Further Math with Ali Hashir

Matrices

Topical Mark Schemes (CIE 9231)

Mark schemes corresponding to the topical questions booklet for this topic, compiled from official CIE 9231 Paper 1 marking schemes (2015–2025). Question numbers match the questions booklet.

About this resource: I teach CIE A Level Further Mathematics, and over the years I found that there simply weren't many well-organised, topic-wise resources available for this subject – it's a niche A Level with relatively few candidates, so it doesn't get much attention from resource makers. These topical booklets are my attempt to fix that: every past-paper question, sorted by topic, with its official mark scheme alongside it.

Help us improve these documents for future students. If you spot a mistake, email us at **vectorspacepk@gmail.com**.

What kind of mistakes might you find? These booklets are compiled by hand from hundreds of past-paper PDFs. I've checked them as carefully as I can, but I'm only human, and the odd error may have slipped through – so please help me catch what I missed. Things to look out for:

- Spelling or typing mistakes
- Wrong final answers
- A mark scheme that doesn't line up with its question (misaligned pages)
- A mark scheme that belongs to the wrong question entirely
- Pre-2020 questions that are technically out of the current syllabus (the syllabus changed in 2020, but some older questions are still included here for practice)
- Missing, extra, or wrongly-labelled marks (e.g. an M1 or A1 that shouldn't be there, or one that's missing)
- Diagrams or images that are unclear, missing, or don't match the question
- Questions placed under the wrong topic

If you spot any of these (or anything else that looks off), please let me know.

This is **Version 1** of this booklet. As readers point out errors, I'll keep correcting and improving these documents, so I'd recommend checking back on the website from time to time and using the latest version rather than relying on a saved copy.

If you think these documents brought you value, consider following me on social media – it helps more than you'd think.

Contents

Click a question number below to jump straight to its mark scheme. Every box is a live link.

For the best hyperlink experience, open this PDF in a dedicated reader – such as Adobe Acrobat, Apple Preview, or your browser's built-in PDF viewer – rather than a basic in-app or embedded preview, where links sometimes don't work.

Q1

9231/11, 9231/12 • May/June 2020 • Question 6

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Mark scheme.

$\det \mathbf{A}^{-1} = (\det \mathbf{A})^{-1} = \frac{1}{2}$	**M1A1**
$d = 15$	**A1**
	(3)
$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2^1 & 0 \\ 2^1 - 1 & 1 \end{pmatrix}$ so true when $n = 1$.	**B1**
Assume that it is true for $n = k$, so $\mathbf{A}^k = \begin{pmatrix} 2^k & 0 \\ 2^k - 1 & 1 \end{pmatrix}$.	**B1**
Then $\mathbf{A}^{k+1} = \begin{pmatrix} 2^k & 0 \\ 2^k - 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2^{k+1} & 0 \\ 2(2^k - 1) + 1 & 1 \end{pmatrix} = \begin{pmatrix} 2^{k+1} & 0 \\ 2^{k+1} - 1 & 1 \end{pmatrix}$	**M1A1**
So, it is also true for $n = k + 1$. Hence, by induction, true for all positive integers.	**A1**
	(5)
$\mathbf{A}^n\mathbf{B} = \begin{pmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 33 & 0 \end{pmatrix} = \begin{pmatrix} 2^n & 0 \\ 2^n + 32 & 0 \end{pmatrix}$	**M1A1**
$\begin{pmatrix} 2^n & 0 \\ 2^n + 32 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2^n x \\ (2^n + 32)x \end{pmatrix}$	**B1**
$(2^n + 32)x = 2^{n+1}x \Rightarrow 2^n = 32 \Rightarrow n = 5$	**M1A1**
	(5)
	Total: 13

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Q2

9231/13 • May/June 2020 • Question 4

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Mark scheme.

$k\begin{vmatrix}-1 & -1\\ 1 & -k\end{vmatrix}+2\begin{vmatrix}0 & -1\\ 1 & 1\end{vmatrix}=0\Rightarrow k^2+k+2=0$	**M1A1**
$1-4(2)=-7<0\Rightarrow$ Non-singular	**A1**
	(3)
$\begin{pmatrix}-3 & -1 & 1\\ 1 & 1 & 2\end{pmatrix}\begin{pmatrix}k & 0 & 2\\ 0 & -1 & -1\\ 1 & 1 & -k\end{pmatrix}\begin{pmatrix}0 & -3\\ -1 & 3\\ 0 & 0\end{pmatrix}=\begin{pmatrix}-3 & -1 & 1\\ 1 & 1 & 2\end{pmatrix}\begin{pmatrix}0 & -3k\\ 1 & -3\\ -1 & 0\end{pmatrix}$	**M1A1**
$=\begin{pmatrix}-2 & 9k+3\\ -1 & -3k-3\end{pmatrix}=\begin{pmatrix}-2 & -\frac{3}{2}\\ -1 & -\frac{3}{2}\end{pmatrix}\Rightarrow k=-\frac{1}{2}$	**A1**
	(3)
$\begin{pmatrix}2 & \frac{3}{2}\\ 1 & \frac{3}{2}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}2x+\frac{3}{2}y\\ x+\frac{3}{2}y\end{pmatrix}$	**B1**
$x+\frac{3}{2}mx=m\left(2x+\frac{3}{2}mx\right)$	**M1A1**
$1+\frac{3}{2}m=2m+\frac{3}{2}m^2\Rightarrow 3m^2+m-2=0$	**A1**
$y=-x$ and $3y-2x=0$	**A1**
	(5)
	Total: 11

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Q3

9231/11, 9231/13 • October/November 2020 • Question 1

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Mark scheme.

One-way stretch followed by a shear.	**B2**
Both named correctly. Award B1 if given in the wrong order.	
	(2)
$\mathbf{M}^{-1} = \frac{1}{a}\begin{pmatrix} 1 & -b \\ 0 & a \end{pmatrix}$	**M1A1**
	(2)
$a = 2$	**B1**
$\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ y \end{pmatrix}$	**B1**
Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$.	
$= \begin{pmatrix} ax - \frac{1}{3}bx \\ -\frac{1}{3}x \end{pmatrix}$	**M1**
Uses $x + 3y = 0$.	
$x = ax - \frac{1}{3}bx \Rightarrow 1 = a - \frac{1}{3}b$	**M1**
Uses that line is invariant (or $X + 3Y = 0$).	
$b = 3$	**A1**
	(5)
	Total: 9

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Q4

9231/12 • October/November 2020 • Question 4

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Mark scheme.

Reflection in the line $y = x$.	**B1**
Only mention reflection (and no other transformation).	
	(1)
Rotation	**B1**
Writes 'rotation' or 'rotate' (and no other transformation)	
$\frac{1}{3}\pi$ anticlockwise about the origin.	**B1**
Or $60°$	
	(2)
$\mathbf{AB} = \begin{pmatrix} \frac{1}{2}\sqrt{3} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2}\sqrt{3} \end{pmatrix}$ or $\det \mathbf{AB} = \det \mathbf{A} \det \mathbf{B}$	**M1**
Finds **AB** or uses product of determinants. Full marks here may be obtained by arguing that both reflection and rotation preserve [absolute] value of area and so also does their combination.	
$\det \mathbf{AB} = -1$	**B1**
Area of $PQR = \lvert -1 \rvert$ Area of DEF	**A1**
	(3)
$(\mathbf{AB})^{-1} = -\begin{pmatrix} -\frac{1}{2}\sqrt{3} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2}\sqrt{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\sqrt{3} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2}\sqrt{3} \end{pmatrix}$	**M1A1**
or using $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$	
	(2)
$\begin{pmatrix} \frac{1}{2}\sqrt{3} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2}\sqrt{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2}x\sqrt{3} + \frac{1}{2}y \\ \frac{1}{2}x - \frac{1}{2}y\sqrt{3} \end{pmatrix}$	**B1**

Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$. Accept $t\begin{pmatrix} 1 \\ m \end{pmatrix}$ instead of $\begin{pmatrix} x \\ y \end{pmatrix}$.	
$\frac{1}{2} - \frac{1}{2}m\sqrt{3} = \frac{1}{2}m\sqrt{3} + \frac{1}{2}m^2$	**M1A1**
Uses $Y = mX$. OE. Allow working with $y = mx + c$ as long as $c = 0$ is stated explicitly.	
$1 - m\sqrt{3} = m\sqrt{3} + m^2 \Rightarrow m^2 + 2m\sqrt{3} - 1 = 0 \Rightarrow m = \pm 2 - \sqrt{3}$	**A1**
$y = (2 - \sqrt{3})x$ and $y + (2 + \sqrt{3})x = 0$	**A1**
OE	
	(5)
	Total: 13

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Q5

9231/11, 9231/12 • May/June 2021 • Question 4

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Mark scheme.

$\begin{pmatrix} \cos 60 & -\sin 60 \\ \sin 60 & \cos 60 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	**B1**
$\begin{pmatrix} d & 0 \\ 0 & 1 \end{pmatrix}$	**B1**
$\mathbf{M} = \begin{pmatrix} d & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{d}{2} & -\frac{d\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	**M1 A1**
	(4)
$d = \frac{1}{2}d^2$	**M1**
$d \neq 0 \Rightarrow d = 2$	**A1**
	(2)
$\mathbf{M}^{-1} = \frac{1}{2} \begin{pmatrix} \frac{1}{2} & \sqrt{3} \\ -\frac{\sqrt{3}}{2} & 1 \end{pmatrix}$	**B1 FT**
$\mathbf{N} = \mathbf{M}^{-1} \begin{pmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \frac{1}{2} & \sqrt{3} \\ -\frac{\sqrt{3}}{2} & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	**M1**
$= \frac{1}{4} \begin{pmatrix} 1+\sqrt{3} & 1+\sqrt{3} \\ 1-\sqrt{3} & 1-\sqrt{3} \end{pmatrix}$	**A1**
	(3)
$\begin{pmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ \frac{1}{2}x + \frac{1}{2}y \end{pmatrix}$	**B1**
$\frac{1}{2}x + \frac{1}{2}mx = m(x + mx)$	**M1 A1**
$1 + m = 2m + 2m^2 \Rightarrow 2m^2 + m - 1 = 0$	**A1**
$y = \frac{1}{2}x$ and $y = -x$ (WWW)	**A1**

(5)

Total: 14

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Q6

9231/13 • May/June 2021 • Question 4

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Mark scheme.

$\begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 2 & k & k \\ 5 & -1 & 3 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 2+k & k \\ 8 & -1 \\ 2 & 0 \end{pmatrix}$ OR $=$ $\begin{pmatrix} 6 & -1 & 4 \\ 8 & -k-2 & -k+6 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$	**M1 A1**
$\begin{pmatrix} 10 & -1 \\ -k+14 & -k-2 \end{pmatrix}$	**A1**
	(3)
$2\begin{vmatrix} -1 & 3 \\ 0 & 1 \end{vmatrix} - k\begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 5 & -1 \\ 1 & 0 \end{vmatrix} = 0$ leading to $-2-2k+k=0$	**M1 A1**
$k=-2$	**A1**
	(3)
$\begin{pmatrix} 10 & -1 \\ 16 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10x-y \\ 16x \end{pmatrix}$	**M1**
$10x - mx = X$ and $16x = mX$	**M1 A1**
$16 = 10m - m^2$ $[m^2 - 10m + 16 = 0]$	**A1**
$y = 2x$ and $y = 8x$	**A1**
	(5)
	Total: 11

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Q7

9231/11, 9231/13 • October/November 2021 • Question 4

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Mark scheme.

One-way stretch [scale factor 3 parallel to the x-axis] followed by a rotation, [anticlockwise centred at the origin, through an angle θ.]	**B2**
Award B1 if given in the wrong order.	
	(2)
$\mathbf{M} = \begin{pmatrix} 3\cos\theta & -\sin\theta \\ 3\sin\theta & \cos\theta \end{pmatrix}$	**B1**
$\begin{pmatrix} 3\cos\theta & -\sin\theta \\ 3\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x\cos\theta - y\sin\theta \\ 3x\sin\theta + y\cos\theta \end{pmatrix}$	**B1**
Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$.	
$3x\sin\theta + mx\cos\theta = 3mx\cos\theta - m^2x\sin\theta$	**M1A1**
Uses $y = mx$ and $Y = mX$.	
$m^2\sin\theta - 2m\cos\theta + 3\sin\theta = 0$	**A1**
$4\cos^2\theta - 12\sin^2\theta = 0$	**M1A1**
Sets discriminant equal to 0.	
$\tan\theta = \pm\frac{1}{\sqrt{3}}$ or $4\cos^2\theta - 12(1-\cos^2\theta) = 0$ leading to $16\cos^2\theta - 12 = 0$ leading to $\cos\theta = \pm\frac{\sqrt{3}}{2}$	**M1**
Uses an appropriate trigonometric identity.	
$\theta = \frac{1}{6}\pi$ and $\theta = \frac{5}{6}\pi$	**A1**
Allow 30° and 150°	
	(9)
	Total: 11

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Q8

9231/12 • October/November 2021 • Question 1

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Mark scheme.

Enlargement, scale factor 6.	**B1**
	(1)
$\det \mathbf{A} = 6 - 8 = -2$	**M1**
Finds $\det \mathbf{A}$.	
Area $= 2 \times 13 = 26\,\text{cm}^2$	**A1**
	(2)
$\mathbf{A}^{-1} = -\frac{1}{2}\begin{pmatrix} 2 & -4 \\ -2 & 3 \end{pmatrix}$	**M1**
Finds $\mathbf{A}^{-1}$.	
$\mathbf{B} = -3\begin{pmatrix} 2 & -4 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} -6 & 12 \\ 6 & -9 \end{pmatrix}$	**A1**
AEF. Could be solved by equations.	
	(2)
$\begin{pmatrix} 3 & 4 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x+4y \\ 2x+2y \end{pmatrix}$	**B1**
Finds $\begin{pmatrix} X \\ Y \end{pmatrix}$.	
$\left.\begin{matrix} 3x+4y=x \\ 2x+2y=y \end{matrix}\right\}$ leading to $\left\{\begin{matrix} 2x+4y=0 \\ 2x+y=0 \end{matrix}\right.$	**M1**
Uses $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} X \\ Y \end{pmatrix}$ to form simultaneous equations.	
$y = -2x$ leading to $-6x = 0$ leading to $x = 0$, $y = 0$	**M1A1**

Solves equations or states that $\det\begin{pmatrix} 2 & 4 \\ 2 & 1 \end{pmatrix} \neq 0$, AG.
(4)
Total: 9

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Q9

9231/11, 9231/12 • May/June 2022 • Question 7

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Mark scheme.

$\begin{vmatrix} k & 6 \\ 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & k \\ 7 & 8 \end{vmatrix} = 0 \Rightarrow -12k + 60 = 0$	**M1 A1**
$k \neq 5$	**A1**
	(3)
$\dfrac{1}{60-12k}\begin{vmatrix} k & 6 \\ 8 & 9 \end{vmatrix} = \dfrac{9k-48}{60-12k} = \dfrac{3k-16}{4(5-k)}$	**M1 A1**
$\dfrac{-1}{60-12k}\begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} = \dfrac{6}{60-12k} = \dfrac{1}{2(5-k)}$	**A1**
$\dfrac{1}{60-12k}\begin{vmatrix} 2 & 3 \\ k & 6 \end{vmatrix} = \dfrac{12-3k}{60-12k} = \dfrac{4-k}{4(5-k)}$	**A1**
	(4)
$\mathbf{BA} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & k & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & k & 6 \end{pmatrix}$	**M1 A1**
$\begin{pmatrix} 1 & 2 & 3 \\ 4 & k & 6 \end{pmatrix}\begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix} = \begin{pmatrix} a+2b+3c & d+2e+3f \\ 4a+kb+6c & 4d+ke+6f \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}$	**M1**
$\mathbf{C} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$ (accept any a, b, c, d, e, f satisfying the above)	**A1**
	(4)
$\begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x+y \\ kx+4y \end{pmatrix}$	**B1**

$kx + 4mx = m(2x + mx)$	**M1 A1**
$m^2 - 2m - k = 0$	**A1**
$4 + 4k > 0$	**M1**
$k > -1 \quad k \neq 8$	**A1**
	(6)
	Total: 17

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Q10

9231/13 • May/June 2022 • Question 5

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Mark scheme.

A shear in the x-direction.	**B1**
	(1)
$\mathbf{A} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ so true when $n = 1$.	**B1**
Assume that it is true for $n = k$, so $\mathbf{A}^k = \begin{pmatrix} 1 & ka \\ 0 & 1 \end{pmatrix}$.	**B1**
Then $\mathbf{A}^{k+1} = \begin{pmatrix} 1 & ka \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & a+ak \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & (k+1)a \\ 0 & 1 \end{pmatrix}$	**M1 A1**
If it is true for [$n = 1$ and] $n = k$ then it is also true for $n = k + 1$. Hence, [by induction,] true for all positive integers.	**A1**
	(5)
$\mathbf{A}^n\mathbf{B} = \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix} \begin{pmatrix} b & b \\ a^{-1} & a^{-1} \end{pmatrix} = \begin{pmatrix} b+n & b+n \\ \frac{1}{a} & \frac{1}{a} \end{pmatrix}$	**M1 A1**
$\begin{pmatrix} b+n & b+n \\ \frac{1}{a} & \frac{1}{a} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (b+n)(x+y) \\ \frac{1}{a}(x+y) \end{pmatrix}$	**B1**
$\frac{1}{a}(1+m) = m(b+n)(1+m)$	**M1**
$y = -x$	**A1**
$y = \dfrac{1}{a(b+n)}x$	**A1**
	(6)
	Total: 12

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Q11

9231/11, 9231/13 • October/November 2022 • Question 5

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Mark scheme.

Shear (in the x-direction) followed by a rotation (anticlockwise about the origin through 45°).	**B2**
	(2)
$\mathbf{M}^{-1} = \frac{1}{1}\begin{pmatrix} \frac{k+1}{\sqrt{2}} & -\frac{k-1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{k+1}{\sqrt{2}} & -\frac{k-1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$	**M1A1**
	(2)
$\mathbf{M} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}(k-1)\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}(k+1)\sqrt{2} \end{pmatrix}$	**B1**
$\frac{1}{2}\sqrt{2}\begin{pmatrix} 1 & k-1 \\ 1 & k+1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\sqrt{2}\begin{pmatrix} x+(k-1)y \\ x+(k+1)y \end{pmatrix}$	**B1**
$x + m(k+1)x = m\big(x + m(k-1)x\big)$	**M1A1**
$(k-1)m^2 - km - 1 = 0$	**A1**
$k^2 + 4k - 4 < 0$	**M1**
$2(-1-\sqrt{2}) < k < 2(-1+\sqrt{2})$	**A1**
	(7)
	Total: 11

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Q12

9231/12 • October/November 2022 • Question 3

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Mark scheme.

Shear [in the y-direction] followed by a stretch, [parallel to the y-axis, scale factor k].	**B2**
	(2)
$\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -k & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{k} \end{pmatrix}$	**B1**
$\mathbf{M}^{-1} = \begin{pmatrix} 1 & 0 \\ -k & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{k} \end{pmatrix}$	**B1**
	(2)
$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ k^2 & k \end{pmatrix}$	**B1**
$\begin{pmatrix} 1 & 0 \\ k^2 & k \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ k^2x + ky \end{pmatrix}$	**B1**
$k^2x + ky = y$	**M1**
$y = \dfrac{k^2}{1-k}x$	**A1**
	(4)
$1 = \lvert k \rvert$	**M1**
$k = -1$	**A1**
	(2)
	Total: 10

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Q13

9231/11, 9231/12 • May/June 2023 • Question 1(b)

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Mark scheme.

$\det \mathbf{A}^n = \det \begin{pmatrix} 3^n & 0 \\ \frac{1}{2}(3^n - 1) & 1 \end{pmatrix} = 3^n$ Or a multiple of $\mathbf{A}^{-n} = 2^{-1}3^{-n} \begin{pmatrix} 2 & 0 \\ 1 - 3^n & 2 \times 3^n \end{pmatrix}$ seen.	**B1**
$\mathbf{A}^{-n} = 3^{-n} \begin{pmatrix} 1 & 0 \\ \frac{1}{2}(1 - 3^n) & 3^n \end{pmatrix}$ OE $\mathbf{A}^{-n} = 2^{-1}3^{-n} \begin{pmatrix} 2 & 0 \\ 1 - 3^n & 2 \times 3^n \end{pmatrix}$	**B1**

Total: 2

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Q14

9231/11, 9231/12 • May/June 2023 • Question 4

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Mark scheme.

$\begin{pmatrix} a & b^2 \\ c^2 & a \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$	**M1**
Uses correct matrix for rotation.	
$b^2 = -c^2$ which is impossible since b and c are real and $b \neq 0$.	**A1**
	(2)
$\begin{pmatrix} a & b^2 \\ c^2 & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + b^2 y \\ c^2 x + ay \end{pmatrix}$ Transforms (x, y) to (X, Y).	**B1**
$c^2 x + amx = m(ax + b^2 mx)$ Uses $y = mx$ and $Y = mX$.	**M1A1**
$c^2 + am = ma + b^2 m^2 \Rightarrow c^2 = b^2 m^2$	**A1**
$y = \frac{c}{b}x$ and $y = -\frac{c}{b}x$	**A1**
	(5)
$\mathbf{M} = \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$ Award M1 if matrices correct but order is wrong.	**M1A1***
$\begin{pmatrix} 5 & 5^2 \\ 0 & 5 \end{pmatrix}$ Dep: previous A1	**DA1**
	(3)
$12 \times \det \mathbf{M}$ Using *their* $\mathbf{M}$.	**M1**
300 cm^2	**A1FT**
	(2)
	Total: 12

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Q15

9231/13 • May/June 2023 • Question 4

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Mark scheme.

Rotation [anticlockwise]	**B1**
about the origin through angle 2θ.	**B1**
Shear [in the x-direction] followed by a rotation [anticlockwise about the origin through angle 2θ].	**B1**
	(3)
$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -k \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}^{-1} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$	**B1**
$\mathbf{M}^{-1} = \begin{pmatrix} 1 & -k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$ Correct order	**B1**
	(2)
$\mathbf{M} - \mathbf{I} = \begin{pmatrix} \cos 2\theta - 1 & k\cos 2\theta - \sin 2\theta \\ \sin 2\theta & k\sin 2\theta + \cos 2\theta - 1 \end{pmatrix}$	**B1**
$(\cos 2\theta - 1)(k\sin 2\theta + \cos 2\theta - 1) - k\sin 2\theta\cos 2\theta + \sin^2 2\theta$ $[= 0]$ Evaluates $\det(\mathbf{M} - \mathbf{I})$	**M1**
$2 - 2\cos 2\theta - k\sin 2\theta = 0$ Brackets removed correctly and $= 0$	**A1**
$4\sin^2\theta = 2k\sin\theta\cos\theta$ Uses $1 - \cos 2\theta = 2\sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$ or all necessary double angle formulae.	**M1**
$\tan\theta = \frac{1}{2}k$	**A1**
	(5)
$\mathbf{M} = \begin{pmatrix} -\frac{1}{2} & -\frac{3}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{5}{2} \end{pmatrix}$	**B1**
$\begin{pmatrix} -\frac{1}{2} & -\frac{3}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{5}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}x - \frac{3}{2}\sqrt{3}y \\ \frac{1}{2}\sqrt{3}x + \frac{5}{2}y \end{pmatrix}$ Transforms (x, y) to (X, Y)	**B1FT**

$-\frac{1}{2}x - \frac{3}{2}\sqrt{3}y = x \Rightarrow -\frac{3}{2}x - \frac{3}{2}\sqrt{3}y = 0 \Rightarrow x + \sqrt{3}y = 0$ and $\frac{1}{2}\sqrt{3}x + \frac{5}{2}y = y \Rightarrow \frac{1}{2}\sqrt{3}x + \frac{3}{2}y = 0 \Rightarrow \sqrt{3}x + 3y = 0$ Sets $(X, Y) = (x, y)$	**M1**
$\sqrt{3}x + 3y = 0$	**A1**
	(4)
	Total: 14

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Q16

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Mark scheme.

$\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} - k\begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 0 \Rightarrow -1-k+3=0 \Rightarrow k=2$	**M1A1**
$\begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}\begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ -1 & -1 \\ -2 & 0 \end{pmatrix}$	**M1**
$\begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}$ ((a) Total: 5)	**M1A1**
$\begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x-7y \\ -9x+3y \end{pmatrix}$	**B1**
$-9x+3mx = m(3x-7mx)$	**M1A1**
$-9+3m = 3m-7m^2 \Rightarrow 7m^2 = 9$	**A1**
$y = \frac{3}{\sqrt{7}}x$ and $y = -\frac{3}{\sqrt{7}}x$ ((b) Total: 5)	**A1**
$\mathbf{D} = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix}$	**B1**
$\mathbf{E} = \begin{pmatrix} \beta & 0 \\ 0 & 1 \end{pmatrix}$	**B1**
$\mathbf{F} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	**B1**
$\begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix} - 9\begin{pmatrix} 0 & \beta \\ 1 & 0 \end{pmatrix}$	**M1**
$\mathbf{D} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$, $\mathbf{E} = \begin{pmatrix} \frac{7}{9} & 0 \\ 0 & 1 \end{pmatrix}$ ((c) Total: 5)	**A1**

Total: 15

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Q17

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Mark scheme.

Shear followed by a stretch. ((a) Total: 2)	**B2**
$\lvert OPQR\rvert = \lvert\det\mathbf{M}\rvert = \lvert k\rvert$	**B1**
$\mathbf{M}^{-1} = \frac{1}{k}\begin{pmatrix} 1 & 0 \\ -1 & k \end{pmatrix}$ ((b) Total: 3)	**M1A1**
$\begin{pmatrix} k & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ \frac{1}{k-1}x \end{pmatrix}$	**B1**
$\begin{pmatrix} k & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ \frac{1}{k-1}x \end{pmatrix} = \begin{pmatrix} kx \\ x + \frac{1}{k-1}x \end{pmatrix} = \begin{pmatrix} kx \\ \frac{k}{k-1}x \end{pmatrix}$	**M1**
$k\begin{pmatrix} x \\ \frac{1}{k-1}x \end{pmatrix}$ ((c) Total: 3)	**A1**
	Total: 8

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Q18

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Mark scheme.

[One way] stretch, rotation	**B1**
Stretch followed by rotation	**B1**
Correct order.	
Stretch parallel to the x-axis, scale factor 14	**B1**
Rotation, $\frac{1}{3}\pi$ [anticlockwise] about the origin.	**B1**
	(4)
$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}, \begin{pmatrix} 14 & 0 \\ 0 & 1 \end{pmatrix}^{-1} = \frac{1}{14}\begin{pmatrix} 1 & 0 \\ 0 & 14 \end{pmatrix}$	**B1**
Both inverses correct.	
$\mathbf{M}^{-1} = \begin{pmatrix} \frac{1}{14} & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$	**B1**
Correct order.	
	(2)
$\mathbf{M} = \begin{pmatrix} 7 & -\frac{1}{2}\sqrt{3} \\ 7\sqrt{3} & \frac{1}{2} \end{pmatrix}$	**B1**
$\begin{pmatrix} 7 & -\frac{1}{2}\sqrt{3} \\ 7\sqrt{3} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7x - \frac{1}{2}\sqrt{3}y \\ 7\sqrt{3}x + \frac{1}{2}y \end{pmatrix}$	**B1**
Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$.	
$7\sqrt{3}x + \frac{1}{2}mx = m\left(7x - \frac{1}{2}\sqrt{3}mx\right)$	**M1**
Uses $y = mx$ and $Y = mX$.	
$7\sqrt{3} + \frac{1}{2}m = 7m - \frac{1}{2}\sqrt{3}m^2 \Rightarrow \frac{1}{2}\sqrt{3}m^2 - \frac{13}{2}m + 7\sqrt{3} = 0$	**A1**

$y = 2\sqrt{3}x$ and $y = \frac{7}{3}\sqrt{3}x$	**A1**
	(5)
$28 = 14 \times \lvert ABC \rvert$	**M1**
Uses $\lvert DEF \rvert = \lvert \det \mathbf{M} \rvert \lvert ABC \rvert$.	
$2\,\mathrm{cm}^2$	**A1**
Allow with units missing.	
	(2)
	Total: 13

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Q19

9231/13 • May/June 2024 • Question 1

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Mark scheme.

$k\begin{vmatrix} 5 & 2 \\ 3 & -k \end{vmatrix} - \begin{vmatrix} 6 & 2 \\ -1 & -k \end{vmatrix} = k(-5k-6) - (-6k+2) = -5k^2 - 2$	**M1 A1**
Evaluates determinant, forms quadratic expression. (Allow 1 slip in the calculation of the determinant for M1 only)	
No real value of $k \Rightarrow$ Non-singular	**A1**
Convincing conclusion using the discriminant or determinant.	
	(3)
$3k+1 = 1$ or $-\frac{1}{2} = -\frac{1}{5k^2+2}$	**M1**
Uses $\mathbf{AA}^{-1} = \mathbf{I}$ or $\det(\mathbf{A}^{-1}) = (\det \mathbf{A})^{-1}$ to find equation in k. Could also use a minor determinant e.g. $-k \times 1 - 3 \times 0 = 0$	
$k = 0$	**A1**
Only $k = 0$, A0 for any additional solutions.	
	(2)
	Total: 5

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Q20

9231/13 • May/June 2024 • Question 3

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Mark scheme.

[One-way] stretch, shear	**B1**
Both types.	
Stretch followed by shear	**B1**
Correct order.	
Stretch parallel to the x-axis, scale factor 7	**B1**
Shear, x-axis fixed, with $(0, 1)$ mapped to $(2, 1)$.	**B1**
	(4)
$\mathbf{M} = \begin{pmatrix} 7 & 2 \\ 0 & 1 \end{pmatrix}$	**B1**
$\begin{pmatrix} 7 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7x + 2y \\ y \end{pmatrix}$	**B1 FT**
Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$.	
$mx = m(7x + 2mx)$	**M1**
Uses $y = mx$ and $Y = mX$.	
$2m^2 + 6m = 0$	**A1**
$y = 0$ and $y = -3x$	**A1**
SCB1 if M0 and both straight lines correct	
	(5)
Area of $PQR = \|7\| \times$ Area of DEF	**M1**
Area of $DEF = 5\,\text{cm}^2$	**A1**

SCB1 for an answer of 245 (35×7)
(2)
Total: 11

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Q21

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Mark scheme.

$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ [Stretch parallel to the x-axis, scale factor k ($k \neq 0$)]. (Allow without identification.)	**B1**
$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ [Shear, x-axis fixed, with $(0,1)$ mapped to $(k,1)$.] (Allow without identification.)	**B1**
$\mathbf{M} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} k & k \\ 0 & 1 \end{pmatrix}$ ((a) Total: 4)	**M1A1**
$\begin{pmatrix} k & k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx+ky \\ y \end{pmatrix}$	**B1**
$kx + ky = x$	**M1**
$y = \frac{1-k}{k}x$ oe ((b) Total: 3)	**A1**
$\mathbf{M}^{-1} = \frac{1}{k}\begin{pmatrix} 1 & -k \\ 0 & k \end{pmatrix}$ ((c) Total: 1)	**B1**
$\lvert k \rvert = 3k^2$	**M1**
$k \neq 0 \Rightarrow k = \pm\frac{1}{3}$ ((d) Total: 2)	**A1**
	Total: 10

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Q22

9231/12 • October/November 2024 • Question 4

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Mark scheme.

$\begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}\begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ -1 & -1 \\ -2 & 0 \end{pmatrix}$	**M1A1**
Or $\begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}\begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} -1 & -3 & -4 \\ 12 & 9 & 21 \end{pmatrix}\begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix}$	
$= \begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}$ ((a) Total: 3)	**B1**
$\begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x - 7y \\ -9x + 3y \end{pmatrix}$	**B1**
$-9x + 3mx = m(3x - 7mx)$	**M1A1**
$-9 + 3m = 3m - 7m^2 \Rightarrow 7m^2 = 9$	**A1**
$y = \frac{3}{\sqrt{7}}x$ and $y = -\frac{3}{\sqrt{7}}x$ ((b) Total: 5)	**A1**
Stretch	**B1**
parallel to the x-axis, scale factor 3. ((c) Total: 2)	**B1**
$\mathbf{M}^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$	**B1**
$\mathbf{N} = \frac{1}{3}\begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ or $\mathbf{N} = \begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{pmatrix}$	**M1**
$= \begin{pmatrix} 1 & -7 \\ -3 & 3 \end{pmatrix}$ ((d) Total: 3)	**A1**
	Total: 13

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Q23

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Mark scheme.

A rotation, followed by a shear.	**B2**
Award B1 if given in the wrong order. [anticlockwise, centred at the origin, through an angle θ], [x-axis fixed, with $(0,1)$ mapped to $(2,1)$.]	
	(2)
$\mathbf{M} = \begin{pmatrix} \cos\theta + 2\sin\theta & 2\cos\theta - \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$	**B1**
$\begin{pmatrix} \cos\theta + 2\sin\theta & 2\cos\theta - \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x\cos\theta + 2x\sin\theta - y\sin\theta + 2y\cos\theta \\ x\sin\theta + y\cos\theta \end{pmatrix}$	**B1**
Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$.	
$x\sin\theta + y\cos\theta = y$ AND $x\cos\theta + 2x\sin\theta - y\sin\theta + 2y\cos\theta = x$	**M1***
Uses $X = x$ and $Y = y$.	
$\Rightarrow x\cos\theta + 2x\sin\theta + \dfrac{x\sin\theta}{1-\cos\theta}(2\cos\theta - \sin\theta) = x$	**A1**
Combines equations correctly to eliminate y or x.	
$(\cos\theta + 2\sin\theta)(1-\cos\theta) + 2\sin\theta\cos\theta - \sin^2\theta = 1 - \cos\theta$	**DM1A1**
$\cos\theta - \cos^2\theta + 2\sin\theta - \sin^2\theta = 1 - \cos\theta \Rightarrow \sin\theta + \cos\theta = 1$	
Simplifies.	
$\theta = \frac{1}{2}\pi$	**A1**
	(7)
	Total: 9

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Q24

9231/13 • May/June 2025 • Question 1

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Mark scheme.

$\begin{pmatrix} 14 & 0 \\ 0 & 1 \end{pmatrix}$	**B1**
Stretch parallel to the x-axis, scale factor k $(k \neq 0)$.	
$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	**B1**
Rotation anticlockwise about the origin through angle $\frac{1}{3}\pi$.	
$\mathbf{M} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 14 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -\frac{\sqrt{3}}{2} \\ 7\sqrt{3} & \frac{1}{2} \end{pmatrix}$	**M1**
Correct order.	
$2\mathbf{M} = \begin{pmatrix} 14 & -\sqrt{3} \\ 14\sqrt{3} & 1 \end{pmatrix}$.	**A1**
AG	
	(4)
$\begin{pmatrix} 7 & -\frac{\sqrt{3}}{2} \\ 7\sqrt{3} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7x - \frac{\sqrt{3}}{2}y \\ 7\sqrt{3}x + \frac{1}{2}y \end{pmatrix}$	**B1**
Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$	
$7\sqrt{3}x + \frac{1}{2}mx = m\left(7x - \frac{\sqrt{3}}{2}mx\right)$	**M1**
Uses $y = mx$ and $Y = mX$.	
$7\sqrt{3} + \frac{1}{2}m = 7m - \dfrac{\sqrt{3}}{2}m^2$	
$\sqrt{3}m^2 - 13m + 14\sqrt{3} = 0$	**A1**
Correct quadratic AEF	

$m = 2\sqrt{3} \quad m = \dfrac{7}{\sqrt{3}}$	**A1**
Correct solutions to the quadratic	
$y = 2\sqrt{3}x \quad y = \dfrac{7}{\sqrt{3}}x$	**A1**
Correct invariant lines. If M0 then SCB1 for two correct lines.	
	(5)
$\mathbf{M}^{-1} = \dfrac{1}{14}\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -7\sqrt{3} & 7 \end{pmatrix} = \dfrac{1}{2}\begin{pmatrix} \frac{1}{14} & \frac{\sqrt{3}}{14} \\ -\sqrt{3} & 1 \end{pmatrix}$	**M1A1**
SCB1 for finding the inverse of $2\mathbf{M}$	
	(2)
	Total: 11

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Q25

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Mark scheme.

Shear, [x-axis fixed]	**B1**
With $(0,1)$ mapped to $\left(\frac{3}{2},1\right)$.	**B1**
Or other suitable example/description.	
	(2)
Shear, [y-axis fixed],	**B1**
with $(1,0)$ mapped to $\left(1,\frac{3}{2}\right)$.	**B1**
Or other suitable example/description.	
	(2)
$\mathbf{AB}=\begin{pmatrix}\frac{13}{4} & \frac{3}{2}\\ \frac{3}{2} & 1\end{pmatrix}$ or $\det\mathbf{AB}=\det\mathbf{A}\det\mathbf{B}$	**M1**
Finds **AB** or uses product of determinants. Or suitable reasoning that area does not change.	
$\det\mathbf{AB}=1$	**A1**
	(2)
$\begin{pmatrix}\frac{13}{4} & \frac{3}{2}\\ \frac{3}{2} & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}\frac{13}{4}x+\frac{3}{2}y\\ \frac{3}{2}x+y\end{pmatrix}$	**B1**
Transforms $\begin{pmatrix}x\\ y\end{pmatrix}$ to $\begin{pmatrix}X\\ Y\end{pmatrix}$.	
$\frac{3}{2}x+mx=m\left(\frac{13}{4}x+\frac{3}{2}mx\right)$	**M1A1**
Uses $Y=mX$.	
$\frac{3}{2}+m=\frac{13}{4}m+\frac{3}{2}m^2\Rightarrow m^2+\frac{3}{2}m-1=0$	**A1**
$y=-2x$ and $y=\frac{1}{2}x$.	**A1**

If only values of m are given, A1 A0.

(5)

Total: 11

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Q26

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Mark scheme.

Stretch	**B1**
parallel to the x-axis, scale factor k.	**B1**
Allow in the x direction, x axis	
	(2)
Enlargement,	**B1**
centre the origin, scale factor k.	**B1**
	(2)
$\begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} k & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & m \\ -k & m \\ k & m \end{pmatrix}\begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$	**M1A1**
Multiplies two matrices correctly. Allow one error for the M mark.	
$\begin{pmatrix} m & m & 2m \\ -2k+m & k+m & -k+2m \\ 2k+m & -k+m & k+2m \end{pmatrix}$	**A1**
$m\begin{vmatrix} k+m & -k+2m \\ -k+m & k+2m \end{vmatrix} - m\begin{vmatrix} -2k+m & -k+2m \\ 2k+m & k+2m \end{vmatrix} + 2m\begin{vmatrix} -2k+m & k+m \\ 2k+m & -k+m \end{vmatrix}$	**M1A1**
Finds determinant. Allow two errors for the M mark.	
$= m(k+m)(k+2m) - m(-k+m)(-k+2m) - m(-2k+m)(k+2m) + m(2k+m)(-k+2m) + 2m(-2k+m)(-k+m) - 2m(2k+m)(k+m)$	**A1**
$= 6m^2k + 6m^2k - 12m^2k = 0$	
AG. There must be evidence to show that the determinant is equal to zero.	
	(6)

Total: 10

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Q27

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Mark scheme.

$\cos\left(\frac{1}{4}\pi - \frac{1}{6}\pi\right) = \cos\left(\frac{1}{4}\pi\right)\cos\left(\frac{1}{6}\pi\right) + \sin\left(\frac{1}{4}\pi\right)\sin\left(\frac{1}{6}\pi\right) = \frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$ (AG)	**B1**
	(1)
$\sin\left(\frac{1}{4}\pi - \frac{1}{6}\pi\right) = \sin\left(\frac{1}{4}\pi\right)\cos\left(\frac{1}{6}\pi\right) - \cos\left(\frac{1}{4}\pi\right)\sin\left(\frac{1}{6}\pi\right) = \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$ (AG)	**B1**
	(1)
Reflection and rotation, correct order (reflection then rotation).	**B1B1**
Reflection in the x-axis.	**B1**
A rotation, $\frac{1}{12}\pi$ (anticlockwise) about the origin.	**B1**
	(4)
$\begin{pmatrix} \frac{1}{4}(\sqrt{6}+\sqrt{2}) & \frac{1}{4}(\sqrt{2}-\sqrt{6}) \\ \frac{1}{4}(\sqrt{6}-\sqrt{2}) & \frac{1}{4}(\sqrt{6}+\sqrt{2}) \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{4}(\sqrt{6}+\sqrt{2}) & \frac{1}{4}(\sqrt{6}-\sqrt{2}) \\ \frac{1}{4}(\sqrt{2}-\sqrt{6}) & \frac{1}{4}(\sqrt{6}+\sqrt{2}) \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	**B1**
Both correct inverses required.	
$\mathbf{M}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{4}(\sqrt{6}+\sqrt{2}) & \frac{1}{4}(\sqrt{6}-\sqrt{2}) \\ \frac{1}{4}(\sqrt{2}-\sqrt{6}) & \frac{1}{4}(\sqrt{6}+\sqrt{2}) \end{pmatrix}$	**B1**
Correct order of their inverses.	
	(2)
$\mathbf{M} = \begin{pmatrix} \cos\left(\frac{\pi}{12}\right) & \sin\left(\frac{\pi}{12}\right) \\ \sin\left(\frac{\pi}{12}\right) & -\cos\left(\frac{\pi}{12}\right) \end{pmatrix}$	**B1**
$\begin{pmatrix} \cos\left(\frac{\pi}{12}\right) & \sin\left(\frac{\pi}{12}\right) \\ \sin\left(\frac{\pi}{12}\right) & -\cos\left(\frac{\pi}{12}\right) \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\pi}{12}\right)x + \sin\left(\frac{\pi}{12}\right)y \\ \sin\left(\frac{\pi}{12}\right)x - \cos\left(\frac{\pi}{12}\right)y \end{pmatrix}$	**B1**

FT their $\mathbf{M}$. Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$.	
$\sin\left(\frac{\pi}{12}\right)x - m\cos\left(\frac{\pi}{12}\right)x = m\left(\cos\left(\frac{\pi}{12}\right)x + \sin\left(\frac{\pi}{12}\right)mx\right)$	**M1**
Uses $y = mx$ and $Y = mX$.	
$\sin\left(\frac{\pi}{12}\right)m^2 + 2m\cos\left(\frac{\pi}{12}\right) - \sin\left(\frac{\pi}{12}\right) = 0$ (AG)	**A1**
$m = \dfrac{-2\cos\left(\frac{\pi}{12}\right) \pm \sqrt{4\cos^2\left(\frac{\pi}{12}\right) + 4\sin^2\left(\frac{\pi}{12}\right)}}{2\sin\left(\frac{\pi}{12}\right)}$	**M1**
Applies quadratic formula and $\cos^2\left(\frac{\pi}{12}\right) + \sin^2\left(\frac{\pi}{12}\right) = 1$. OE.	
$m = -\cot\left(\frac{1}{12}\pi\right) \pm \operatorname{cosec}\left(\frac{1}{12}\pi\right)$ (i.e. $a = -1$, $b = \pm 1$)	**A1**
	(6)
	Total: 14

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Written by Ali Hashir.

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