Further Math with Ali Hashir

Matrices

Topical Questions (CIE 9231)

Compiled from CIE 9231 Further Mathematics Paper 1 past papers (2015–2025), sorted by topic. Each question is numbered for this booklet, with its original paper code and session shown beneath.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Q1

9231/11, 9231/12 • May/June 2020 • Question 6

Let $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$.

(a) The transformation in the x-y plane represented by $\mathbf{A}^{-1}$ transforms a triangle of area $30\,\text{cm}^2$ into a triangle of area $d\,\text{cm}^2$.

Find the value of d. [3]

(b) Prove by mathematical induction that, for all positive integers n,

$$\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{pmatrix}.$$

[5]

(c) The line $y = 2x$ is invariant under the transformation in the x-y plane represented by $\mathbf{A}^n\mathbf{B}$, where

$$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 33 & 0 \end{pmatrix}.$$

Find the value of n. [5]

Q2

9231/13 • May/June 2020 • Question 4

The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} k & 0 & 2 \\ 0 & -1 & -1 \\ 1 & 1 & -k \end{pmatrix},$$

where k is a real constant.

(a) Show that **A** is non-singular. [3]

The matrices **B** and **C** are given by

$$\mathbf{B} = \begin{pmatrix} 0 & -3 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} -3 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

It is given that $\mathbf{CAB} = \begin{pmatrix} -2 & -\frac{3}{2} \\ -1 & -\frac{3}{2} \end{pmatrix}$.

(b) Find the value of k. [3]

(c) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **CAB**. [5]

Q3

9231/11, 9231/13 • October/November 2020 • Question 1

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}$, where a and b are positive constants.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

The unit square in the x-y plane is transformed by $\mathbf{M}$ onto parallelogram $OPQR$.

(b) Find, in terms of a and b, the matrix which transforms parallelogram $OPQR$ onto the unit square. [2]

It is given that the area of $OPQR$ is $2\,\text{cm}^2$ and that the line $x + 3y = 0$ is invariant under the transformation represented by $\mathbf{M}$.

(c) Find the values of a and b. [5]

Q4

9231/12 • October/November 2020 • Question 4

The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}.$$

(a) Give full details of the geometrical transformation in the x-y plane represented by **A**. [1]

(b) Give full details of the geometrical transformation in the x-y plane represented by **B**. [2]

The triangle DEF in the x-y plane is transformed by **AB** onto triangle PQR.

(c) Show that the triangles DEF and PQR have the same area. [3]

(d) Find the matrix which transforms triangle PQR onto triangle DEF. [2]

(e) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **AB**. [5]

Q5

9231/11, 9231/12 • May/June 2021 • Question 4

The matrix $\mathbf{M}$ represents the sequence of two transformations in the x-y plane given by a rotation of 60 anticlockwise about the origin followed by a one-way stretch in the x-direction, scale factor d ($d \neq 0$).

(a) Find $\mathbf{M}$ in terms of d. [4]

(b) The unit square in the x-y plane is transformed by $\mathbf{M}$ onto a parallelogram of area $\frac{1}{2}d^2$ units^2.

Show that $d = 2$. [2]

The matrix $\mathbf{N}$ is such that $\mathbf{MN} = \begin{pmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

(c) Find $\mathbf{N}$. [3]

(d) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by $\mathbf{MN}$. [5]

Q6

9231/13 • May/June 2021 • Question 4

The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 2 & k & k \\ 5 & -1 & 3 \\ 1 & 0 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \quad \mathbf{C} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 0 \end{pmatrix},$$

where k is a real constant.

(a) Find **CAB**. [3]

(b) Given that **A** is singular, find the value of k. [3]

(c) Using the value of k from part (b), find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **CAB**. [5]

Q7

9231/11, 9231/13 • October/November 2021 • Question 4

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

(b) Find the values of θ, for $0 \leqslant \theta \leqslant \pi$, for which the transformation represented by $\mathbf{M}$ has exactly one invariant line through the origin, giving your answers in terms of π. [9]

Q8

9231/12 • October/November 2021 • Question 1

(a) Give full details of the geometrical transformation in the x-y plane represented by the matrix $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$. [1]

Let $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & 2 \end{pmatrix}$.

(b) The triangle DEF in the x-y plane is transformed by $\mathbf{A}$ onto triangle PQR.
Given that the area of triangle DEF is $13\,\text{cm}^2$, find the area of triangle PQR. [2]

(c) Find the matrix $\mathbf{B}$ such that $\mathbf{AB} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$. [2]

(d) Show that the origin is the only invariant point of the transformation in the x-y plane represented by $\mathbf{A}$. [4]

Q9

9231/11, 9231/12 • May/June 2022 • Question 7

The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & k & 6 \\ 7 & 8 & 9 \end{pmatrix}$.

(a) Find the set of values of k for which $\mathbf{A}$ is non-singular. [3]

(b) Given that $\mathbf{A}$ is non-singular, find, in terms of k, the entries in the top row of $\mathbf{A}^{-1}$. [4]

(c) Given that $\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, give an example of a matrix $\mathbf{C}$ such that $\mathbf{BAC} = \begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}$. [4]

(d) Find the set of values of k for which the transformation in the x-y plane represented by $\begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}$ has two distinct invariant lines through the origin. [6]

Q10

9231/13 • May/June 2022 • Question 5

Let $\mathbf{A} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$, where a is a positive constant.

(a) State the type of the geometrical transformation in the x–y plane represented by $\mathbf{A}$. [1]

(b) Prove by mathematical induction that, for all positive integers n,

$$\mathbf{A}^n = \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}.$$

[5]

Let $\mathbf{B} = \begin{pmatrix} b & b \\ a^{-1} & a^{-1} \end{pmatrix}$, where b is a positive constant.

(c) Find the equations of the invariant lines, through the origin, of the transformation in the x–y plane represented by $\mathbf{A}^n\mathbf{B}$. [6]

Q11

9231/11, 9231/13 • October/November 2022 • Question 5

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$, where k is a constant.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

(b) The triangle ABC in the x–y plane is transformed by $\mathbf{M}$ onto triangle DEF.

Find, in terms of k, the single matrix which transforms triangle DEF onto triangle ABC. [2]

(c) Find the set of values of k for which the transformation represented by $\mathbf{M}$ has no invariant lines through the origin. [7]

Q12

9231/12 • October/November 2022 • Question 3

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$, where k is a constant and $k \neq 0$ or 1.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

(b) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

(c) Show that the invariant points of the transformation represented by $\mathbf{M}$ lie on the line $y = \dfrac{k^2}{1-k}x$. [4]

(d) The triangle ABC in the x-y plane is transformed by $\mathbf{M}$ onto triangle DEF.

Find the value of k for which the area of triangle DEF is equal to the area of triangle ABC. [2]

Q13

9231/11, 9231/12 • May/June 2023 • Question 1(b)

Let $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$.

Find, in terms of n, the inverse of $\mathbf{A}^n$. [2]

Q14

9231/11, 9231/12 • May/June 2023 • Question 4

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & b^2 \\ c^2 & a \end{pmatrix}$, where a, b, c are real constants and $b \neq 0$.

(a) Show that $\mathbf{M}$ does not represent a rotation about the origin. [2]

(b) Find the equations of the invariant lines, through the origin, of the transformation in the x–y plane represented by $\mathbf{M}$. [5]

It is given that $\mathbf{M}$ represents the sequence of two transformations in the x–y plane given by an enlargement, centre the origin, scale factor 5 followed by a shear, x-axis fixed, with $(0, 1)$ mapped to $(5, 1)$.

(c) Find $\mathbf{M}$. [3]

(d) The triangle DEF in the x–y plane is transformed by $\mathbf{M}$ onto triangle PQR.

Given that the area of triangle DEF is $12\,\text{cm}^2$, find the area of triangle PQR. [2]

Q15

9231/13 • May/June 2023 • Question 4

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$, where $0 < \theta < \pi$ and k is a non-zero constant. The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations, one of which is a shear.

(a) Describe fully the other transformation and state the order in which the transformations are applied. [3]

(b) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

(c) Find, in terms of k, the value of $\tan\theta$ for which $\mathbf{M} - \mathbf{I}$ is singular. [5]

(d) Given that $k = 2\sqrt{3}$ and $\theta = \frac{1}{3}\pi$, show that the invariant points of the transformation represented by $\mathbf{M}$ lie on the line $3y + \sqrt{3}x = 0$. [4]

Q16

9231/11, 9231/13 • October/November 2023 • Question 5

Let k be a constant. The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by

$$\mathbf{A} = \begin{pmatrix} 1 & k & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \qquad \text{and} \qquad \mathbf{C} = \begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

It is given that $\mathbf{A}$ is singular.

(a) Show that $\mathbf{CAB} = \begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}$. [5]

(b) Find the equations of the invariant lines, through the origin, of the transformation in the x–y plane represented by $\mathbf{CAB}$. [5]

The matrices $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{F}$ represent geometrical transformations in the x–y plane.

- $\mathbf{D}$ represents an enlargement, centre the origin.
- $\mathbf{E}$ represents a stretch parallel to the x-axis.
- $\mathbf{F}$ represents a reflection in the line $y = x$.

(c) Given that $\mathbf{CAB} = \mathbf{D} - 9\mathbf{EF}$, find $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{F}$. [5]

Q17

9231/12 • October/November 2023 • Question 3

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, where k is a constant and $k \neq 0$ and $k \neq 1$.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations. State the type of each transformation, and make clear the order in which they are applied. [2]

The unit square in the x–y plane is transformed by $\mathbf{M}$ onto parallelogram $OPQR$.

(b) Find, in terms of k, the area of parallelogram $OPQR$ and the matrix which transforms $OPQR$ onto the unit square. [3]

(c) Show that the line through the origin with gradient $\dfrac{1}{k-1}$ is invariant under the transformation in the x–y plane represented by $\mathbf{M}$. [3]

Q18

9231/11, 9231/12 • May/June 2024 • Question 4

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 14 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x-y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

(b) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

(c) Find the equations of the invariant lines, through the origin, of the transformation represented by $\mathbf{M}$. [5]

(d) The triangle ABC in the x-y plane is transformed by $\mathbf{M}$ onto triangle DEF.

Given that the area of triangle DEF is $28\,\text{cm}^2$, find the area of triangle ABC. [2]

Q19

9231/13 • May/June 2024 • Question 1

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} k & 1 & 0 \\ 6 & 5 & 2 \\ -1 & 3 & -k \end{pmatrix},$$

where k is a real constant.

(a) Show that $\mathbf{A}$ is non-singular. [3]

(b) Given that $\mathbf{A}^{-1} = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 0 & 0 \\ -\frac{23}{2} & \frac{1}{2} & 3 \end{pmatrix}$, find the value of k. [2]

Q20

9231/13 • May/June 2024 • Question 3

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 7 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x-y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

(b) Find the equations of the invariant lines, through the origin, of the transformation represented by $\mathbf{M}$. [5]

The triangle DEF in the x-y plane is transformed by $\mathbf{M}$ onto triangle PQR.

(c) Given that the area of triangle PQR is $35\,\text{cm}^2$, find the area of triangle DEF. [2]

Q21

9231/11, 9231/13 • October/November 2024 • Question 1

The matrix $\mathbf{M}$ represents the sequence of two transformations in the x-y plane given by a stretch parallel to the x-axis, scale factor k ($k \neq 0$), followed by a shear, x-axis fixed, with $(0, 1)$ mapped to $(k, 1)$.

(a) Show that $\mathbf{M} = \begin{pmatrix} k & k \\ 0 & 1 \end{pmatrix}$. [4]

(b) The transformation represented by $\mathbf{M}$ has a line of invariant points.

Find, in terms of k, the equation of this line. [3]

The unit square S in the x-y plane is transformed by $\mathbf{M}$ onto the parallelogram P.

(c) Find, in terms of k, a matrix which transforms P onto S. [1]

(d) Given that the area of P is $3k^2$ units^2, find the possible values of k. [2]

Q22

9231/12 • October/November 2024 • Question 4

The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \qquad \text{and} \qquad \mathbf{C} = \begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

(a) Show that $\mathbf{CAB} = \begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}$. [3]

(b) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **CAB**. [5]

Let $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$.

(c) Give full details of the transformation represented by **M**. [2]

(d) Find the matrix **N** such that $\mathbf{NM} = \mathbf{CAB}$. [3]

Q23

9231/11, 9231/12 • May/June 2025 • Question 4

The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, where $0 < \theta < 2\pi$.

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x–y plane.

State the type of each transformation, and make clear the order in which they are applied. [2]

(b) Find the value of θ for which the transformation represented by $\mathbf{M}$ has a line of invariant points. [7]

Q24

9231/13 • May/June 2025 • Question 1

The matrix $\mathbf{M}$ represents the sequence of two transformations in the x-y plane given by a stretch parallel to the x-axis, scale factor 14, followed by a rotation anticlockwise about the origin through angle $\frac{1}{3}\pi$.

(a) Show that $2\mathbf{M} = \begin{pmatrix} 14 & -\sqrt{3} \\ 14\sqrt{3} & 1 \end{pmatrix}$. [4]

(b) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by $\mathbf{M}$. [5]

The unit square S in the x-y plane is transformed by $\mathbf{M}$ onto the rectangle P.

(c) Find the matrix which transforms P onto S. [2]

Q25

9231/11, 9231/13 • October/November 2025 • Question 2

The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix}.$$

(a) Give full details of the geometrical transformation in the x-y plane represented by $\mathbf{A}$. [2]

(b) Give full details of the geometrical transformation in the x-y plane represented by $\mathbf{B}$. [2]

The triangle DEF in the x-y plane is transformed by $\mathbf{AB}$ onto triangle PQR.

(c) Show that the triangles DEF and PQR have the same area. [2]

(d) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by $\mathbf{AB}$. [5]

Q26

9231/12 • October/November 2025 • Question 4

Let k and m be non-zero constants. The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} k & 0 \\ 0 & m \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

(a) Give full details of the geometrical transformation in the x-y plane represented by the matrix $\mathbf{B}$ in each of the following cases.

(i) $m = 1$ [2]

(ii) $m = k$ [2]

(b) Show that the matrix $\mathbf{ABC}$ is singular. [6]

Q27

9231/14 • October/November 2025 • Question 5

(i) By writing $\cos\left(\frac{1}{12}\pi\right)$ as $\cos\left(\frac{1}{4}\pi - \frac{1}{6}\pi\right)$, show that $\cos\left(\frac{1}{12}\pi\right) = \frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$. [1]

(ii) Show also that $\sin\left(\frac{1}{12}\pi\right) = \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$. [1]

The matrix $\mathbf{M}$ is such that

$$\mathbf{M} = \frac{1}{4}\begin{pmatrix} \sqrt{6}+\sqrt{2} & \sqrt{2}-\sqrt{6} \\ \sqrt{6}-\sqrt{2} & \sqrt{6}+\sqrt{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x-y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

(b) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is $\mathbf{I}$. [2]

(c) Given that $y = mx$ is an invariant line of the transformation represented by $\mathbf{M}$, show that

$$m^2\sin\left(\tfrac{1}{12}\pi\right) + 2m\cos\left(\tfrac{1}{12}\pi\right) - \sin\left(\tfrac{1}{12}\pi\right) = 0$$

and find the values of m in the form $a\cot\left(\frac{1}{12}\pi\right) + b\,\mathrm{cosec}\left(\frac{1}{12}\pi\right)$, where a and b are integers to be determined. [6]