pi\right)=\frac{1}{4}a^2\left(\frac{1}{2}\pi-1\right)$	**M1A1 FT**
	(2)
	Total: 11

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Q19

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Mark scheme.

$r^5 = 4xy(x^2 - y^2)$	**B1**
Uses $r^2 = x^2 + y^2$.	
$r^5 = 4r^4 \cos\theta \sin\theta(\cos^2\theta - \sin^2\theta)$	**B1**
Uses $x = r\cos\theta$ and $y = r\sin\theta$.	
$r = 2\sin 2\theta \cos 2\theta$	**M1**
Applies at least one double angle formula.	
$r = \sin 4\theta$	**A1**
Applies both double angle formulae, AG.	
	(4)
[Sketch: initial line drawn and one loop in the first quadrant; correct shape at extremities.]	**B1B1**
$\theta = \frac{1}{8}\pi$	**B1**
States the equation of the line of symmetry.	
	(3)
$\frac{1}{2}\int_0^{\frac{1}{4}\pi} \sin^2 4\theta \, \mathrm{d}\theta$	**M1**
Uses $\frac{1}{2}\int r^2 \, \mathrm{d}\theta$ with correct limits.	
$= \frac{1}{4}\int_0^{\frac{1}{4}\pi} 1 - \cos 8\theta \, \mathrm{d}\theta = \frac{1}{4}\left[\theta - \frac{1}{8}\sin 8\theta\right]_0^{\frac{1}{4}\pi}$	**M1A1**
Applies double angle formula and integrates.	
$= \frac{1}{16}\pi$	**A1**
	(4)
$x = 4(\sin\theta\cos^4\theta - \sin^3\theta\cos^2\theta)$	**B1**

Uses $x = r\cos\theta$.	
$-4\sin^2\theta\cos^3\theta + \cos^5\theta + 2\sin^4\theta\cos\theta - 3\sin^2\theta\cos^3\theta = 0$	**M1**
Differentiates and sets equal to 0.	
$\cos\theta(2\sin^4\theta - 7\sin^2\theta\cos^2\theta + \cos^4\theta) = 0$	**A1**
$\cos\theta = 0$ or $2\tan^4\theta - 7\tan^2\theta + 1 = 0$	**M1**
Forms quadratic in $\tan^2\theta$. Must see consideration of $\cos\theta = 0$.	
$\tan^2\theta = \frac{1}{4}(7 \pm \sqrt{41}) \Rightarrow \theta = \pm 0.369, \pm 1.071$	**B1**
Allow use of decimals.	
$\theta = 0.369 \Rightarrow x = 0.93$ (or $\theta = -0.369 \Rightarrow \lvert x\rvert = 0.93$)	**A1**
Substituting $\theta = \pm 0.369$ gives maximum value of $\lvert x\rvert$.	
	(6)
	Total: 17

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Q20

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Mark scheme.

[Sketch: $\theta = 0$; correct shape, r decreasing; section $\frac{1}{2}\pi \leqslant \theta \leqslant \pi$ correct, becoming tangential at $\theta = \pi$.]	**B1B1**
$(\ln(1+\pi), 0)$	**B1**
May be seen on their diagram. Allow $(1.42, 0)$	
	(3)
$\frac{1}{2}\int_0^{\pi} \ln^2(1+\pi-\theta)\,\mathrm{d}\theta = \frac{1}{2}\int_1^{1+\pi} \ln^2 u\,\mathrm{d}u$	**M1**
Uses correct formula with correct limits.	
$= \left[\frac{1}{2}u\ln^2 u\right]_1^{1+\pi} - \int_1^{1+\pi} \ln u\,\mathrm{d}u$	**M1A1**
Integrates by parts once.	
$= \left[\frac{1}{2}u\ln^2 u\right]_1^{1+\pi} - \left([u\ln u]_1^{1+\pi} - \int_1^{1+\pi} 1\,\mathrm{d}u\right)$	**M1**
Integrates by parts again (or uses known result for integral of $\ln u$)	
$= \left[\frac{1}{2}u\ln^2 u - u\ln u + u\right]_1^{1+\pi}$	**A1**
$\frac{1}{2}(1+\pi)\ln^2(1+\pi) - (1+\pi)\ln(1+\pi) + \pi = \frac{1}{2}(1+\pi)\ln(1+\pi)(\ln(1+\pi)-2) + \pi$	**A1**
AG	
	(6)
$y = \ln(1+\pi-\theta)\sin\theta$	**B1**
Uses $y = r\sin\theta$	
$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \ln(1+\pi-\theta)\cos\theta - \dfrac{\sin\theta}{1+\pi-\theta} = 0$	**M1A1**
Sets derivative equal to zero.	
$\cos\theta \neq 0 \Rightarrow (1+\pi-\theta)\ln(1+\pi-\theta) - \tan\theta = 0$	**A1**
AG	

$(1+\pi-1.2)\ln(1+\pi-1.2)-\tan 1.2 = 0.602$ and $(1+\pi-1.3)\ln(1+\pi-1.3)-\tan 1.3 = -0.634$	**B1**
Shows sign change. Values correct to 2sf should be shown.	
	(5)
	Total: 14

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Q21

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Mark scheme.

Initial line with correct position and shape of curve. SCB1 if correct shape with wrong starting position. Pole position must be clear.	**B1 B1**
$a\cot\frac{1}{6}\pi = 2\sqrt{3} \Rightarrow a = 2$	**B1**
	(3)
$\frac{1}{2}\int_0^{\frac{1}{6}\pi} 4\cot^2\left(\frac{1}{3}\pi - \theta\right)\,\mathrm{d}\theta$	**M1**
$2\int_0^{\frac{1}{6}\pi} \csc^2\left(\frac{1}{3}\pi - \theta\right) - 1\,\mathrm{d}\theta = 2\left[\cot\left(\frac{1}{3}\pi - \theta\right) - \theta\right]_0^{\frac{1}{6}\pi}$	**M1 A1**
$2\left(\sqrt{3} - \frac{1}{6}\pi - \frac{1}{3}\sqrt{3}\right) = \frac{4}{3}\sqrt{3} - \frac{1}{3}\pi$ (OE, must be exact)	**A1**
	(4)
$r = 2\dfrac{\cos\frac{\pi}{3}\cos\theta + \sin\frac{\pi}{3}\sin\theta}{\sin\frac{\pi}{3}\cos\theta - \cos\frac{\pi}{3}\sin\theta}$	**M1**
$\sqrt{x^2+y^2} = 2\dfrac{r\cos\theta + \sqrt{3}r\sin\theta}{\sqrt{3}r\cos\theta - r\sin\theta}$	**M1**
$2(x + y\sqrt{3}) = (x\sqrt{3} - y)\sqrt{x^2+y^2}$ (AG)	**A1**
	(3)
	Total: 10

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Q22

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Mark scheme.

Approximately correct curve passing through the pole, O, with correct domain.	**B1**
r strictly increasing over the domain 0 to $\frac{\pi}{2}$.	**B1**
Correct form at O. Approx tangential to initial line at O.	**B1**
	(3)
$\frac{1}{2}\int_0^{\frac{1}{2}\pi}\left(\frac{1}{\pi-\theta}-\frac{1}{\pi}\right)^2 \mathrm{d}\theta$	**M1**
$\frac{1}{2}\int_0^{\frac{1}{2}\pi}\frac{1}{(\pi-\theta)^2}-\frac{2}{\pi(\pi-\theta)}+\frac{1}{\pi^2}\mathrm{d}\theta$	**M1**
$\frac{1}{2}\left[\frac{1}{\pi-\theta}+\frac{2}{\pi}\ln(\pi-\theta)+\frac{\theta}{\pi^2}\right]_0^{\frac{1}{2}\pi}$	**M1 A1**
$\frac{1}{2}\left(\frac{2}{\pi}+\frac{2}{\pi}\ln\frac{\pi}{2}+\frac{1}{2\pi}-\left(\frac{1}{\pi}+\frac{2}{\pi}\ln\pi\right)\right)=\frac{1}{2}\left(\frac{3}{2\pi}+\frac{2}{\pi}\ln\frac{1}{2}\right)$	**M1**
$=\frac{3-4\ln 2}{4\pi}$ (AG)	**A1**
	(6)
	Total: 9

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Q23

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Mark scheme.

$2\left(c(c)-(1+s)s\right)=0$	**M1**
Finds $\dfrac{\mathrm{d}r}{\mathrm{d}\theta}$ and sets equal to 0.	
$1-s-2s^2=0$	**A1**
$\sin\theta=-1$, $\sin\theta=\frac{1}{2}$ leading to $\theta=\frac{1}{6}\pi$	**M1A1**
Solves quadratic in $\sin\theta$.	
$\left(\frac{3}{2}\sqrt{3},\frac{1}{6}\pi\right)$	**A1**
Allow $(2.60,\frac{1}{6}\pi)$ 3sf	
	(5)
Correct shape. Tangential to $\theta=\frac{\pi}{2}$ at pole, r strictly increasing to $\frac{\pi}{6}$ then decreasing.	**B1**
Correct position. All in first quadrant.	**B1**
	(2)
$\frac{1}{2}\int_0^{\frac{1}{2}\pi}4c^2(1+s)^2\,\mathrm{d}\theta=2\int_0^{\frac{1}{2}\pi}c^2+2sc^2+s^2c^2\,\mathrm{d}\theta$	**M1A1**
Uses $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ with correct limits.	
$2\int_0^{\frac{1}{2}\pi}\frac{1}{2}(\cos 2\theta+1)+2\sin\theta\cos^2\theta+\frac{1}{4}\sin^2 2\theta\,\mathrm{d}\theta$	**M1**
Uses double angle formulae on first and last terms.	
$\int_0^{\frac{1}{2}\pi}\cos 2\theta+1+4\sin\theta\cos^2\theta+\frac{1}{4}-\frac{1}{4}\cos 4\theta\,\mathrm{d}\theta$	**A1**
Obtains integrable form.	
$\left[\frac{5}{4}\theta+\frac{1}{2}\sin 2\theta-\frac{4}{3}\cos^3\theta-\frac{1}{16}\sin 4\theta\right]_0^{\frac{1}{2}\pi}$	**M1**
Integrates.	
$\frac{5}{8}\pi+\frac{4}{3}$	**A1**

(6)

Total: 13

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Q24

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Mark scheme.

Correct symmetrical shape, closed loop.	**B1**
	(1)
Line l parallel to initial line and correct side of pole.	**B1**
$2 = 3\sin\theta + 2\sin^2\theta$	**M1**
Forms quadratic in $\sin\theta$. Or in r: $r = 3 + \dfrac{4}{r}$	
$\sin\theta = \frac{1}{2}$	**M1**
Solves for $\sin\theta$.	
$\left(4, \frac{1}{6}\pi\right)$	**A1**
$\left(4, \frac{5}{6}\pi\right)$	**A1**
SC1 for finding both angles correctly	
	(5)
$2 \times \frac{1}{2}\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} (3 + 2\sin\theta)^2 \, \mathrm{d}\theta$	**M1**
Finds the part of the required area enclosed by the curved outer edge and two line segments from the pole. Limits must be correct.	
$\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} 9 + 12\sin\theta + 2(1 - \cos 2\theta) \, \mathrm{d}\theta$	**M1**
Uses double angle formula and integrates.	
$\left[11\theta - 12\cos\theta - \sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{6}} = \frac{22}{3}\pi - \frac{13}{2}\sqrt{3}$	**A1A1**
$\frac{22}{3}\pi - \frac{13}{2}\sqrt{3} + \left(4\cos\frac{\pi}{6}\right) \times 2$	**M1**
Adds area of triangle.	
$\frac{22}{3}\pi - \frac{5}{2}\sqrt{3}$	**A1**
	(6)

Total: 12

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Q25

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Mark scheme.

Sketch: correct domain, r strictly increasing.	**B1**
Sketch: correct gradient when $\theta = 0$ and $\theta = 2$.	**B1**
Maximum distance of C from the pole is $\sqrt{\frac{1}{4}\pi}$.	**B1**
	(3)
$\frac{1}{2}\int_0^2 \tan^{-1}\left(\frac{1}{2}\theta\right) \mathrm{d}\theta$	**M1**
$\left[\frac{1}{2}\theta\tan^{-1}\left(\frac{1}{2}\theta\right)\right]_0^2 - \int_0^2 \frac{\theta}{\theta^2+4}\,\mathrm{d}\theta$	**M1 A1**
$\frac{1}{2}\left[\theta\tan^{-1}\left(\frac{1}{2}\theta\right) - \ln(\theta^2+4)\right]_0^2$	**A1**
$\frac{1}{4}\pi - \frac{1}{2}\ln 8 + \frac{1}{2}\ln 4 = \frac{1}{4}\pi - \frac{1}{2}\ln 2$	**A1**
	(5)
$x = \left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}}\cos\theta$	**B1**
$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -\left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}}\sin\theta + \cos\theta\left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{-\frac{1}{2}}(\theta^2+4)^{-1} = 0$	**M1 A1**
$(\theta^2+4)\tan^{-1}\left(\frac{1}{2}\theta\right)\sin\theta - \cos\theta = 0$	**A1**
$(0.6^2+4)\tan^{-1}(0.3)\sin 0.6 - \cos 0.6 = -0.108$ and $(0.7^2+4)\tan^{-1}(0.35)\sin 0.7 - \cos 0.7 = 0.209$	**B1**
	(5)
	Total: 13

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Q26

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Mark scheme.

$x = r\cos\theta \quad y = r\sin\theta \quad x^2+y^2=r^2$	**B1**
$r^2(1+\sin\theta\cos\theta) = r^2\left(1+\frac{1}{2}\sin 2\theta\right) = a$	**M1**
$r^2 = \dfrac{2a}{2+\sin 2\theta}$	**A1**
	(3)
Sketch: polar graph with curve in correct domain. Graph needs gradient ≤ 0 and $r > 0$ for all θ in the domain.	**B1**
r strictly decreasing such that r at $\theta = \frac{\pi}{4}$ is greater than half r at $\theta = 0$. Concave graph. Gradient at $\theta = 0$ and $\frac{\pi}{4}$ must not be vertical or horizontal, respectively.	**B1**
	(2)
$R = \dfrac{1}{2}\displaystyle\int_0^{\frac{1}{4}\pi} r^2\,\mathrm{d}\theta = a\int_0^{\frac{1}{4}\pi} \frac{1+\tan^2\theta}{2(1+\tan^2\theta)+2\tan\theta}\,\mathrm{d}\theta$	**B1**
$t = \tan\theta$ leading to $\dfrac{\mathrm{d}t}{\mathrm{d}\theta} = \sec^2\theta$	**M1**
$\dfrac{\mathrm{d}t}{\mathrm{d}\theta} = \sec^2\theta = 1+t^2$	**M1**
$R = \dfrac{1}{2}a\displaystyle\int_0^1 \frac{1}{t^2+t+1}\,\mathrm{d}t$	**A1**
$t^2+t+1 = \left(t+\frac{1}{2}\right)^2+\frac{3}{4}$	**B1**
$R = \dfrac{1}{2}a\displaystyle\int_0^1 \frac{1}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}\,\mathrm{d}t = \frac{1}{\sqrt{3}}a\left[\tan^{-1}\left(\frac{2}{\sqrt{3}}t+\frac{1}{\sqrt{3}}\right)\right]_0^1$	**M1 A1**
$\dfrac{\pi a}{6\sqrt{3}}$	**A1**
	(8)
	Total: 13

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Q27

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Mark scheme.

$x^2 + y^2 = r^2$, $x = r\cos\theta$, $y = r\sin\theta$	**B1**
$r^4 = 36r^2\left(\cos^2\theta - \sin^2\theta\right) = 36r^2\cos 2\theta$	**M1**
$r^2 = 36\cos 2\theta$	**A1**
	(3)
Closed curve. Correct position and symmetrical about initial line.	**B1**
Single correct loop.	**B1**
Maximum distance from pole is 6.	**B1**
	(3)
$18\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos 2\theta \, \mathrm{d}\theta = 9\left[\sin 2\theta\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$	**M1**
18	**A1**
	(2)
$y = 6\cos^{\frac{1}{2}} 2\theta \sin\theta$	**B1**
$\cos^{\frac{1}{2}} 2\theta\cos\theta - \cos^{-\frac{1}{2}} 2\theta\sin 2\theta\sin\theta = 0$	**M1A1**
$\cos 2\theta\cos\theta - \sin 2\theta\sin\theta = 0$ leading to $1 = \dfrac{2\tan^2\theta}{1-\tan^2\theta}$	**M1**
$\theta = \pm\frac{1}{6}\pi$	**A1**
$\frac{3}{2}\sqrt{2}$	**A1**
	(6)
	Total: 14

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Q28

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Mark scheme.

Correct position and domain and r strictly increasing.	**B1**
Decreasing positive gradient.	**B1**
$(a, 0)$, $\left(2a, \frac{1}{4}\pi\right)$	**B1**
	(3)
$y = (2a)\sin\left(\frac{1}{4}\pi\right)$	**M1**
$a\sqrt{2}$	**A1**
	(2)
$\frac{1}{2}a^2 \int_0^{\frac{\pi}{4}} \sec^4\theta \, \mathrm{d}\theta$	**M1**
$= \frac{1}{2}a^2 \int_0^{\frac{\pi}{4}} \sec^2\theta\left(\tan^2\theta + 1\right)\mathrm{d}\theta$	**M1**
$= \frac{1}{2}a^2 \left[\frac{1}{3}\tan^3\theta + \tan\theta\right]_0^{\frac{\pi}{4}}$	**A1**
$= \frac{2}{3}a^2$	**A1**
	(4)
$x^2 + y^2 = r^2$, $x = r\cos\theta$	**M1**
$r^2\cos^2\theta = ar$ leading to $x^2 = a\sqrt{x^2+y^2}$ leading to $x^4 = a^2(x^2+y^2)$	**M1**
$y = \sqrt{a^{-2}x^4 - x^2} = x\sqrt{a^{-2}x^2 - 1}$	**A1**
	(3)
	Total: 12

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Q29

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Mark scheme.

Correct shape and domain, polar graph with r strictly decreasing but condone if not strictly decreasing close to $\theta = \pi$.	**B1***
Fully correct including shape at $\theta = 0$ correct, shape at $\theta = \pi$ correct.	**DB1**
$(1, 0)$ identified as point furthest from the pole and given as coordinates.	**B1**
	(3)
$\frac{1}{2}\int_0^{\pi} \frac{1}{\theta^2+1}\,\mathrm{d}\theta$	**M1**
$\frac{1}{2}\left[\tan^{-1}\theta\right]_0^{\pi}$	**M1A1**
$\frac{1}{2}\tan^{-1}\pi = 0.631$	**A1**
	(4)
$y = \frac{\sin\theta}{\sqrt{\theta^2+1}}$, uses $y = r\sin\theta$	**B1**
$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{(\theta^2+1)^{\frac{1}{2}}\cos\theta - \theta(\theta^2+1)^{-\frac{1}{2}}\sin\theta}{\theta^2+1} = 0$, sets derivative equal to zero.	**M1A1**
$\theta \neq 0 \Rightarrow \left(\theta + \frac{1}{\theta}\right)\cot\theta - 1 = 0$	**A1**
$\left(1.1 + \frac{1}{1.1}\right)\cot 1.1 - 1 = 0.02\ldots$ and $\left(1.2 + \frac{1}{1.2}\right)\cot 1.2 - 1 = -0.209\ldots$, shows sign change (1sf or better).	**B1**
	(5)
	Total: 12

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Q30

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Mark scheme.

$r^2(\cos^2\theta - \sin^2\theta) = a$, uses $x = r\cos\theta$ and/or $y = r\sin\theta$.	**B1**
$r^2\cos 2\theta = a$, applies relevant double angle formulae.	**M1**
$r^2 = a\sec 2\theta$	**A1**
	(3)
Initial line drawn. Correct domain and position, r strictly increasing.	**B1***
Also sloping to right, concave on opposite side to pole, correct gradient when $\theta = 0$ and $\theta \to \pi/4$.	**dB1**
$\sqrt{a}$	**B1**
	(3)
$\frac{1}{2}a\int_0^{\frac{1}{12}\pi} \sec 2\theta \, d\theta$, uses $\frac{1}{2}\int r^2 \, d\theta$ with correct limits.	**M1**
$\frac{1}{4}a\left[\ln\tan\left(\theta + \frac{1}{4}\pi\right)\right]_0^{\frac{1}{12}\pi}$ or $\frac{1}{4}a\left[\ln(\tan 2\theta + \sec 2\theta)\right]_0^{\frac{1}{12}\pi}$, integrates.	**M1A1**
$\frac{1}{4}a\ln\sqrt{3} = \frac{1}{8}a\ln 3$	**A1**
	(4)
	Total: 10

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Q31

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Mark scheme.

$x^2 - x + \frac{1}{4} + y^2 = \frac{1}{4} \Rightarrow r^2 - r\cos\theta + \frac{1}{4} = \frac{1}{4}$	**B1**
$r(r - \cos\theta) = 0$	**M1**
$[r \neq 0 \Rightarrow] r = \cos\theta$ ((a) Total: 3)	**A1**
$\sin 2\theta = \cos\theta \Rightarrow 2\sin\theta\cos\theta = \cos\theta$	**M1**
$\cos\theta \neq 0 \Rightarrow \sin\theta = \frac{1}{2}$	**A1**
$\left(\frac{1}{2}\sqrt{3}, \frac{1}{6}\pi\right)$ ((b) Total: 3)	**A1**
Initial line drawn and one curve correct.	**B1**
Other curve correct.	**B1**
Intersection marked in correct position and both curves labelled. ((c) Total: 3)	**B1**
$\frac{1}{2}\int_0^{\frac{1}{6}\pi} \sin^2 2\theta \, \mathrm{d}\theta + \frac{1}{2}\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \cos^2\theta \, \mathrm{d}\theta$	**M1**
$\frac{1}{2}\int_0^{\frac{1}{6}\pi} \sin^2 2\theta \, \mathrm{d}\theta = \frac{1}{4}\int_0^{\frac{1}{6}\pi} 1 - \cos 4\theta \, \mathrm{d}\theta$	**M1**
$= \frac{1}{4}\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\frac{1}{6}\pi}$	**A1**
$\frac{1}{2}\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \cos^2\theta \, \mathrm{d}\theta = \frac{1}{4}\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} 1 + \cos 2\theta \, \mathrm{d}\theta$	**M1**
$= \frac{1}{4}\left[\theta + \frac{1}{2}\sin 2\theta\right]_{\frac{1}{6}\pi}^{\frac{1}{2}\pi}$	**A1**
$\frac{1}{4}\left(\frac{1}{6}\pi - \frac{1}{8}\sqrt{3}\right) + \frac{1}{4}\left(\frac{1}{2}\pi - \frac{1}{6}\pi - \frac{1}{4}\sqrt{3}\right) = \frac{1}{8}\left(\pi - \frac{3}{4}\sqrt{3}\right)$ ((d) Total: 6)	**A1**
	Total: 15

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Q32

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Mark scheme.

Initial line drawn. Correct shape, r strictly decreasing.	**B1**
Correct shape at extremities.	**B1**
$1-e^{-\frac{1}{2}\pi}$ ((a) Total: 3)	**B1**
$\frac{1}{2}\int_0^{\frac{1}{2}\pi}\left(e^{-\theta}-e^{-\frac{1}{2}\pi}\right)^2 d\theta$	**M1**
$\frac{1}{2}\int_0^{\frac{1}{2}\pi} e^{-2\theta}-2e^{-\theta-\frac{1}{2}\pi}+e^{-\pi}\, d\theta$	**A1**
$\frac{1}{2}\left[-\frac{1}{2}e^{-2\theta}+2e^{-\theta-\frac{1}{2}\pi}+e^{-\pi}\theta\right]_0^{\frac{1}{2}\pi}$	**M1A1**
$\frac{1}{2}\left(-\frac{1}{2}e^{-\pi}+2e^{-\pi}+\frac{1}{2}\pi e^{-\pi}+\frac{1}{2}-2e^{-\frac{1}{2}\pi}\right)=\frac{3}{4}e^{-\pi}+\frac{1}{4}\pi e^{-\pi}-e^{-\frac{1}{2}\pi}+\frac{1}{4}$ ((b) Total: 5)	**A1**
$y=\left(e^{-\theta}-e^{-\frac{1}{2}\pi}\right)\sin\theta$	**B1**
$\frac{dy}{d\theta}=\left(e^{-\theta}-e^{-\frac{1}{2}\pi}\right)\cos\theta+\sin\theta\left(-e^{-\theta}\right)=0$	**M1A1**
$\left[\theta\neq\frac{1}{2}\pi\Rightarrow\right]1+\left(\frac{-e^{-\theta}}{e^{-\theta}-e^{-\frac{1}{2}\pi}}\right)\tan\theta=0\Rightarrow 1-e^{\theta-\frac{1}{2}\pi}-\tan\theta=0$	**A1**
$1-e^{0.56-\frac{1}{2}\pi}-\tan 0.56=0.00912$ and $1-e^{0.57-\frac{1}{2}\pi}-\tan 0.57=-0.00856$ ((c) Total: 5)	**B1**
	Total: 13

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Q33

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Mark scheme.

[Graph: correct shape of C]	**B1**
Correct shape.	
[Section $\frac{1}{2}\pi \leqslant \theta \leqslant \pi$ correct]	**B1**
Section $\frac{1}{2}\pi \leqslant \theta \leqslant \pi$ correct.	
$\left(\sqrt{\pi \tan^{-1}\pi},\ 0\right) = (1.99,\ 0)$	**B1** **May be seen on their diagram.**
	(3)
$A = \frac{1}{2}\int_0^{\pi} (\pi-\theta)\tan^{-1}(\pi-\theta)\,\mathrm{d}\theta$ or $A = -\frac{1}{2}\int_{\pi}^{0} (u)\tan^{-1}(u)\,\mathrm{d}u$	**M1**
Uses correct formula. or $A = \frac{1}{2}\int_0^{\pi} (u)\tan^{-1}(u)\,\mathrm{d}u$	
$\frac{1}{2}\left[\frac{1}{2}(u)^2\tan^{-1}(u)\right]_0^{\pi} - \frac{1}{4}\int_0^{\pi} \frac{u^2}{1+u^2}\,\mathrm{d}u$	**M1 A1**
Integrates by parts.	
$\int_0^{\pi} \frac{u^2}{1+u^2}\,\mathrm{d}u = \int_0^{\pi} \frac{1+u^2-1}{1+u^2}\,\mathrm{d}u = \int_0^{\pi} 1 - \frac{1}{1+u^2}\,\mathrm{d}u$	**M1**
Rearranges integral into form which can be integrated.	
$\left[u - \tan^{-1}u\right]_0^{\pi}$	**A1**
$A = \frac{1}{2}\left[\frac{1}{2}(u)^2\tan^{-1}(u)\right]_0^{\pi} - \frac{1}{4}\left[u - \tan^{-1}(u)\right]_0^{\pi}$	**A1**
$A = \frac{1}{4}\pi^2\tan^{-1}\pi - \frac{1}{4}(\pi - \tan^{-1}\pi) = \frac{1}{4}(\pi^2+1)\tan^{-1}\pi - \frac{1}{4}\pi = 2.65$	**A1**
	(7)
$y = \sqrt{(\pi-\theta)\tan^{-1}(\pi-\theta)}\sin\theta$	**B1** **Uses** $y = r\sin\theta$

$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \sqrt{(\pi-\theta)\tan^{-1}(\pi-\theta)}\cos\theta - \dfrac{(\pi-\theta)\left(1+(\pi-\theta)^2\right)^{-1} + \tan^{-1}(\pi-\theta)}{2\sqrt{(\pi-\theta)\tan^{-1}(\pi-\theta)}}\sin\theta$	**M1 A1 Finds derivative.**
$[\theta \neq 0], \pi \Rightarrow 2(\pi-\theta)\tan^{-1}(\pi-\theta)\cot\theta - \dfrac{\pi-\theta}{1+(\pi-\theta)^2} - \tan^{-1}(\pi-\theta) = 0$	**A1 Puts** $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=0$ **and forms equation. AG.**
$2(\pi-1.2)\tan^{-1}(\pi-1.2)\cot 1.2 - \dfrac{\pi-1.2}{1+(\pi-1.2)^2} - \tan^{-1}(\pi-1.2) = 0.151$	**B1**
Shows sign change.	
and $2(\pi-1.3)\tan^{-1}(\pi-1.3)\cot 1.3 - \dfrac{\pi-1.3}{1+(\pi-1.3)^2} - \tan^{-1}(\pi-1.3) = -0.395$	
	(5)
	Total: 15

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Q34

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Mark scheme.

[Graph: two-lobed curve with correct form at the pole]	
One loop correct with initial line.	**B1**
Second loop correct with correct form at the pole.	**B1**
$\theta = \frac{1}{2}\pi$	**B1**
May be seen on their diagram.	
	(3)
$r^5 = 2(r\sin\theta)(r\cos\theta)^2$	**M1**
Use of $\sin 2\theta = 2\sin\theta\cos\theta$, and $x = r\cos\theta$ or $y = r\sin\theta$.	
$r^5 = 2yx^2$	**A1**
$(x^2+y^2)^{\frac{5}{2}} = 2x^2y$	**A1 AEF**
	(3)
$\frac{1}{2}\int_0^{\pi} \sin(2\theta)\cos\theta\, \mathrm{d}\theta$	**M1**
Applies $\frac{1}{2}\int r^2\, \mathrm{d}\theta$.	
$\frac{1}{2}\int \sin(2\theta)\cos\theta\, \mathrm{d}\theta$	**M1**
Attempt to integrate in a valid way. May apply $\sin(2\theta) = 2\sin\theta\cos\theta$.	
$= \int \sin\theta\cos^2\theta\, \mathrm{d}\theta = -\frac{1}{3}\cos^3\theta + [c]$	**A1**
Correct answer (there are alternative forms).	
$= \left[-\frac{1}{3}\cos^3\theta\right]_0^{\pi} = \frac{2}{3}$	**A1**
CAO	
	(4)

$\frac{\mathrm{d}r}{\mathrm{d}\theta} = \frac{1}{2}(\sin 2\theta \cos\theta)^{-\frac{1}{2}}(2\cos 2\theta\cos\theta - \sin 2\theta\sin\theta)$	***M1 A1 Differentiates with respect to θ.**
$2\cos 2\theta\cos\theta - \sin 2\theta\sin\theta = 6\cos^3\theta - 4\cos\theta$	**dM1**
Correct use of relevant identities to express in terms of a single trig function.	
$6\cos^3\theta - 4\cos\theta = 0 \Rightarrow \cos^2\theta = \frac{2}{3}$	**dM1 A1**
Sets derivative equal to 0 and solves to find one of $\sin^2\theta = \frac{1}{3}$, $\cos^2\theta = \frac{2}{3}$, $\tan^2\theta = \frac{1}{2}$ or $\theta = 0.6155$	
$r = \sqrt{\frac{4}{3\sqrt{3}}} = \sqrt{\frac{4\sqrt{3}}{9}} = 0.877$	**A1**
Alternative method:	
$2r\frac{\mathrm{d}r}{\mathrm{d}\theta} = 2\left(-2\sin^2\theta\cos\theta + \cos^3\theta\right)$	***M1 A1 Differentiates [RHS] with respect to θ.**
$2\left(-2(1-\cos^2\theta)\cos\theta + \cos^3\theta\right) = 6\cos^3\theta - 4\cos\theta$	**dM1**
Correct use of relevant identities in terms of a single trig function	
$6\cos^3\theta - 4\cos\theta = 0 \Rightarrow \cos^2\theta = \frac{2}{3}$	**dM1 A1**
$r = \sqrt{\frac{4}{3\sqrt{3}}} = \sqrt{\frac{4\sqrt{3}}{9}} = 0.877$	**A1**
	(6)
	Total: 16

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Q35

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Mark scheme.

$r^4 = 6r^2 \sin\theta \cos\theta$	**M1**
$r^2 = 3\sin 2\theta$ ((a) Total: 2)	**A1**
[Sketch: single loop, correct position and symmetrical about $\theta = \frac{1}{4}\pi$.]	**B1**
[Single correct loop.]	**B1**
$\sqrt{3}$ ((b) Total: 3)	**B1**
$\frac{3}{2}\int_0^{\frac{\pi}{2}} \sin 2\theta \, \mathrm{d}\theta = \frac{3}{2}\left[-\frac{1}{2}\cos 2\theta\right]_0^{\frac{\pi}{2}}$	**M1**
$\dfrac{3}{2}$ ((c) Total: 2)	**A1**
$y = 3^{\frac{1}{2}} \sin^{\frac{1}{2}} 2\theta \sin\theta$	**B1**
$\sin^{\frac{1}{2}} 2\theta \cos\theta + \sin^{-\frac{1}{2}} 2\theta \cos 2\theta \sin\theta = 0$	**M1A1**
$\sin 2\theta \cos\theta + \cos 2\theta \sin\theta = 0 \Rightarrow \tan 2\theta = -\tan\theta \Rightarrow \dfrac{2\tan\theta}{1-\tan^2\theta} = -\tan\theta$	**M1**
$\theta = \frac{1}{3}\pi$	**A1**
$\dfrac{3^{\frac{5}{4}}}{2^{\frac{3}{2}}} = 1.40$ ((d) Total: 6)	**A1**
	Total: 13

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Q36

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Mark scheme.

$r = a\left(\frac{x}{r}+\frac{y}{r}\right) \Rightarrow r^2 = ax+ay$	**M1**
$x^2 - ax + y^2 - ay = 0$	**M1A1**
$\left(x-\frac{a}{2}\right)^2+\left(y-\frac{a}{2}\right)^2=\frac{a^2}{2} \Rightarrow$ Centre $\left(\frac{a}{2},\frac{a}{2}\right)$ and radius $\frac{a}{\sqrt{2}}$ ((a) Total: 4)	**B1**
[Sketch: circle, correct position, passing through O.]	**B1**
[Correct position, passing through O.]	**B1**
$a\sqrt{2}$ ((b) Total: 3)	**B1**
$\cos\phi+\sin\phi-\phi=0$	**M1**
$\cos 1.25+\sin 1.25-1.25=0.01$, $\cos 1.26+\sin 1.26-1.26=-0.002$ ((c) Total: 2)	**A1**
$\frac{a^2}{2}\int_0^{\phi}\theta^2\,\mathrm{d}\theta+\frac{a^2}{2}\int_{\phi}^{\frac{3}{4}\pi}(\cos\theta+\sin\theta)^2\,\mathrm{d}\theta$	**M1**
[Forms area of smaller region enclosed by C_1 and C_2 with correct limits.]	**A1**
$\frac{a^2}{2}\int_0^{\phi}\theta^2\,\mathrm{d}\theta+\frac{a^2}{2}\int_{\phi}^{\frac{3}{4}\pi}1+2\sin\theta\cos\theta\,\mathrm{d}\theta$	**M1**
$\frac{a^2}{2}\left[\frac{1}{3}\theta^3\right]_0^{\phi}+\frac{a^2}{2}\left[\theta+\sin^2\theta\right]_{\phi}^{\frac{3}{4}\pi}$	**A1**
$\frac{a^2\phi^3}{6}+\frac{a^2}{2}\left(\frac{3}{4}\pi+\frac{1}{2}-\phi-\sin^2\phi\right)=\frac{a^2}{2}\left(\frac{\phi^3}{3}+\frac{3}{4}\pi-\phi+\frac{1}{2}\cos 2\phi\right)$	**A1**
$\frac{a^2}{2}\pi-\frac{a^2}{2}\left(\frac{\phi^3}{3}+\frac{3}{4}\pi-\phi+\frac{1}{2}\cos 2\phi\right)$	**M1**
$\frac{a^2}{2}\left(-\frac{\phi^3}{3}+\frac{1}{4}\pi+\phi-\frac{1}{2}\cos 2\phi\right)$ ((d) Total: 7)	**A1**
	Total: 16

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Q37

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Mark scheme.

[Spiral shape on $[0, 2\pi]$, with correct orientation, starting at pole and with one other point of intersection with the initial line.]	**B1**
[r strictly increasing, correct gradient at points of intersection with the initial line.]	**B1**
	(2)
$\frac{1}{2}\int_0^{2\pi} \theta^2 e^{\frac{1}{4}\theta}\, \mathrm{d}\theta$	**B1**
$\frac{1}{2}\int_0^{2\pi} r^2\, \mathrm{d}\theta$	
$= \frac{1}{2}\left[4\theta^2 e^{\frac{1}{4}\theta}\right]_0^{2\pi} - \frac{1}{2}\int_0^{2\pi} 8\theta e^{\frac{1}{4}\theta}\, \mathrm{d}\theta$	**M1A1**
Integrates by parts.	
$= \frac{1}{2}\left[4\theta^2 e^{\frac{1}{4}\theta}\right]_0^{2\pi} - 4\left[4\theta e^{\frac{1}{4}\theta}\right]_0^{2\pi} + 16\int_0^{2\pi} e^{\frac{1}{4}\theta}\, \mathrm{d}\theta = \left[2\theta^2 e^{\frac{1}{4}\theta} - 16\theta e^{\frac{1}{4}\theta} + 64 e^{\frac{1}{4}\theta}\right]_0^{2\pi}$	**M1A1**
Integrates by parts again.	
$= \left(8\pi^2 - 32\pi + 64\right) e^{\frac{1}{2}\pi} - 64$	**A1**
	(6)
$y = \theta e^{\frac{1}{8}\theta} \sin\theta$	**B1**
Uses $y = r\sin\theta$	
$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \theta e^{\frac{1}{8}\theta}\cos\theta + \sin\theta\left(\frac{1}{8}\theta e^{\frac{1}{8}\theta} + e^{\frac{1}{8}\theta}\right)$	**M1A1**
Differentiates.	
$e^{\frac{1}{8}\theta} \neq 0 \Rightarrow \theta\cos\theta + \left(\frac{1}{8}\theta + 1\right)\sin\theta = 0$	**A1**
AG.	
$5\cos 5 + \left(\frac{5}{8} + 1\right)\sin 5 = -0.1399$	**B1**
and $5.05\cos 5.05 + \left(\frac{5.05}{8} + 1\right)\sin 5.05 = 0.1336$	
Shows sign change.	

(5)

Total: 13

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Q38

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Mark scheme.

$-e^{\sin\theta}\sin\theta + e^{\sin\theta}\cos^2\theta = 0$	**M1**
Finds $\dfrac{\mathrm{d}r}{\mathrm{d}\theta}$ and sets equal to 0.	
$\cos^2\theta - \sin\theta = 0$	**A1**
$1 - \sin^2\theta - \sin\theta = 0$	**M1**
Makes it a quadratic in $\sin\theta$ and attempts to solve	
$\sin\theta = \frac{1}{2}\left(-1+\sqrt{5}\right)$	**A1**
Correct solution. Condone 0.666	
(1.208, 0.666)	**A1**
Must be to 3 decimal places	
	(5)
$x = e^{\frac{1}{2}\sin\theta}\cos^{\frac{3}{2}}\theta$	**M1**
$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = -\frac{3}{2}e^{\frac{1}{2}\sin\theta}\cos^{\frac{1}{2}}\theta\sin\theta + \frac{1}{2}\cos^{\frac{5}{2}}\theta e^{\frac{1}{2}\sin\theta} = 0$	
Uses $x = r\cos\theta$. Finds $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ and sets equal to 0.	
$-3\sin\theta + \cos^2\theta = 0$	**A1**
$-3\sin\theta + 1 - \sin^2\theta = 0$	**M1**
Makes it a quadratic in $\sin\theta$ and attempts to solve.	
$\sin\theta = \frac{1}{2}\left(-3+\sqrt{13}\right)$	**A1**
Correct solution. Condone 0.308	
(1.136, 0.308)	**A1**
If already penalised for rounding to 3 decimal places in part (a) then allow to 3 s.f.	

	(5)
[Closed loop passing through O and one other point on the initial line. Does not go into 2nd and 3rd quadrants. More in the 1st than 4th quadrant.]	**B1**
[$-\frac{1}{2}\pi \leqslant \theta \leqslant 0.666$ correct. Tangent needs to have positive gradient at the intersection $(1, 0)$.]	**B1**
[$0.666 \leqslant \theta \leqslant \frac{1}{2}\pi$ correct. Graph should approach the line $\theta = \frac{\pi}{2}$.]	**B1**
	(3)
$\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\sin\theta}\cos\theta\,\mathrm{d}\theta$	**B1**
Condone absence of $\mathrm{d}\theta$ but must include the limits.	
$= \frac{1}{2}\left[e^{\sin\theta}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$	**M1**
Recognises integral.	
$= \frac{1}{2}\left(e - e^{-1}\right)$	**A1**
Must be exact. ISW after correct answer seen.	
	(3)
	Total: 16

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Q39

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Mark scheme.

[Sketch: initial line drawn, pole clear and one loop in the first quadrant, symmetrical with correct shape at extremities.]	**B1*DB1**
Initial line drawn, pole clear and one loop in the first quadrant.	
Symmetrical with correct shape at extremities.	
$\theta = \frac{1}{6}\pi$.	**B1**
States the equation of the line of symmetry.	
	(3)
$\frac{1}{2}\int_0^{\frac{1}{3}\pi} \sin^2 3\theta \, d\theta$	**M1**
Uses $\frac{1}{2}\int r^2 \, d\theta$ with correct limits.	
$= \frac{1}{4}\int_0^{\frac{1}{3}\pi} 1 - \cos 6\theta \, d\theta = \frac{1}{4}\left[\theta - \frac{1}{6}\sin 6\theta\right]_0^{\frac{1}{3}\pi}$	**M1A1**
Applies double angle formula correctly and integrates.	
$= \frac{1}{12}\pi$	**A1**
CWO	
	(4)
$y = 3\sin^2\theta - 4\sin^4\theta$	**B1**
Uses $y = r\sin\theta$.	
$6\sin\theta\cos\theta - 16\sin^3\theta\cos\theta = 0$	**M1***
Differentiates correct function and sets equal to 0. Might see completing the square.	
$\sin\theta\cos\theta \neq 0 \Rightarrow 6 - 16\sin^2\theta = 0$	**A1**
$\sin^2\theta = \frac{3}{8}$	**DM1**
Solves quadratic in $\sin\theta$.	

$y = \dfrac{9}{16}$.	**A1**
	(5)
$r^4 = r\sin\theta\left(3r^2 - 4r^2\sin^2\theta\right)$	**M1**
Uses $y = r\sin\theta$	
$r = \dfrac{3y}{r} - 4\dfrac{y^3}{r^3}$	
$\left(x^2 + y^2\right)^2 = y\left(3x^2 - y^2\right)$	**M1**
Uses $r^2 = x^2 + y^2$	
Rearranges to a form with no fractions.	**A1**
	(3)
	Total: 15

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Q40

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Mark scheme.

[Sketch: initial line drawn and correct shape.]	**B1**
Initial line drawn and correct shape.	
Maximum distance of C from the pole is 1.	**B1**
	(2)
$\frac{1}{2}\int_0^{\frac{1}{8}\pi} \tan 2\theta \, d\theta$	**M1**
Uses $\frac{1}{2}\int r^2 \, d\theta$ with correct limits.	
$= \frac{1}{2}\int_0^{\frac{1}{8}\pi} \frac{\sin 2\theta}{\cos 2\theta} \, d\theta = -\frac{1}{4}\left[\ln \cos 2\theta\right]_0^{\frac{1}{8}\pi}$	**M1A1**
Integrates a correct function to get $-\frac{1}{2}[\ln \cos 2\theta]$ condoning the lack of limits and factor of $\frac{1}{2}$ for M1A1.	
$= -\frac{1}{4}\ln\frac{\sqrt{2}}{2} = \frac{1}{8}\ln 2$	**A1**
OE, must be exact. ISW	
	(4)
$r^2 = \frac{2\tan\theta}{1-\tan^2\theta}$	**M1**
Uses relevant identity for $\tan 2\theta$.	
$x^2+y^2 = \frac{2\left(\frac{y}{x}\right)}{1-\frac{y^2}{x^2}} = \frac{2xy}{x^2-y^2}$	**M1**
For the M mark uses either $r=\sqrt{x^2+y^2}$ or $\tan\theta = \frac{y}{x}$.	
Use both $r=\sqrt{x^2+y^2}$ and $\tan\theta=\frac{y}{x}$ correctly.	**A1**
$\left(x^2+y^2\right)\left(x^2-y^2\right) = 2xy$	**A1**
$x^4-y^4=2xy$	
$x^4-2xy-y^4=0$.	

At least one intermediate step before the answer given. Needs to be in this form as answer given.	
	(4)
$\frac{1}{2}\left(\cos\frac{1}{8}\pi\right)\left(\sin\frac{1}{8}\pi\right)-\frac{1}{8}\ln 2$	**M1**
Area of triangle minus *their* (b).	
$\frac{1}{8}\sqrt{2}-\frac{1}{8}\ln 2$	**A1**
By double angle formula or otherwise.	
	(2)
	Total: 12

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Q41

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Mark scheme.

Initial line drawn and $(1, 0)$ joined to O by a single arc lying in both of the first two quadrants and no others.	**B1**
Fully correct shape at extremities and r decreasing.	**B1**
	(2)
$\frac{1}{2}\int_0^{\pi} \cos^2 \frac{1}{2}\theta \,\mathrm{d}\theta$	**M1**
Uses $\frac{1}{2}\int r^2 \,\mathrm{d}\theta$ with correct limits.	
$= \frac{1}{4}\int_0^{\pi} \cos\theta + 1 \,\mathrm{d}\theta = \frac{1}{4}\left[\sin\theta + \theta\right]_0^{\pi}$	**M1A1**
Applies given identity and integrates.	
$= \frac{1}{4}\pi$	**A1**
	(4)
$y = \cos\frac{1}{2}\theta \sin\theta$	**B1**
Uses $y = r\sin\theta$.	
$\cos\frac{1}{2}\theta\cos\theta - \frac{1}{2}\sin\theta\sin\frac{1}{2}\theta = 0$, correct and equal to zero	**M1A1**
Differentiates.	
$\cos\frac{1}{2}\theta\left(2\cos^2\frac{1}{2}\theta - 1\right) - \sin^2\frac{1}{2}\theta\cos\frac{1}{2}\theta = 0$	**M1**
$\cos\frac{1}{2}\theta \neq 0 \Rightarrow 2\cos^2\frac{1}{2}\theta - 1 - \sin^2\frac{1}{2}\theta = 0 \Rightarrow 3\cos^2\frac{1}{2}\theta - 2 = 0$	
Forms an equation in one trig ratio using correct formulae. (May be in terms of sin or tan)	
$\cos^2\frac{1}{2}\theta = \dfrac{2}{3}$	**A1**
Solving equation to find one of $\sin^2\frac{1}{2}\theta = \dfrac{1}{3}$, $\tan^2\frac{1}{2}\theta = \dfrac{1}{2}$, $\sin\theta = \dfrac{2\sqrt{2}}{3}$, $\cos\theta = \dfrac{1}{3}$	
$y = 2\cos^2\frac{1}{2}\theta\sin\frac{1}{2}\theta = 2\left(\frac{2}{3}\right)\sqrt{1-\frac{2}{3}} = \frac{4}{3}\sqrt{\frac{1}{3}}$ (accept $\frac{4}{9}\sqrt{3}$)	**A1**
	(6)

Total: 12

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